SOLUTION- SAMPLE PAPER- XI

PART -A

SECTION -I (One Mark Questions)

1. . All possible ordered pairs are (1, 12), (12, 1), (2, 6), (6, 2) (3, 4) and (4, 3). Hence, possible orders are $1\times12, 12\times1, 2\times6, 6\times2, 3\times4, 4\times3$. Possible orders of 7 elements are$1\times7, 7\times1$.

2. $\int_0^{\frac{\pi}{2}} \sin^2 x dx = \int_0^{\frac{\pi}{2}} \frac{1-\cos 2x}{2} dx = \frac{1}{2}\int_0^{\frac{\pi}{2}} 1-\cos 2x dx = \frac{x}{2} + \left.\frac{\sin 2x}{4}\right|_0^{\frac{\pi}{2}} = \frac{\pi}{4} + \frac{\sin 2\frac{\pi}{2}}{4} = \frac{\pi}{4} + \frac{\sin\pi}{4} - (0+0) = \frac{\pi}{4}$

3. Given: $A = \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$ and B is a square matrix of order 2 such that $AB = I$

   This implies, $B = A^{-1}$ Therefore, $A = \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$ $\det A = (3)(2) - (-4)(-1) = 6-4 = 2$ $B = A^{-1} = \frac{1}{\det A}\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ \frac{1}{2} & \frac{3}{2} \end{bmatrix}$

4. $P(AUBUC) = P(A)+P(B)+P(C)-P(AB)-P(BC)-P(CA)+P(ABC)$
   $= (1/2)+(1/3)+(1/6)-((1/2)*(1/3))-((1/3)*(1/6))-((1/6)*(1/2))+((1/2)*(1/3)*(1/6))$
   $= (1/2)+(1/3)+(1/6)-(1/6)-(1/18)-(1/12)+(1/36) = (5/6)-(5/36)+(1/36) = 26/36 = 13/18$

5. Unit vector of $\vec{a} = 2\hat{i}-3\hat{j}+6\hat{k}$ $\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{2\hat{i}-3\hat{j}+6\hat{k}}{\sqrt{2^2+(-3)^2+6^2}} = \frac{2\hat{i}-3\hat{j}+6\hat{k}}{\sqrt{49}} = \frac{2\hat{i}-3\hat{j}+6\hat{k}}{7}$

6. $f(x) = \left\{ \begin{array}{c} \frac{\sin 5x}{3x}, \text{if} x \neq 0 \\ k, \text{if} x = 0 \end{array} \right\}$

   Since $f(x)$ is continuous. Left hand limit is equal to right hand limit. $\lim_{x\to0} f(x) = \lim_{x\to0}\frac{\sin 5x}{3x} = \lim_{x\to0}\left(\frac{\sin 5x}{5x}\right)\times\frac{5x}{3x} = \frac{5}{3}\times1 = \frac{5}{3}$

7. Let A: Event that both children are boys, and B: Event that at least one of them is a boy $A: \{BB\}$ and $B: \{BG, GB, BB\} \to P(A\cap B) = \{BB\}$

   Sample space: $S = \{BB, BG, GB, GG$ where B= boy and G= girl. $P(A) = \frac{1}{4}$ $P(B) = \frac{3}{4}$ $P(A\cap B) = \frac{1}{4}$

   Probability that both are boys, if it is known that one of them is a boy is $P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$

8. The possible outcomes are as follows: $5H, 5T, (H,4T), (T,4H), (2H,3T)(3H,2T)$i.e., Total number of outcomes is 6 In only three outcomes out of the six outcomes; tail appears an odd number of times, that is, Number of favorable outcomes are 3. Therefore, the probability that tail appears an odd number of times $= \frac{\text{Numberoffavourableoutcomes}}{\text{Totalnumberofoutcomes}} = \frac{3}{6} = \frac{1}{2}$

9. Given planes $2x-y+2z-3=0$.(1) and $6x-2y+3z-5=0$..(2) Direction ratios of plane (1) is $<2,-1,2>$ that is, $\vec{r_1} = 2\hat{i}-\hat{j}+2\hat{k}$

   Direction ratio of plane 2 is $<6,-2,3>$ that is, $\vec{r}_2 = 6\hat{i}-2\hat{j}-5\hat{k}$ $-\cos\theta = \frac{\overline{r_1}\cdot\overline{r_2}}{r_1|\cdot|\overline{r_2}|} = \frac{(2\hat{i}-\hat{j}+2\hat{k})\cdot(6\hat{i}-2\hat{j}+3\hat{k})}{\sqrt{2^2+(-1)^2+2^2}\cdot\sqrt{6^2+(-2)^2+(3)^2}} = \frac{12+2+6}{3\cdot7}$

   $= \frac{20}{21} \Rightarrow \cos\theta = \frac{20}{21}$

10. $\int_2^3 3^x dx = \left[\frac{3^x}{\log 3}\right]_2^3 = \frac{1}{\log 3}\left[3^3-3^2\right] = \frac{18}{\log 3}$

11. The direction ratios of $\vec{r}$ are 6, 2,- 3 and, $\sqrt{6^2+2^2+(-3)^2} = \sqrt{36+4+9} = \sqrt{49} = 7$

    Hence, the direction cosines of $\vec{r}$ are $\frac{6}{7}, \frac{2}{7}, \frac{-3}{7}$

12. Order : 2 Degree : 1

13. . $\cot\left(\sec^{-1}x+\sin^{-1}\frac{1}{x}\right)=\cot(\sec^{-1}x+\text{cosec}^{-1}x)=\cot\frac{\pi}{2}=0.$

14. When a die is rolled, the sample space is given by $S=\{1,2,3,4,5,6\}$ Let A = event of getting a prime number and B = event of getting an odd number

Then, $A=\{2,3,5\}, A=\{2,3,5\}, B=\{1,3,5\}$ and $A\cap B=\{3,5\}$ $P(A)=\frac{P(A)}{P(S)}=\frac{3}{6}=\frac{1}{2}$ $P(B)=\frac{P(B)}{P(S)}=\frac{3}{6}=\frac{1}{2}$

Conditional probability of event A given that event B has already occurred, P(A|B) $\text{P(A|B)}=\frac{\text{P}(A\cap B)}{\text{P(B)}}=\frac{\frac{2}{6}}{\frac{3}{6}}=\frac{2}{3}$ Hence, the probability of prime is $\frac{2}{3}$

15. We have, $R(x)=13x^2+26x+15$.(i) Differentiating (i) wr.t. x, we have.

Marginal revenue $=\frac{dR}{dx}=13\times 2x+26=26x+26$

$\therefore \left(\frac{dR}{dx}\right)_{x=7}=26\times 7+26=208\Rightarrow$

Marginal revenue (when $x=7$)= $Rs208$.

16. We have, $f(x)=3x+17$ (i) $f(x)$ being a polynomial function, is continuous and derivable on R. Differentiating (i), w.r.t. x, we get $f(x)=3>0\forall x\in R$

$\Rightarrow f$ is strictly increasing on R.

SECTION -II (CASE STUDY)

CASE STUDY -I

17. (a) Probability of Varun hitting the target = $\frac{4}{5}$

Probability of Abhay hitting the target = $\frac{3}{4}$,

Probability of Rohit to hit the target = $\frac{2}{3}$

If all may hit the target , then required probabilty = $\frac{4}{5}\times\frac{3}{4}\times\frac{2}{3}$

$=\frac{2}{5}$

(b) Required probability = $(1-\frac{4}{5})\times\frac{3}{4}\times\frac{2}{3}=\frac{1}{10}$

(c) Required probability = $\frac{4}{3}\times\frac{3}{4}\times\frac{1}{3}+\frac{1}{5}\times\frac{3}{4}\times\frac{2}{3}+\frac{4}{5}\times\frac{1}{4}\times\frac{2}{3}=\frac{13}{30}$

(d) For none of them will hit the target , we have to the find when they will not hit the target individually and then multiply each = $\frac{1}{5}\times\frac{1}{4}\times\frac{1}{3}=\frac{1}{60}$

(e) Total probability can't be more than one. Hence false

CASE STUDY : II

18. (a) where x is the radius of the base of the cylinder.

Let VAB is the given cone with height h, semi vertical angle α

$tan\alpha=\frac{x}{VO'}$

$\Rightarrow OO'=VO-VO'=h-xcot\alpha$ ..(i)

Let V be the volume of cyliner. Then, $V=\pi(O'B')^2(OO')$

$\Rightarrow V=\pi x^2(h-xcot\alpha)..(ii)$

(b) $\Rightarrow\frac{dV}{dx}=2\pi xh-3\pi x^2cot\alpha$

(c) $\Rightarrow\frac{dV}{dx}=2\pi xh-3\pi x^2cot\alpha$

$\frac{d^2V}{dx^2}=2\pi h-6\pi xcot\alpha$

(d) $\Rightarrow \frac{dV}{dx} = 2\pi xh - 3\pi x^2 cot\alpha = 0$

$\Rightarrow x = \frac{2h}{3} tan\alpha$

(e) By putting x = $\frac{2h}{3} tan\alpha$ in (ii) we get :

$V = \frac{4}{27}\pi h^3 tan^2\alpha$

Putting x = $\frac{2h}{3} tan\alpha$ in (i) we get

$OO' = h - xcot\alpha = h - \frac{2h}{3} = \frac{h}{3}$

PART -B

SECTION-III(Each question carries 2 Marks)

19. $\vec{d}_1 = (2\hat{\imath} - \hat{\jmath} + \hat{k})$ $\vec{d}_2 = (3\hat{\imath} + 4\hat{\jmath} - \hat{k})$ Then, vector area of the parallelogram is $\frac{1}{2}\left|\vec{d}_1 \times \vec{d}_2\right|$ $\vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -1 & 1 \\ 3 & 4 & -1 \end{vmatrix} = (1-4)\hat{\imath} - (-2-3)\hat{\jmath} + (8+3)\hat{k} = -3\hat{\imath} + 5\hat{\jmath} + 11\hat{k}$ Required area $= \frac{1}{2}\left|\vec{d}_1 \times \vec{d}_2\right| = \sqrt{(\text{co}-\text{efficientof}\hat{\imath})^2 + (\text{co}-\text{efficientof}\hat{\jmath})^2 + (\text{co}-\text{efficientof}\hat{k})^2}$
$= \frac{1}{2}\sqrt{(-3)^2 + 5^2 + (11)^2} = \frac{1}{2}\sqrt{155}$ sq. units

20. $(x+2)\frac{dy}{1} = x^2 + 5x - 3 (x \neq -2)$

Given that, $(x+2)\frac{dy}{dx} = x^2 + 5x - 3 dy = \left(\frac{x^2+5x-3}{(x+2)}\right) dx \int dy = \int \left(\frac{x^2+5x-3}{(x+2)}\right) dx$

It is of the form $\frac{f(x)}{g(x)} =$ degree $f(x) >$ degree $g(x)$

Therefore, divide $f(x)$ by$g(x)$,

$y = \int \left(x + 3 - \frac{9}{(x+2)}\right) dx + C$ $y = \frac{x^2}{2} + 3x - 9\log|x+2| + C$ Hence, $y = \frac{x^2}{2} + 3x - 9\log|x+2| + \text{C}$ is the required solution

21. Let A = event that 4 appears at least once B = event that the sum of the numbers appearing is 6 S = sample space when 2 dies are thrown twicely. Then, $n(\text{S}) = 2^2 = 36$ $A = \{(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),(1,4),(3,4),(3,4),(5,4),(6,4)\}$

And $\text{B} = \{(1,5),(2,4),(3,3),(4,2),(5,1)\}$ $A \cap B = \{(2,4),(4,2)\}$ So, $P(A) = \frac{n(A)}{n(S)} = \frac{11}{36}$ $P(B) = \frac{n(B)}{n(S)} = \frac{5}{36}$ $\text{P}(\text{A}\cap\text{B}) = \frac{n(\text{A}\cap\text{B})}{n(\text{S})} = \frac{2}{36}$

Conditional probability of event A given that event B has already occurred,

$P(A|B)$ Required probability $= \text{P}(\text{A}|\text{B}) = \frac{\text{P}(\text{A}\cap\text{B})}{\text{P}(\text{B})} = \frac{\frac{2}{3}}{\frac{5}{36}} = \frac{2}{5}$

22. Let, $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

Matrix AB $= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1-1 & -1+1 \\ 1-1 & -1+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Matrix BA $= \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1-1 & 1-1 \\ -1+1 & -1+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Thus, $A \neq 0, B \neq 0$ But AB = BA = O

23. $f(x) = \left(\sin^4 x + \cos^4 x\right)$ $f'(x) = 4\sin^3 x\cos x - 4\cos^3 x\sin x = -4\sin x\cos x\left(\cos^2 x - \sin^2 x\right) = -2\sin 2x\cos 2x = -\sin 4x$ $f''(x) = -4\cos 4x$

To find the local minima. put $f'(x) = 0$

Now, $f'(x) = 0 \Rightarrow -\sin x 4x = 0 \Rightarrow 4x = \pi \Rightarrow x = \frac{\pi}{4}$ $x = \frac{\pi}{4}$ is a point of local maximum or local minimum. $f''\left(\frac{\pi}{4}\right) = -4\cos\pi = 4 > 0$ $x = \frac{\pi}{4}$ is a point of local minimum.

Local minimum value $= f\left(\frac{\pi}{4}\right) = \frac{1}{2}$

24. $\int \frac{dx}{(1+5\sin^2 x)}$

$I=\int \frac{dx}{(1+5\sin^2 x)}$ $I=\int \frac{dx}{(1+5\sin^2 x)}=\int \frac{\left(\frac{1}{\cos^2 x}\right)}{\left(\frac{1}{\cos^2 x}+5\frac{\sin^2 x}{\cos^2 x}\right)}dx$ Divide numerator and denominator by $\cos^2 x=\int \frac{\sec^2 x}{\sec^2 x+5\tan^2 x}dx=$

$\int \frac{\sec^2 x}{\{(1+\tan^2 x)+5\tan^2 x\}}dx=\int \frac{\sec^2 x}{(1+6\tan^2 x)}dx$

Let $\tan x=t=\int \frac{dt}{(1+6t^2)}=\frac{1}{6}\int \frac{dt}{\left(\frac{1}{6}+t^2\right)}=\frac{1}{6}\int \frac{dt}{\left\{\left(\frac{1}{\sqrt{6}}\right)^2+t^2\right\}}$

$=\frac{1}{6}\frac{1}{\left(\frac{1}{\sqrt{6}}\right)}\tan^{-1}\frac{1}{\left(\frac{1}{\sqrt{6}}\right)}+C$

$=\frac{1}{\sqrt{6}}\tan^{-1}(\sqrt{6}t)+C=\frac{1}{\sqrt{6}}\tan^{-1}(\sqrt{6}\tan x)+C$

25. $\frac{dy}{dx}=e^{ax}\cdot\frac{d}{dx}|\cos(bx+c)\Big\}+\cos(bx+c)\cdot\frac{d}{dx}(e^{ax})$

$=e^{ax}\cdot\{-b\sin(bx+c)\}+\cos(bx+c)\cdot ae^{ax}=e^{ax}\cdot\{a\cos(bx+c)-b\sin(bx+c)\}$

26. Let, $\sin^{-1}\frac{3}{5}=A$ and $\sin^{-1}\frac{5}{13}=B$

Then, $A,B\in\left[\frac{-\pi}{2},\frac{\pi}{2}\right]\Rightarrow\cos A>0$ and $B>0\therefore\sin A=\frac{3}{5}$ and $B=\frac{5}{13}$ $\cos A=\sqrt{1-\sin^2 A}=\sqrt{1-\frac{9}{25}}=\sqrt{\frac{16}{25}}=\frac{4}{5}$ $\cos B=\sqrt{1-\sin^2 B}$

$=\sqrt{1-\frac{25}{169}}=\sqrt{\frac{144}{169}}=\frac{12}{13}$ $\cos\left(\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{5}{13}\right)=\cos(\mathrm{A}+\mathrm{B})$

$=\cos A\cos B-\sin A\sin B=\left(\frac{4}{5}\times\frac{12}{13}\right)-\left(\frac{3}{5}\times\frac{5}{13}\right)=\left(\frac{48}{65}-\frac{15}{65}\right)=\frac{33}{65}$

OR

$\tan^{-1}\left\{2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right\}=\tan^{-1}\left\{2\cos\left(2\times\frac{\pi}{6}\right)\right\}$

$=\tan^{-1}\left\{2\cos\frac{\pi}{3}\right\}=\tan^{-1}\left(2\times\frac{1}{2}\right)=\tan^{-1}1=\frac{\pi}{4}$

27. $f(x)=\tan^{-1}(\cos x+\sin x)$ $f'(x)=\frac{1}{1+(\cos x+\sin x)^2}\cdot\frac{d}{dx}(\cos x+\sin x)$

$=\frac{(-\sin x+\cos x)}{(1+\cos^2 x+\sin^2 x+2\sin x\cos x)}$ $f'(x)=\frac{(\cos x-\sin x)}{(2+\sin 2x)}$ Now, when $0<x<\frac{\pi}{4}$, we have $0<\tan x<\tan\frac{\pi}{4}$ $0<\frac{\sin x}{\cos x}<1$

$\cos x>\sin x$ and $\sin 2x>0$ $(\cos x-\sin x)>0$ and $(2+\sin 2x)>0$ Therefore, $f'(x)>0$ for all x when $0<x<\frac{\pi}{4}$ Hence, $f(x)$ is strictly increasing in $\left(0,\frac{\pi}{4}\right)$

28. Let, $\mathrm{I}=\int\frac{dx}{x(x^n+1)}$

Let, $x^n=t\Rightarrow nx^{n-1}dx=dt$ $I=\int\frac{dt}{nt(t+1)}=\frac{1}{n}\cdot\int\frac{dt}{t(t+1)}$

Let $\frac{1}{t(t+1)}=\frac{\mathrm{A}}{t}+\frac{\mathrm{B}}{(t+1)}$ Then, $1=A(t+1)+Bt$

Compare the constant term on both sides we get, $1=\mathrm{A}$, compare the coefficient of t on both sides we get, $0=\mathrm{A}+\mathrm{B}$ Put $A=1$, we get $B=-1$

$\therefore\frac{1}{t(t+1)}=\frac{1}{t}-\frac{1}{(t+1)}$ $I=\frac{1}{n}\left[\int\frac{1}{t}dt-\int\frac{1}{(t+1)}dt\right]=\frac{1}{n}[\log|t|-\log|t+1|]+C=\frac{1}{n}\log\left|\frac{t}{t+1}\right|+\mathrm{C}=\frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right|+C$

SECTION -IV (Three Mark Questions)

29. We have, $I=\int\frac{\cos 7x-\cos 8x}{1+2\cos 5x}dx \therefore I=\int\frac{2\sin\frac{15}{2}x\sin\frac{x}{2}}{1+2\cos 5x}dx$

Multiplying and dividing by $\sin\frac{5x}{2}$,we get $I=\int\frac{2\sin\frac{15x}{2}\sin\frac{x}{2}\sin\frac{5x}{2}}{\sin\frac{5x}{2}+2\sin\frac{5x}{2}\cos 5x}dx=\int\frac{2\sin\frac{15x}{2}\sin\frac{x}{2}\sin\frac{5x}{2}}{\sin\frac{5x}{2}+\sin\frac{5x}{2}-\sin\frac{5x}{2}}dx=\frac{2\sin\frac{15x}{2}\sin\frac{x}{2}\sin\frac{5x}{2}}{\sin\frac{15x}{2}}dx$

$=\int 2\sin\frac{x}{2}\sin\frac{5x}{2}dx=\int(\cos 2x-\cos 3x)\,dx=\frac{\sin 2x}{2}-\frac{\sin 3x}{3}+C$

30. . We have $x=\theta+\sin\theta$ and $y=1+\cos\theta \Rightarrow \frac{dx}{d\theta}=1+\cos\theta$ and $\frac{dy}{d\theta}=-\sin\theta$

$\therefore \frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{-\sin\theta}{1+\cos\theta}\left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{4}}=\frac{-\sin\frac{\pi}{4}}{1+\cos\frac{\pi}{4}}=\frac{\frac{-1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}=\frac{-1}{\sqrt{2}+1}=-(\sqrt{2}-1)$

And $x=\theta+\sin\theta=\frac{\pi}{4}+\sin\frac{\pi}{4}=\frac{\pi}{4}+\frac{1}{\sqrt{2}}\,y=1+\cos\theta=1+\frac{1}{\sqrt{2}}$

Equation of tangent line at $\theta=\frac{\pi}{4}$ is $y-\left(1+\frac{1}{\sqrt{2}}\right)=-(\sqrt{2}-1)\left(x-\left(\frac{\pi}{4}+\frac{1}{\sqrt{2}}\right)\right)$

$\Rightarrow y-1-\frac{1}{\sqrt{2}}=-(\sqrt{2}-1)x+(\sqrt{2}-1)\left(\frac{\pi}{4}+\frac{1}{\sqrt{2}}\right)\Rightarrow(\sqrt{2}-1)x+y=(\sqrt{2}-1)\left(\frac{\pi}{4}+\frac{1}{\sqrt{2}}\right)+1+\frac{1}{\sqrt{2}}$

$\Rightarrow(\sqrt{2}-1)x+y=\frac{\pi}{2\sqrt{2}}-\frac{\pi}{4}+1-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}+1\Rightarrow(\sqrt{2}-1)x+y=\frac{\pi}{2\sqrt{2}}-\frac{\pi}{4}+2$

31. Let $I=\int_{-\infty}^{\infty}xe^{-x^2}dx$, Consider $f(x)=xe^{-x^2}\Rightarrow f(-x)=-xe^{-(-x)^2}=-xe^{-x^2}=-f(x)$

So, $f(x)$ is an odd function $\therefore \int_{-\infty}^{\infty}xe^{-x^2}dx=0$

32. Take LHS. LHS $=2\left(\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}\right)=2\tan^{-1}\frac{1}{4}+2\tan^{-1}\frac{2}{9}$

Apply the formula: $2\tan^{-1}A=\tan^{-1}\left(\frac{2A}{1-A^2}\right)$ LHS $=\tan^{-1}\frac{\left(2\times\frac{1}{4}\right)}{\left\{1-\left(\frac{1}{4}\right)^2\right\}}+\tan^{-1}\frac{\left(2\times\frac{2}{9}\right)}{\left\{1-\left(\frac{2}{9}\right)^2\right\}}$

$=\tan^{-1}\frac{\left(\frac{1}{2}\right)}{\left(\frac{15}{16}\right)}+\tan^{-1}\frac{\left(\frac{4}{9}\right)}{\left(\frac{77}{81}\right)}=\tan^{-1}\left(\frac{1}{2}\times\frac{16}{15}\right)+\tan^{-1}\left(\frac{4}{9}\times\frac{81}{77}\right)=\tan^{-1}\frac{8}{15}+\tan^{-1}\frac{36}{77}$

Apply the formula, $\tan^{-1}A+B=\tan^{-1}\left(\frac{A+B}{1-AB}\right)$ $\tan^{-1}\frac{8}{15}+\tan^{-1}\frac{36}{77}=\tan^{-1}\frac{\left(\frac{8}{15}+\frac{36}{77}\right)}{\left(1-\frac{8}{15}\times\frac{36}{77}\right)}=\tan^{-1}\left(\frac{1156}{867}\right)=\tan^{-1}\frac{4}{3}=$ RHS

33. Given a line $\vec{r}=(\hat{i}+2\hat{j}-\hat{k})+\lambda(2\hat{i}+\hat{j}+2\hat{k})(1)$ is parallel to the plane $\vec{r}\cdot(3\hat{i}-2\hat{j}+m\hat{k})=12.(2)$
After comparing (1) with standard equation of line $\vec{r}=\vec{a}+\lambda\vec{b}$ and comparing (2) with standard vector form of plane $\vec{r}\cdot\vec{n}=q$ we get, $\vec{b}=(2\hat{i}+\hat{j}+2\hat{k})\ \vec{n}=3\hat{i}-2\hat{j}+m\hat{k}\ \vec{a}=\hat{i}+2\hat{j}-\hat{k}\ q=12$
Now, line (1) is parallel to the plane (2) this implies, The unit vector of (2) is perpendicular to $\vec{b}$ of line (2)
Therefore, $\vec{b}\cdot\vec{n}=0\ (2\hat{i}+\hat{j}+2\hat{k})\cdot(3\hat{i}-2\hat{j}+m\hat{k})=0\ (2\times3)+1\times(-2)+2\times m=0\ 2m=-4\ m=-2$ The value of m is-2.

34. Let the three student be named$A$, B and C respectively. Let, E_1, E_2, E_3 be the events that the problem is solved byA, B, Crespectively.

Then, $P(E_1)=\frac{1}{3}$, $P(E_2)=\frac{2}{7}$, $P(E_3)=\frac{3}{8}\,P(\text{not}E_1)=\left(1-\frac{1}{3}\right)=\frac{2}{3}\,P(\text{not}E_2)=\left(1-\frac{2}{7}\right)=\frac{5}{7}\,P(\text{not}E_3)=\left(1-\frac{3}{8}\right)=\frac{5}{8}$

$=P[(\text{not } E_1) \text{ and } (\text{not } E_2) \text{ and } (\text{not } E_3)] = \mathrm{P}\left(\overline{\mathrm{E}}_1 \cap \overline{\mathrm{E}}_2 \cap \overline{\mathrm{E}}_3\right)$

$= \mathrm{P}\left(\overline{\mathrm{E}}_1\right) \times \mathrm{P}\left(\overline{\mathrm{E}}_2\right) \times \mathrm{P}\left(\overline{\mathrm{E}}_3\right)$

$= \left(\frac{2}{3} \times \frac{5}{7} \times \frac{5}{8}\right)$

$= \frac{25}{84}$ P(thattheproblemsolved)

$= 1 - \mathrm{P}(\text{ none solve the problem}) = \left(1 - \frac{25}{84}\right) = \frac{59}{84}$

Hence, the required probability is $\frac{59}{84}$

35. $(\sqrt{a+x})\frac{dy}{dx} + x = 0$

Given that, $(\sqrt{a+x})\frac{dy}{dx} + x = 0$ $\frac{dy}{dx} = \frac{-x}{\sqrt{a+x}}$ $dy = \frac{-x}{\sqrt{a+x}}dx$

$\int dy = \int \frac{-x}{\sqrt{a+x}}dx$ $y = -\int \frac{[(a+x)-a]}{\sqrt{a+x}}dx$ $y = -\int \left(\sqrt{a+x} - \frac{a}{\sqrt{a+x}}\right)dx$

$y = -\int \sqrt{a+x}dx + a\int (a+x)^{-\frac{1}{2}}dx$ $y = -\frac{2}{3}(a+x)^{\frac{3}{2}} + 2a\sqrt{a+x} + \mathrm{C}$

OR

Given that, $(1+x)\left(1+y^2\right)dx + (1+y)\left(1+x^2\right)dy = 0$ $\frac{(1+x)}{(1+x^2)}dx + \frac{(1+y)}{(1+y^2)}dy = 0$

$\int \frac{(1+x)}{(1+x^2)}dx + \int \frac{(1+y)}{(1+y^2)}dy = 0$ $\int \left(\frac{1}{(1+x^2)} + \frac{x}{(1+x^2)}\right)dx + \int \left(\frac{1}{(1+y^2)} + \frac{y}{(1+y^2)}\right)dy = 0$

$\int \frac{1}{(1+x^2)}dx + \frac{1}{2}\int \frac{2x}{(1+x^2)}dx + \int \frac{1}{(1+y^2)}dy + \frac{1}{2}\int \frac{2y}{(1+y^2)}dy = 0$

Let $\mathrm{I}_1 = \frac{1}{2}\int \frac{2x}{(1+x^2)}$ and let $\mathrm{I}_2 = \frac{1}{2}\int \frac{2y}{(1+y^2)}dy$ $\int \frac{1}{(1+x^2)}dx + I_1 + \int \frac{1}{(1+y^2)}dy + I_2 = 0$ $I_1 = \frac{1}{2}\int \frac{2x}{(1+x^2)}$

Put $x^2 = t$ $2x = dt = \frac{1}{2}\int \frac{dt}{(1+t)} = \frac{1}{2}\log|1+t| + c$

$= \frac{1}{2}\log\left|1+x^2\right| + c_1$ Similarly, $\mathrm{I}_2 = \frac{1}{2}\int \frac{2y}{(1+y^2)}dy = \frac{1}{2}\log\left|1+y^2\right| + c_2$

Put the values of I_1 and I_2 in (1), we get, $\int \frac{1}{(1+x^2)}dx + \frac{1}{2}\log\left(1+x^2\right) + c_1 + \int \frac{1}{(1+y^2)}dy + \frac{1}{2}\log\left(1+y^2\right) + c_2 = C$

Let $c_1 + c_2 = c'$ and solve it. $\tan^{-1}x + \frac{1}{2}\log\left(1+x^2\right) + \tan^{-1}y + \frac{1}{2}\log\left(1+y^2\right) + c' + c'' = C$

$\tan^{-1}x + \frac{1}{2}\log\left(1+x^2\right) + \tan^{-1}y + \frac{1}{2}\log\left(1+y^2\right) = C - c' - c'' = C''$ (lct) $\tan^{-1}x + \tan^{-1}y + \frac{1}{2}\left[\log\left(1+x^2\right) + \log\left(1+y^2\right)\right] = C''$

$\tan^{-1}x + \frac{1}{2}\log\left(1+x^2\right) + \tan^{-1}y + \frac{1}{2}\log\left(1+y^2\right) = C$

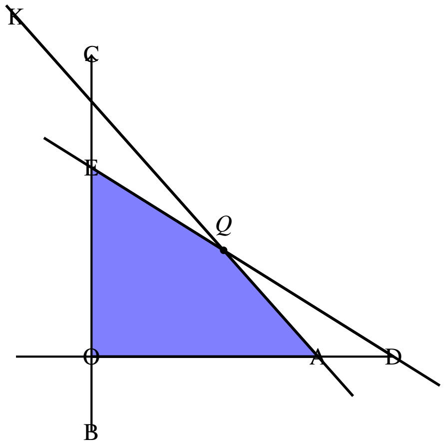
$\tan^{-1}x + \tan^{-1}y + \frac{1}{2}\left[\log\left(1+x^2\right) + \log\left(1+y^2\right)\right] = C$

SECTION -V (Five Mark Questions)

36. Suppose x be the number of pieces of Model A and y be the number of pieces of Model B. Then, Total profit (in Rs.) $= 8000x + 12000y$ Let $Z = 8000x + 12000y$

We now have the following mathematical model for the given problem. Maximize $Z = 8000x + 12000y$ ...(i) subject to the constraints : $9x + 12y \leq 180$ (Fabricating constraint) i.e. $3x + 4y \leq 60$ ...(ii) $x + 3y \leq 30$ (Finishing constraint) ...(iii) $x \geq 0, y \geq 0$ Non negative constraint) (iv)

The feasible region (shaded) OAQE determined by the linear inequalities (ii) to (iv) is shown in the figure. Note that the feasible region is bounded.



Let us evaluate the objective function Z at each corner point as shown below :

Corner Point	$Z = 8000x + 12000y$
$O(0,0)$	0
$A(20,0)$	160000
$Q(12,6)$	168000 $\rightarrow$ Maximum
$E(0,10)$	120000

We find that maximum value of Z is 1,68,000 at 5(12, 6). Hence, the company should produce 12 pieces of Model A and 6 pieces of Model 6 to realise maximum profit and maximum profit then will be Rs. 1,68,000.

37. Let x and y be the lengths of two sides of rectangle of given area A and P the perimeter, then we have, $A = xy \Rightarrow\ y = \frac{A}{x}$

Now, $P = 2(x+y) = 2\left(x + \frac{A}{x}\right) \Rightarrow \frac{dP}{dx} = 2\left(1 - \frac{A}{x^2}\right)$

For maxima or minima, $\frac{dP}{dx} = 0 \Rightarrow 2\left(1 - \frac{A}{x^2}\right) = 0 \Rightarrow A = x^2$

Also, $\frac{d^2P}{dx^2} = \frac{4A}{x^3} > 0$

$\therefore$ Perimeter of rectangle is minimum, when $x = \sqrt{A}$

and So, perimeter of rectangle is minimum, when$y = x$ i.e., Rectangle is square.

38. Any plane parallel to the plane $\vec{r}\cdot(2\hat{i} - \hat{j} + 2\hat{k}) = 5$ is given by $(x\hat{i} + y\hat{j} + z\hat{k})\cdot(2\hat{i} - \hat{j} + 2\hat{k}) = q 2x - y + 2z = q\ 2x - y + 2z - q = 0$........(1) for some constant q since (1)

passes through a point having position vector $(\hat{i} + \hat{j} + \hat{k})$, that is point (1,1,1).

Put 1 for x, 1 for y and 1 for z in (1), we get, $2(1) - (1) + 2(1) - q = 0\ 2 - 1 + 2 - q = 0\ 3 = q$

Substitute $q = 3$ in the equation (1) we get, $2x - y + 2z - 3 = 0$ or, $(x\hat{i} + y\hat{j} + z\hat{k})\cdot(2\hat{i} - \hat{j} + 2\hat{k}) = 3\ \vec{r}\cdot(2\hat{i} - \hat{j} + 2\hat{k}) = 3$

OR

Given equation of parallel planes are $2x - 2y + z + 3 = 0$......(1) and $2x - 2y + z + 9 = 0$...........(2)

Let the required equation of the plane mid parallel to plane (1) and (2) be $y + z + k = 0$............(3)

Plane (3) equidistant from each of the given planes. Let, $\mathrm{P}(x_1, y_1, z_1)$ be any point on the plane (3) i.e. $2x - 2y + z + k = 0$

This implies, the point satisfy plane (3). Substitute x_1 for x, y_1 for y and z_1 for z we get, $2x_1 - 2y_1 + z_1 + k = 0 \Rightarrow 2x_1 - 2y_1 + z_1 = -k$.........(4)

Let d_1 and d_2 be the shortest distance of point $\mathrm{P}(x_1, y_1, z_1)$ to plane (1) and (2) Also, $\mathrm{P}(x_1, y_1, z_1)$ is equidistant to the planes (1) and (2) Therefore, distance of point $\mathrm{P}(x_1, y_1, z_1)$ to plane (1) and (2) are same.

This implies, $d_1 = d_2$ ..(5) Now apply the formula i.e. Shortest distance, d of point $\mathrm{A}(x_y, y_1 z_1)$ to plane $ax + by + cz + q = 0$ is $d = \frac{|ax_1 + by_1 + cz_1 + q|}{\sqrt{a^2 + b^2 + c^2}}$.(6)

From (5) and (6) Distance of point, $\mathrm{P}(x_1, y_1, z_1)$ to plane (1) and (2) is $\frac{|2x_1 - 2y_1 + z_1 + 3|}{\sqrt{2^2 + (-2)^2 + 1^2}} = \frac{|2x_1 - 2y_1 + z_1 + 9|}{\sqrt{2^2 + (-2)^2 + 1^2}}\ |2x_1 - 2y_1 + z_1 + 3| = |2x_1 - 2y_1 + z_1 + 9|\ |-k + 3| = |-k + 9|\ (-k + 3) = (-k + 9)$ or $-(-k + 3) = (-k + 9)\ k - 3 = -k + 9\ 2k = 12\ k = 6$

Put the value of $k = 6$ in (4) we get, The required equation of the plane is $2x - 2y + z + 6 = 0$

SOLUTION- SAMPLE PAPER- XII

PART -A

SECTION -I (One Mark Questions)

1. We have, $2y+x^2=3$ (i) Differentiating (i) w.r.t. x, we get $2\frac{dy}{dx}+2x=0 \Rightarrow \frac{dy}{dx}=-x$ As, $\left(\frac{dy}{dx}\right)_{(1,1)}=-1$. $\therefore$ Slope of normal $=1$ The equation of normal is $y-1=1\cdot(x-1)\Rightarrow x-y=0$.
2. Here, vector product that is, $\vec{a}\times\vec{b}=\hat{i}-\hat{j}+\hat{k}$ $|\vec{a}\times\vec{b}|=\sqrt{1^2+(-1)^2+1^2}=\sqrt{3}$ $\frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}=\sin\theta$ $\frac{\sqrt{3}}{\sqrt{2}\cdot\sqrt{2}}=\sin\theta$ $\frac{\sqrt{3}}{2}=\sin\theta$ We know that $\cos^2\theta=\sqrt{1-\sin^2\theta}=\sqrt{1-\frac{3}{4}}=\sqrt{\frac{4-3}{4}}=\sqrt{\frac{1}{4}}\cos^2\theta=\sqrt{\frac{1}{4}}\cos\theta=\pm\frac{1}{\sqrt{2}}$ Accept positive value, Therefore, $\theta=\frac{\pi}{4}$
3. Let $I=\int_0^1\log\left(\frac{1}{x}-1\right)dx$ (i) $\Rightarrow I=\int_0^1\log\left(\frac{1}{1-x}-1\right)dx$
$\Rightarrow I=\int_0^1\log\left(\frac{1-1+x}{1-x}\right)dx\Rightarrow I=\int_0^1\log\left(\frac{x}{1-x}\right)dx$ (ii) Adding (i) and (ii) we get $2I=\int_0^1\left[\log\left(\frac{1-x}{x}\right)+\log\left(\frac{x}{1-x}\right)\right]dx=\int_0^1\log 1\cdot dx=0$
$\Rightarrow I=0$
4. Order (highest derivative) $=3$ and degree (power of highest derivative) $=4$
5. $2\begin{bmatrix}1&3\\0&x\end{bmatrix}+\begin{bmatrix}y&0\\1&2\end{bmatrix}=\begin{bmatrix}5&6\\1&8\end{bmatrix}$ $\begin{bmatrix}2&6\\0&2x\end{bmatrix}+\begin{bmatrix}y&0\\1&2\end{bmatrix}=\begin{bmatrix}5&6\\1&8\end{bmatrix}$ $\begin{bmatrix}2+y&6\\0&2x+2\end{bmatrix}=\begin{bmatrix}5&6\\1&8\end{bmatrix}$ $2+y=5$ $y=3$ $2x+2=8$ $x=3$
6. $P\left(\frac{B}{A}\right)=\frac{P(B\cap A)}{P(A)}=\frac{\mathrm{P(A)}}{\mathrm{P(A)}}=1$
7. $Z=x+y$
At point $\left(\frac{50}{3},\frac{40}{3}\right)$ gives maximum value ofZ.
8. Diagonal matrix is that matrix whose all elements are zero except the diagonal elements.
9. $P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}=\frac{1}{3}=\frac{P(A\cap B)}{\frac{6}{13}}$ $\frac{2}{13}=P(A\cap B)$ $\mathrm{P(A\cup B)=P(A)+P(B)-P(A\cap B)}=\frac{1}{13}+\frac{6}{13}-\frac{2}{13}$ $\frac{5}{13}=\mathrm{P(A\cup B)}$
10. $\mathrm{P(A)=0.8, P(B/A)=0.4}$, then find $\mathrm{P(A\cap B)}$ We know that $\frac{\mathrm{P(A\cap B)}}{\mathrm{P(A)}}=\mathrm{P(B|A)}$ $\mathrm{P(A\cap B)=P(A)\times P(B|A)}=0.8\times0.4$ $\mathrm{P(A\cap B)}=0.32$
11. As the x, y and z intercept are 3,6 and -4. applying the intercept form, we get, $\frac{x}{3}+\frac{y}{6}+\frac{z}{-4}=1$ Taking L.C.M. $=12$ $\frac{4x+2y-3z}{12}=1$ $4x+2y-3z=12$ $4x+2y-3z-12=0$
12. If the lines are perpendicular to each other the dot product is equal to 0 $(-3\hat{i}+2\hat{j}+3k\hat{k})\cdot(3k\hat{i}+\hat{j}-5\hat{k})=0$ $-9k+2-15k=0$ $-24k=-2$ $k=12$
13. $3\mathrm{A}-(\mathrm{I}+\mathrm{A})^2=3\mathrm{A}-\left(\mathrm{I}^2+\mathrm{A}^2+2\mathrm{IA}\right)=3\mathrm{A}-(\mathrm{I}+\mathrm{A}+2\mathrm{A})=3\mathrm{A}-(\mathrm{I}+3\mathrm{A})=-\mathrm{I}$
14. Function $f(x)=\begin{cases}\frac{x}{x^2+1} & x\neq-1\\ k & x=-1\end{cases}$ is given to be continuous at $x=-1$ then $f(-1)=k$
It is given for $x=-1$ f$(x)=k$) $\frac{-1}{(-1)^2+1}=k$
$\frac{-1}{2}=k$
15. Given that an urn has 5 Red balls and 2 Black balls. Number of ways in which two balls can be represented are {(RR), (RB), (BR), (BB)
Let X represents the number of black balls. Possible values of$X$are: $X(RR)=0$, $X(RB)=1$, $X(BR=1)$ and $X(BB)=2$
Therefore, the possible values of X are 0, 1 and 2.
16. $|\vec{a}|=2$ and $|\vec{b}|=3$ $|\vec{a}-\vec{b}|=|\vec{a}|^2+|\vec{b}|^2-2|\vec{a}|\cdot|\vec{b}|\cos\theta=2^2+3^2-2\cdot2\cdot3\cdot1=4+9-12=1$

SECTION -II (CASE STUDY)

CASE STUDY -I

17. (a) From the graph we can see, graph of the function reaches at 3 when value of x =5
    (b) We can easily see that graph is continuous in interval [1,7] therefore continuous in [2,6]
    (c) We can see from the graph, that slope of graph reduces in (5,6) therefore decreasing in this interval
    (d) If we see the graph, graph breaks at point (1,1) therefore discontinuous at this point and domain expand from (-1,7] and 1 not included in it. Also range of the function is [1,4) as graph starts from point (1,1) and does not reach to y=4
    (e) statement c is false, as the value at 1 is 1.

CASE STUDY : II

18. (a) Given integral can be written as $\int_0^{\pi/2} \frac{1-cos2x}{2}dx$
    = $[\frac{x}{2}+\frac{sin2x}{4}]_0^{\pi/2}$
    $=\frac{\pi}{4}+0=\frac{\pi}{4}$
    (b) The given function is odd, the value of the given integral is zero.
    (c) Given integral can be written as : $\int_0^{\pi/2} \sqrt{cosx}(1-cos^2x)sinxdx$
    Let cosx = t $\Rightarrow -sinxdx = dt$
    $\int_0^{\pi/2} \sqrt{t}(1-t^2)dt$
    $\int_0^{\pi/2} (\sqrt{t}-t^{5/2})dt$
    = $[\frac{t^{3/2}}{3/2}-\frac{t^{7/2}}{7/2}]$
    =$[\frac{2}{3}cos^{3/2}x-\frac{2}{7}cos^{7/2}x]_0^{\pi/2}$
    $=-\frac{2}{3}+\frac{2}{7}$ = $\frac{-8}{21}$
    (d) The given function is odd , therefore the value of the given integral is zero
    (e) Given function is odd, therefore the value of the given integral is zero.

PART -B

SECTION-III(Each question carries 2 Marks)

19. Unit vector of $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$ is $\frac{\vec{a}}{|\vec{a}|}=\frac{\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{1^2+2^2+3^2}}=\frac{\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{14}}$

20. $\int 8^x+x^8dx=\frac{8^x}{\log 8}+\frac{x^9}{9}+c$

21. $\int \frac{1}{\sqrt{1-4x^2}}dx=\int \frac{1}{\sqrt{1-(2x)^2}}dx=\sin^{-1}\left(\frac{2x}{1}\right)=\sin^{-1}(2x)$

22. $\frac{d}{dr}(\mathrm{A})=\frac{d}{dr}\left(\pi r^2\right)$ Now $\frac{d\mathrm{A}}{dr}=2\pi r$ $r=21$ $\frac{d\mathrm{A}}{dr}=2\pi(21)=132\mathrm{cm}^2$

OR

$f(x)=x^2+kx+1$ $f'(x)=2x+k\geq 0$ For least value $f'(x)=2x+k=0$At $x=1$ $2(1)+k=0k=-2$

23. $y=\log_x 4\ \frac{dy}{dx}=\frac{d}{dx}\left[\frac{\log 4}{\log x}\right]$

$=\log 4\frac{d}{dx}\left[\frac{1}{\log x}\right]=\log 4\left[\frac{\log x\cdot 0-x\cdot\frac{1}{x}}{(\log x)^2}\right]$

$=\log 4\left[\frac{-1}{x(\log x)^2}\right]=\frac{-\log 4}{x(\log x)^2}$

24. The anti-derivative of $\sin 3x$ is a function of x whose derivative is $\sin 3x\ \frac{d}{dx}(\cos 3x)=-3\sin 3x$

Now, $\sin 3x=\frac{-1}{3}\frac{d}{dx}(\cos 3x)\ \sin 3x=\frac{d}{dx}\left(\frac{-1}{3}\cos 3x\right)$ The anti-derivative of $\sin 3x$ is $\frac{-1}{3}\cos 3x$

25. Put $x=\tan\theta$ in $\sin^{-1}\frac{2x}{x^2+1}$ We get, L.H.S $=\sin^{-1}\frac{2x}{x^2+1}=\sin^{-1}\left(\frac{2\tan\theta}{\tan^2\theta+1}\right)$

$=\sin^{-1}\left(\frac{2\tan\theta}{\sec^2\theta}\right)=\sin^{-1}(2\sin\theta\cdot\cos\theta)=\sin^{-1}(\sin 2\theta)=2\theta$

$=2\tan^{-1}\left(\frac{1}{x}\right)=$ R.H.S. L.H.S. = R.H.S Hence proved.

OR

$\tan^{-1}\sqrt{3}=\frac{\pi}{3}\ \sec^{-1}(-2)=\frac{2\pi}{3}$

Principle value is $\frac{\pi}{3}-\frac{2\pi}{3}=-\frac{\pi}{3}$

26. $x=\sqrt{a^{\sin^{-1}t}}$ and $y=\sqrt{a^{\cos^{-1}t}}$ Differentiating both sides we get, $\frac{dx}{dt}=\frac{1}{2\sqrt{a^{\sin^{-1}t}}}\cdot\frac{d}{dt}\left(a^{\sin^{-1}t}\right)$ and

$\frac{dy}{dt}=\frac{1}{2\sqrt{a^{\cos^{-1}t}}}\cdot\frac{d}{dt}\left(a^{\cos^{-1}t}\right)$

$\frac{dx}{dt}=\frac{1}{2\sqrt{a^{\sin^{-1}t}}}\cdot\left(a^{\sin^{-1}t}\right)(\log_e a)\cdot\frac{d}{dt}\left(\sin^{-1}t\right)$ and $\frac{dy}{dt}=\frac{1}{2\sqrt{a^{\cos^{-1}t}}}\cdot\left(a^{\cos^{-1}t}\right)(\log_e a)\cdot\frac{d}{dt}(\cos^{-1}t)$

$\frac{dx}{dt}=\frac{1}{2\sqrt{a^{\sin^{-1}t}}}\cdot\left(a^{\sin^{-1}t}\right)(\log_e a)\frac{1}{\sqrt{1-t^2}}$ and $\frac{dy}{dt}=\frac{1}{2\sqrt{a^{\cos^{-1}t}}}\cdot\left(a^{\cos^{-1}t}\right)(\log_e a)\left(\frac{-1}{\sqrt{1-t^2}}\right)$

$\frac{dx}{dt}=\frac{\sqrt{a^{\sin^{-1}t}}\cdot\log_e a}{2\sqrt{1-t^2}}$ and $\frac{dy}{dt}=\frac{-\sqrt{a^{\cos^{-1}t}}\cdot\log_e a}{2\sqrt{1-t^2}}$

Now divide $\frac{dy}{dt}$ by $\frac{dx}{dt}$ we get $\frac{dy}{dt}=\frac{dy}{dx}=\frac{-\frac{\sqrt{a^{\cos^{-1}t}}\cdot\log_e a}{2\sqrt{1-t^2}}}{\frac{2\sqrt{1-t^2}}{2\sqrt{1-t^2}}}$

$\frac{dy}{dx}=\frac{-\sqrt{a^{\cos^{-1}t}}}{\sqrt{a^{\sin^{-1}t}}}=-\frac{y}{x}\ \frac{dy}{dx}=-\frac{y}{x}$ Hence proved.

Short Cut Method: xy = $\sqrt{a^{sin^{-1}t}}\sqrt{a^{cos^{-1}t}}=\sqrt{a^{sin^{-1}t+cos^{-1}t}}=\sqrt{a^{\pi/2}}$

$\Rightarrow xy=\sqrt{a^{\pi/2}}$

Differentiating borth sides we get :

$xy'+y=0\Rightarrow y'=\frac{-y}{x}$ Hence proved

27. $f(x)=3+|x|$ where $x\in$ R Since value of $|x|>0$ Therefore, minimum value of $|x|=0$
This implies, minimum value of $f(x)=3+$ minimum value of $|x|=3+0=3$ Hence, minimum value of $f(x)=3$

OR

$f(x)=\log x$ Differentiating with respect to x we get, $f'(x)=\frac{1}{x}$when $f'(x)=0\ f'(x)=\frac{1}{x}=0$

Hence $f(x)$ is not defined for any real value of x does not have maxima and minima.

28. Let $I=\int\limits_0^{\pi}\frac{x}{a^2\cos^2x+b^2\sin^2x}dx$.(i)

$\Rightarrow I=\int\limits_0^{\pi}\frac{\pi-x}{a^2\cos^2(\pi-x)+b^2\sin^2(\pi-x)}dx$

$\Rightarrow I=\int\limits_0^{\pi}\frac{\pi-x}{a^2\cos^2x+b^2\sin^2x}dx$ ..(ii)

Adding (i) and (ii) we get $2I=\int\limits_0^{\pi}\frac{\pi}{a^2\cos^2x+b^2\sin^2x}dx=\pi\int\limits_0^{\pi}\frac{dx}{a^2\cos^2x+b^2\sin^2x}$

$\Rightarrow 2I=\pi\int\limits_0^{\pi}\frac{\sec^2xdx}{a^2+b^2\tan^2x}=\frac{\pi}{b^2}\int\limits_0^{\pi}\frac{\sec^2xdx}{\frac{a^2}{b^2}+\tan^2x}$

$\Rightarrow 2I=\frac{\pi}{b^2}2\cdot\int\limits_0^{\frac{\pi}{2}}\frac{\sec^2xdx}{\frac{a^2}{b^2}+\tan^2x}$

$\Rightarrow I=\frac{\pi}{b^2}\int\limits_0^{\frac{\pi}{2}}\frac{\sec^2xdx}{\frac{a^2}{b^2}+\tan^2xdx}$

Put $\tan x=t\Rightarrow\sec^2xdx=dt$;$x=0,t=0,x=\frac{\pi}{2},t=\infty$

$\Rightarrow I=\frac{\pi}{b^2}\int\limits_0^{\infty}\frac{dt}{\frac{a^2}{b^2}+t^2}=\frac{\pi}{b^2}\times\frac{b}{a}\left[\tan^{-1}\frac{bt}{a}\right]_0^{\infty}$

$=\frac{\pi}{ab}\left[\frac{\pi}{2}-0\right]=\frac{\pi^2}{2ab}$

SECTION -IV (Three Mark Questions)

29. Let a be any real number, then $(1+a\cdot a)=\left(1+a^2\right)>0$ It shows that $(a,a)\varepsilon R$ Hence, R is reflexive. $(a,b)\varepsilon$R
This implies $(1+ab)>0$ $(1+ba)>0$ Also, $(b,a)\varepsilon$R Hence, R is symmetric. Now, in order to prove R is not transitive.
Consider $(-1,0)$ and $(0,2)>0$ $(-1,0)\varepsilon$R This implies, $(1+-1.0)>0$
Now, $(0,2)\varepsilon$R This implies, $(1+0.2)>0$ But,
$(-1,2)$ & R $(1+-1\cdot 2)<0$ So, R is reflexive, symmetric but not transitive.

30. We have, $y^2-2x^3-4y+8=0$ (i) Differentiating (i) w.r.t x , we get $2y\frac{dy}{dx}-6x^2-4\frac{dy}{dx}=0\Rightarrow\frac{dy}{dx}=\frac{3x^2}{y-2}$

Let $(h,\ k)$ be coordinates of the point of contact of the tangent to the curve $y^2-2x^3-4y+8=0$, then $\left(\frac{dy}{dx}\right)_{(h.k)}=\frac{3h^2}{k-2}$

Therefore the equation of tangent is $(y-k)=\frac{3\cdot h^2}{k-2}(x-h)$ (ii)

Since it passes through the point $(1,\ 2)$, so we have $(2-k)=\frac{3h^2}{k-2}(1-h)\Rightarrow 3h^3-3h^2-k^2+4k-4=0$ (iii)

Also, $(h,\ k)$ lies on the curve $y^2-2x^3-4y+8=0$ So, $k-2.h^3-4k+8=0$ (iv)
Adding (iii) and (iv), we get $h^3-3h^2+4=0\Rightarrow(h-2)^2(h+1)=0\Rightarrow h=-1,2$
Putting $h=2$ in (iii), we get $24-12-k^2+4k-4=0\Rightarrow k^2-4k-8=0\Rightarrow k=2\pm 2\sqrt{3}$
and $h=-1$ gives imaginary value of k.
Thus, the points are $(2,\ 2\pm 2\sqrt{3})$.
Equation of tangent at $(2,\ 2+2\sqrt{3})$ is $y-(2+2\sqrt{3})=2\sqrt{3}(x-2)\Rightarrow 2\sqrt{3}x-y=2\sqrt{3}-2$
Equation of tangent at $(2,\ 2-2\sqrt{3})$ is $y-(2-2\sqrt{3})=2\sqrt{3}(x-2)\Rightarrow 2\sqrt{3}x-y=6\sqrt{3}-2.$

31. Take, $\sin x = \dfrac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

We get $\displaystyle\int \frac{1}{5+4\sin x}dx = \int \frac{1}{5+4\left(\dfrac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}dx$

$$\int \frac{1}{5+4\sin x}dx = \int \frac{1+\tan^2\frac{x}{2}}{5\left(1+\tan^2\frac{x}{2}\right)+8\tan\frac{x}{2}}dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{5\left(1+\tan^2\frac{x}{2}\right)+8\tan\frac{x}{2}}dx..(2)$$

Put, $\tan\frac{x}{2} = t$

Differentiating w.r.t. x $\frac{1}{2}\sec^2\frac{x}{2}dx = dt$ Put in (2), we get, $= \dfrac{1}{2}\displaystyle\int \frac{dt}{5+5t^2+8t} = \frac{1}{10}\int \frac{dt}{1+t^2+\frac{8}{5}t}$

$$= \frac{1}{10}\int \frac{dt}{t^2+\frac{8}{5}t+1}$$

$$= \frac{1}{10}\int \frac{dt}{\left(t+\frac{4}{5}\right)^2-\left(\frac{3}{5}\right)^2}$$

$$= \frac{1}{10}\left[\frac{1}{2\left(\frac{3}{5}\right)}\log\frac{t+\frac{4}{5}-\frac{3}{5}}{t+\frac{4}{5}+\frac{3}{5}}\right]+c$$

$$= \frac{1}{3}\log\left(\frac{t+\frac{1}{5}}{t+\frac{7}{5}}\right)+c$$

$$= \frac{1}{3}\log\left(\frac{\tan\frac{x}{2}+\frac{1}{5}}{\tan\frac{x}{2}+\frac{7}{5}}\right)+c$$

32. $\dfrac{dy}{dx} = \sin^{-1}\sqrt{x}$

Separating the variables, we get, $dy = \left(\sin^{-1}\sqrt{x}\right)dx$Integrating both sides, we get, $\int dy = \int\left(\sin^{-1}\sqrt{x}\right)\cdot 1dx$Integrate it by parts.

$$y = \left(\sin^{-1}\sqrt{x}\right)\int 1\cdot dx - \int\left[\frac{d}{dx}\left(\sin^{-1}\sqrt{x}\right)\cdot\int 1\cdot dx\right]dx$$

$$= x\cdot\left(\sin^{-1}\sqrt{x}\right)-\int\left(\frac{1}{\sqrt{1-x}}\cdot x\right)dx$$

$$= x\cdot\left(\sin^{-1}\sqrt{x}\right)-\int\left(\frac{x}{\sqrt{1-x}}\right)dx = x\cdot\left(\sin^{-1}\sqrt{x}\right)+\int\left(\frac{(1-x)-1}{\sqrt{1-x}}\right)dx = x\cdot\left(\sin^{-1}\sqrt{x}\right)+\int\frac{(1-x)}{\sqrt{1-x}}-\frac{1}{\sqrt{1-x}}dx$$

$$= x\cdot\left(\sin^{-1}\sqrt{x}\right)+\int(\sqrt{1-x})dx-\int\frac{1}{\sqrt{1-x}}dx = x\cdot\left(\sin^{-1}\sqrt{x}\right)+\frac{(1-x)^{\frac{3}{2}}}{-\frac{3}{2}}-\sin^{-1}\sqrt{x}$$

$$y = x\cdot\left(\sin^{-1}\sqrt{x}\right)-\frac{2(1-x)^{\frac{3}{2}}}{3}-\sin^{-1}\sqrt{x}$$

33. $y=\log(\log x)$ $\frac{dy}{dx}=\frac{d}{dx}[\log(\log x)]$ $\frac{dy}{dx}=\frac{1}{\log x}\cdot\frac{1}{x}$

$\frac{d^2y}{dx^2}=-\frac{1}{x\log x}\frac{d}{dx}[x(\log x)]=-\frac{1}{x\log x}[1+\log x]$

OR

$f(x)=\begin{cases}\frac{1-\cos 6x}{18x^2} & \text{if} x\neq 0\\ k & \text{if} x=0\end{cases}$ Now, $f(x)$ is continuous at $x=0$ then $(\text{L.H.L})_{x=0}=(R.H.L)_{x=0}=f(0)$.........(1)

L.H.L. $=\lim_{x\to 0^-}\frac{1-\cos 6x}{18x^2}$

Put, $x=0-h$ where $x\to 0$ and $h\to 0=\text{Lt}\sigma_{h\to 0}\frac{1-\cos(-6h)}{18(-h)^2}$

$=\text{Lt}_{h\to 0^-}\frac{2\sin^2 3h}{18h^2}=\lim_{h\to 0^-}\left(\frac{\sin 3h}{3h}\right)^2=1$ At $\quad f(0)=k$

By (1) we have, $(\text{L.H.L})_{x=0}=f(0)=k$ So $k=1$

34. Let E be the event that number 4 appears at least once and F be the event that the sum of the numbers appearing is 6.

Then, $E=\{(4,1),\ (4,2),(4,3),(4,4\ (4,5),(4,6),(1,4),\ (2,4),\ (3,4),\ (5,4),(6,4)\}$ and $F=\{(1,5),(2,4),(3,3),\ (4,2),(5,1)\}$

We have, $P(E)=\frac{11}{36}$ and $P(F)=\frac{5}{36}$ Also, $E\cap F=\{(2,4),\ (4,2)\}$

Therefore, $P(E\cap F)=\frac{2}{36}$ Hence, the required probability $P(E|F)=\frac{P(E\cap F)}{P(F)}=\frac{\frac{2}{36}}{\frac{5}{36}}=\frac{2}{5}$

For the conditional probability discussed above, we have considered the elementary events of the experiment to be equally likely and the corresponding definition of the probability of an event was used.

However, the same definition can also be used in the general case where the elementary events of the sample space are not equally likely the probabilities $P(E\cap F)$ and $P(F)$ king calculated accordingly

35. Directions cosines of (3, 2, -6) are $\left(\frac{3}{7},\frac{2}{7},\frac{-}{6}7\right)$ Direction cosines of $(1,2,2)$ are $\left(\frac{1}{3},\frac{2}{3},\frac{2}{3}\right)$ Let θ be the angle between the two lines.

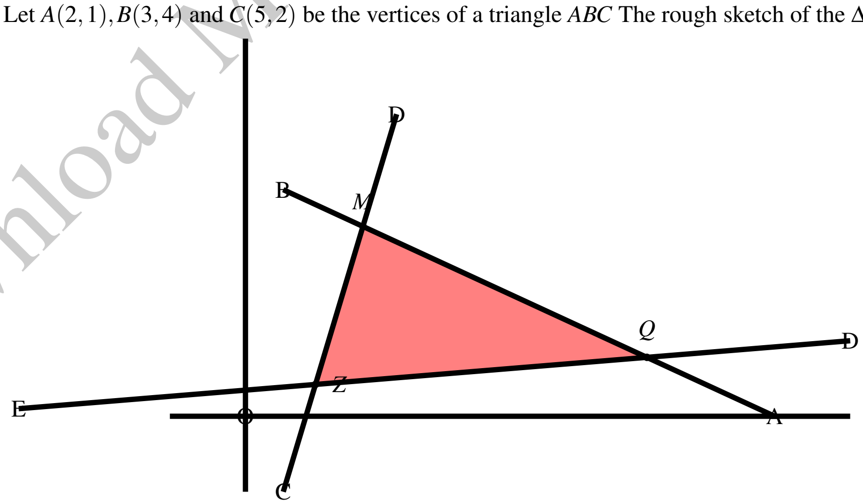
If (l_1,m_1,n_1) and (l_2,m_2,n_2) are the direction cosines of two lines then the angle between these two lines is given by $\cos\theta=l_1l_2+m_1m_2+n_1n_2$

$\cos\theta=\left|\left(\frac{3}{7}\times\frac{1}{3}\right)+\left(\frac{2}{7}\times\frac{2}{3}\right)+\left(\frac{2}{3}\times\frac{-6}{7}\right)\right|$

$=\left|\frac{1}{7}+\left(\frac{4}{21}\right)+\left(\frac{-12}{21}\right)\right|=\frac{5}{21}$ $\theta=\cos^{-1}\left(\frac{5}{21}\right)$

SECTION -V (Five Mark Questions)

36. Let $A(2,1),B(3,4)$ and $C(5,2)$ be the vertices of a triangle ABC The rough sketch of the ΔABC as shown in figure



Equation of line $ZM: y-1=\frac{4-1}{3-2}(x-2)\Rightarrow y-1=3(x-2)\Rightarrow y=3x-5$

Similarly, equation of line $MQ : y = 7 - x$ and equation of line : $ZQ : y = \frac{1}{3}(x+1)$

The required area of shaded region $= \int_2^3 y_1 dx + \int_3^5 y_2 dx - \int_2^5 y_3 dx$ where $y_1 = 3x-5, y_2 = 7-x$ and $y_3 = \frac{1}{3}(x+1)$ $\therefore A = \int_2^3 (3x-5)dx + \int_3^5 (7-x)dx - \frac{1}{3}\int_2^5 (x+1)dx \Rightarrow A = \left[\frac{3x^2}{2} - 5x\right]_2^3 + \left[7x - \frac{x^2}{2}\right]_3^5 - \frac{1}{3}\left[\frac{x^2}{2}+x\right]_2^5 = \left[\left(\frac{27}{2} - 15\right) - (6-10)\right] + \left[\left(35 - \frac{25}{2}\right) - \left(21 - \frac{9}{2}\right)\right] - \frac{1}{3}\left[\left(\frac{25}{2}+5\right) - (2+2)\right]$

$= \left(-\frac{3}{2}+4\right) + \left(35 - \frac{25}{2} - 21 + \frac{9}{2}\right) - \frac{1}{3}\left(\frac{35}{2} - 4\right) = \frac{5}{2} + 6 - \frac{9}{2} = 4$ sq. units

37. Let x and y be the length and breadth of the rectangle whose perimeter is given (P) $\therefore P = 2(x+y) \Rightarrow y = \frac{P}{2} - x$

Area of rectangle $(A) = xy \Rightarrow A = xy \Rightarrow A = x\left(\frac{P}{2} - x\right)$

$\Rightarrow A = \frac{Px}{2} - x^2 \Rightarrow \frac{dA}{dx} = \frac{P}{2} - 2x$

For maxima/minima, $\frac{dA}{dx} = 0$

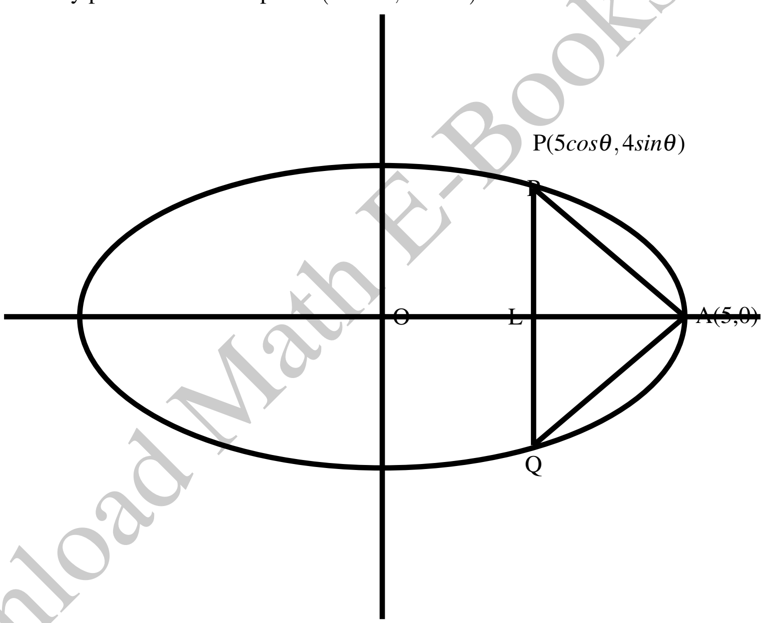
$\frac{P}{2} - 2x = 0 \Rightarrow x = \frac{P}{4}$ Also, $\frac{d^2A}{dx^2} = -2 < 0$

Area is maximum, when $x = \frac{P}{4}$

Now, $y = \frac{P}{2} - \frac{P}{4} = \frac{P}{4}$ Area is maximum, when $x = y$ i.e., rectangle is square.

OR

Let the equation of the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$,
then any point P on the ellipse is $(5\cos\theta,\ 4\sin\theta)$



From P, draw PQ parallel to y-axis and produce it to meet the ellipse at Q then PAQ is an isosceles triangle.

Let A be its area, then $A = \frac{1}{2}PQ \cdot AL = \frac{1}{2}(2 \cdot 4\sin\theta)(5 - 5\cos\theta) \Rightarrow A = (5 - 5\cos\theta) \times 4\sin\theta \Rightarrow A = 5 \cdot 4(\sin\theta - \sin\theta\cos\theta)$

$\Rightarrow A = 5 \cdot 4\left(\sin\theta - \frac{1}{2}\sin 2\theta\right)$

(i)

Differentiating (i) wr.t. θ, we get $\frac{dA}{d\theta} = 5 \cdot 4(\cos\theta - \cos 2\theta)$ (ii)

For maxima/minima, $\frac{dA}{d\theta} = 0 \Rightarrow \cos\theta = \cos 2\theta$

$\Rightarrow \cos 2\theta = \cos(2\pi - \theta) \Rightarrow 2\theta = 2\pi - \theta \Rightarrow \theta = \frac{2\pi}{3}$

Differentiating (ii) w.r.t. θ, we get $\frac{d^2A}{d\theta^2} = 5 \cdot 4(-\sin\theta + 2\sin 2\theta)$

(iii)

Now, $\left(\frac{d^2A}{d\theta^2}\right)_{\theta=\frac{2\pi}{3}} = 5 \cdot 4\left(-\sin\frac{2\pi}{3} + 2\sin\frac{4\pi}{3}\right)$

$= 5 \cdot 4\left(-\frac{\sqrt{3}}{2} - \sqrt{3}\right) = \frac{-3\sqrt{3}}{2} 5 \cdot 4 < 0 \therefore A$ is maximum, when $\theta = \frac{2\pi}{3}$

Also, the maximum area is$A = 20\left(\sin\frac{2\pi}{3} - \frac{1}{2}\sin\frac{4\pi}{3}\right)$

$= 20\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\left(-\frac{\sqrt{3}}{2}\right)\right) = 15\sqrt{3}$sq.units.

38. $\frac{x-1}{1} = \frac{y-3}{2} = \frac{z-4}{3}$.............(1) $\frac{x+2}{3} = \frac{y-2}{-1} = \frac{z+1}{2}$.............(2)

The general solution of a plane passing through the point $(4,-1,2)$ is given by $a(x-4)+b(y+1)+c(z-2) = 0$.............(3) where $< a,b,c >$ are the direction ratios of the plane

Equation (3) is parallel to each of the given lines only when the normal to the plane (3) is perpendicular to each of the given lines.

Apply the condition for two perpendicular planes, then sum of product of direction ratios those two planes is zero. $\therefore a + 2b + 3c = 0(4)$

(Using direction ratios of (2) and (3) (5) $\therefore 3a - b + 2c = 0.(5)$

(Using direction ratios of (1) and (3) (condition for two planes perpendicular) Solving equations (4) and (5) we get, $\frac{a}{4+3} = \frac{b}{9-2} = \frac{c}{-1-6} = \lambda$ (say) $\frac{a}{7} = \frac{b}{7} = \frac{c}{-7} = \lambda$ $\frac{a}{1} = \frac{b}{1} = \frac{c}{-1} = \lambda$

$a = \lambda$ $b = \lambda$ $c = -\lambda$ Putting these values in equation (3) we get, $\lambda(x-4) + \lambda(y+1) - \lambda(z-2) = 0$ $x + y = z = 1$

Hence, $x + y = z = 1$ is the required equation

SOLUTION- SAMPLE PAPER- XIII

PART -A

SECTION -I (One Mark Questions)

1. Putting $x = \sec\theta$, *we get* $\tan^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) = \tan^{-1}\left(\frac{1}{\sqrt{\sec^2\theta-1}}\right) = \tan^{-1}\left(\frac{1}{\tan\theta}\right) = \tan^{-1}(\cot\theta) = \tan^{-1}\left\{\tan\left(\frac{\pi}{2}-\theta\right)\right\}$
$= \frac{\pi}{2} - \theta = \frac{\pi}{2} - \sec^{-1}x \therefore \tan^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) = \frac{\pi}{2} - \sec^{-1}x$

2. we have , $I = \int \frac{dx}{x\log x\, log\,(\log x)}$ Put $\log(\log x) = t$
$\Rightarrow \frac{1}{x\log x}dx = dt \therefore I = \int \frac{dt}{t} = \log t + C = \log(\log(\log x)) + C$

3. Perpendicular distance from the point (x_1, y_1, z_1) to the plane $ax+by+cz+d=0$ is $\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}$
Substituting point (4, 5, 6) for (x_1, y_1, z_1), $< 2, -1, 1 >$ for $< a, b, c;$ in (1), we get, $\frac{2(4)+(-1)5+1(6)+5}{\sqrt{4^2+1^2+1^2}} = \frac{14}{\sqrt{6}}$
$= \frac{14}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{14\sqrt{6}}{6} = \frac{7}{3}\sqrt{6}$

4. $A(3,-1,2)$ and terminal point $B(-5,4,3)$ Scalar component is AB is $-5-3, 4+1, 3-2 = -8$, 5, 1

5. The given function is $f(x) = ax+1$ if $x \le 3$ and $f(x) = bx+3$ for $x > 3$ If f is continuous at $x = 3$ then, $\lim_{x\to3^-} f(x) = \lim_{x\to3^+} f(x) = f(3)$
Also, $\lim_{x\to3^-} f(x) = \lim_{x\to3^-} ax+1 = 3a+1$ $\lim_{x\to3^+} f(x) = \lim_{x\to3^+} bx+3 = 3b+3$ $F(3) = 3a+1$
From (1), we obtain $3a+1 = 3b+3 = 3a+1$ This implies, $3b+3 = 3a+1$ This implies, $3a = 3b+2$ $a = b+\frac{2}{3}$

6. $\begin{vmatrix} 5 & 3 & -1 \\ -7 & x & 2 \\ 9 & 6 & -2 \end{vmatrix} = 0$ $5\begin{vmatrix} x & 2 \\ 6 & -2 \end{vmatrix} - 3\begin{vmatrix} -7 & 2 \\ 9 & -2 \end{vmatrix} - 1\begin{vmatrix} -7 & x \\ 9 & 6 \end{vmatrix} = 0$
$5(-2\times x - 6\times 2) - 3(-7\times -2 - 9\times 2) - 1(-7\times 6 - x\times 9) = 0$ $5(-2x-12) - 3(14-18) - 1(-42-9x) = 0$ $-10x - 60 + 12 + 42 + 9x = 0$
$-x = 60 - 54$ $x = -6$

7. If A and B are mutually exclusive events then, There is nothing in common, that is, $P(A\cap B) = 0$ $P(A) = \frac{1}{2}$, $P(B) = p$, $P(A\cup B) = \frac{3}{5}$
We know that $P(A\cup B) = P(A) + P(B) - P(A\cap B)$ $P(A\cup B) = P(A) + P(B) - P(A\cap B)$
$\frac{3}{5} = \frac{1}{2} + p - 0$ $\frac{3}{5} - \frac{1}{2} = p$ $\frac{1}{10} = p$

8. We know that, If $f(2a-x) = f(x)$ then $\int_0^{2\pi} f(x)dx = 2\int_0^{\pi} f(x)dx$.......(1) And if $f(2a-x) = -f(x)$ then $\int_0^{2\pi} f(x)dx = 0$.....(2)
Let $I = \int_0^{2\pi} \cos^5 x dx$ Apply (1), we get, $\int_0^{2\pi} \cos^5 x dx = 2\int_0^{\pi} \cos^5 x dx$
Now, $\cos^5(\pi - x) = -\cos^5 x$ ($\cos x$ is negative in 2^{nd} quadrant)
Apply (2), we get, $\int_0^{2\pi} \cos^5 x dx = 2\int_0^{\pi} \cos^5 x dx = 0$

9. It is given that : $A = \begin{bmatrix} 2 & -3 \\ 1 & 5 \end{bmatrix}$ and B is a square matrix of order 2 such that $AB = I$ This implies, $B = A^{-1}$
Therefore, $A = \begin{bmatrix} 2 & -3 \\ 1 & 5 \end{bmatrix}$ $\det A = (2)(5) - (-3)(1) = 10 + 3 = 13$ $B = A^{-1} = \frac{1}{\det A}\begin{bmatrix} 2 & -3 \\ 1 & 5 \end{bmatrix} = \frac{1}{13}\begin{bmatrix} 2 & -3 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} \frac{2}{13} & \frac{-3}{13} \\ \frac{1}{13} & \frac{5}{13} \end{bmatrix}$

10. We already know that there are exactly 2 possible outcomes for each and every toss of coins that is head, H and tail, T Therefore, total no. of possible outcomes for 8 coins is $2^8 = 256$
Now, we need to count no. of favorable events, Number of outcomes in which there are 6 heads, or 7 heads or 8 heads. Number of ways to select 6 coins out of 8+ number of ways to select 7 coins out of 8+ number of ways to select 8 coins out of 8 $= {}^8C_6 + {}^8C_7 + {}^8C_8 = 28 + 8 + 1 = 37$
Therefore, the probability of getting at least 6 heads is $\frac{37}{256}$.

11. The direction ratios of $\vec{r}$ are 6, 2, -3 Apply formula for direction cosines of direction ratios $<a,b,c>$ are $<\frac{a}{\sqrt{a^2+b^2+c^2}},\frac{b}{\sqrt{a^2+b^2+c^2}},\frac{c}{\sqrt{a^2+b^2+c^2}}>$.

Substitute 6 for a, 2 for b and -3 for c in (1), we get $=<\frac{6}{\sqrt{6^2+2^2+(-3)^2}},\frac{2}{\sqrt{6^2+2^2+(-3)^2}},\frac{-3}{\sqrt{6^2+2^2+(-3)^2}}>=<\frac{6}{\sqrt{49}},\frac{2}{\sqrt{49}},\frac{-3}{\sqrt{49}}>$

$=<\frac{6}{7},\frac{2}{7},\frac{-3}{7}>$

Hence, the direction cosines of $\vec{r}$ are $\frac{6}{7},\frac{2}{7},\frac{-3}{7}$

12. Area of triangle $=\frac{1}{2}\begin{vmatrix} k & 4 & 1 \\ k & -6 & 1 \\ 5 & 4 & 1 \end{vmatrix}=\pm 35=$

$\Rightarrow \frac{1}{2}[k(-6-4)-4(k-5)+1(4k+30)]=\pm 35 \Rightarrow \frac{1}{2}[-10k-4k+20+4k+30]=\pm 35 \Rightarrow \frac{1}{2}(50-10k)=\pm 35 \Rightarrow 50-10k=\pm 70 \Rightarrow 50-10k=70 or 50-10k=-70 \Rightarrow 10k=-20 or 10k=120$

$\Rightarrow k=-2$ or $k=12$

13. We have $\begin{vmatrix} \sin 10^\circ & -\cos 10^\circ \\ \sin 80^\circ & \cos 80^\circ \end{vmatrix}$

$=(\sin 10^\circ)(\cos 80^\circ)-(\sin 80^\circ)(-\cos 10^\circ)$

$=(\sin 10^\circ \cos 80^\circ+\cos 10^\circ \sin 80^\circ)$

$=\sin(10^\circ+80^\circ)$

$=\sin 90^\circ=1$

14. we have , $I=\int[\sin(\log x)+\cos(\log x)]\,dx$

Put $\log x=t$, $x=e^t \Rightarrow dx=e^t dt \therefore I=\int(\sin t+\cos t)e^t dt$

Consider, $f(t)=\sin t \Rightarrow f'(t)=\cos t \therefore$ Integrand is in the form $e^t\left(f(t)+f'(t)\right) \therefore \int e^t(\sin t+cos t)\,dt=e^t\sin t+C=x\sin(\log x)+C$

15. Given that, $\vec{a}=(5\hat{i}+7\hat{j})\ \vec{b}=(7i-5\hat{j})\ |\vec{a}|=\sqrt{5^2+7^2}=\sqrt{74}\ |\vec{b}|=\sqrt{7^2-(-5)^2}=\sqrt{74}$

But, $(5\hat{i}+7\hat{j})\neq(7\hat{i}-5\hat{j})$ and therefore, $\vec{a}\neq\vec{b}$

16. In the given equation, the highest order derivative is $\frac{d^3y}{dx^3}$ and its power is 2. Its order = 3 and degree = 2

SECTION -II (CASE STUDY)

CASE STUDY -I

17. (a) Solving the given equations $y=\frac{3x^2}{4}....(1); 3x-2y+12=0...(2)$
Putting the value of y from equation (1) in equation (2)
we get x = -2, 4 and y = 3, 12
point in the second quadrant is (-2,3)

(b) point of intersection in first quadrant is (4,12)

(c) If the coefficient of x^2 is negative then parabola will open downwards.

(d) the given statement is true.

(e) Required area is A = $\int_{-2}^{4}(\frac{3x+12}{2}-\frac{3}{4}x^2)dx$
$=[\frac{3}{4}x^2+6x-\frac{x^3}{4}]_{-2}^{4}=27$ sq units

CASE STUDY : II

18. (a) Sum of all probability distribution is always 1

(b) k+4k+2k+k=1 $\Rightarrow \frac{1}{8}$

(c) when x = 1 probability is $\frac{1}{8}$

(d) In case of atmost two colleges, we have to find cases = 0 college + 1 college + 2 college
$=\frac{1}{8}+\frac{1}{2}=\frac{5}{8}$

(e) To find atleast two colleges , we have to consider cases = 2 colleges + 3 colleges + 4 colleges
$= \frac{1}{2}+\frac{1}{4}+\frac{1}{8}=\frac{7}{8}$

PART -B

SECTION-III(Each question carries 2 Marks)

19. Given that, $|\vec{a}|=\sqrt{37}$, $|\vec{b}|=8$ and $|\vec{a}\times\vec{b}|=48$ $|\vec{a}\times\vec{b}|=|\vec{a}||\vec{b}|\sin\theta$ $48=|\vec{a}||\vec{b}|\sin\theta$ $\sin\theta=\frac{48}{|\vec{a}||\vec{b}|}$ $48\sin\theta=\frac{48}{\sqrt{37}\times 8}$ $\sin\theta=\frac{6}{\sqrt{37}}$ Now, $\cos\theta=\sqrt{1-\sin^2\theta}=\sqrt{1-\frac{36}{37}}$
$=\frac{1}{\sqrt{37}}\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos\theta=\left(\sqrt{37}\times 8\times\frac{1}{\sqrt{37}}\right)=8$

20. Sample space for 3 coins are tossed is $\langle$HHH, HHT, HTH, TTH, THH, THT, HT, TTT$\}$ where H: head and T: tail Let $P(A)$ be the probability of getting 3 tails i.e., $|TTT|$ $P(A)=\frac{1}{8}$ Probability of getting no tail $=\frac{1}{8}$ P(B) = Probability of getting at least one tail = 1 Probability of getting no head $=1-\frac{1}{8}=\frac{7}{8}$ P(B) $=\frac{7}{8}$ Or we can say that Probability that the throw is either all tail or at least one tail i.e. P(AUB) $=\frac{7}{8}$ $P(A\cup B)=P(A)+P(B)-P(A\cap B)$ $P(A\cap B)=P(A)+P(B)-P(A\cup B)$ Putting the value of P(A), P(B) and P(AUB) we get $P(A\cap B)=\frac{1}{8}+\frac{7}{8}-\frac{7}{8}=\frac{1}{8}$ The probability that all coins show tails if at least one of the coins shows a tail is $P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{\frac{1}{8}}{\frac{7}{8}}=\frac{1}{7}$

21. Given that, $|A|=\begin{bmatrix}2&5\\1&3\end{bmatrix}$ The co factors of the elements of |A| are given by, $A_{11}=3, A_{12}=-1$ $A_{21}=-5, A_{22}=2$ adjA $=\begin{bmatrix}3&-1\\-5&2\end{bmatrix}'$ $=\begin{bmatrix}3&-5\\-1&2\end{bmatrix}$

22. Let the required point be (x_1,y_1) Then, slope of the tangent at (x_1,y_1) = slope of the given line. $\left(\frac{dy}{dx}\right)_{(x_1,y_1)}=-1$ Now, $2y=\left(3-x^2\right)$
$\Rightarrow 2\frac{dy}{dx}=-2x\Rightarrow\frac{dy}{dx}=-x$ $\left(\frac{dy}{dx}\right)_{(x_1y_2)}=-x_1$ $-x_1=-1\Rightarrow x_1=1$
$f(x)=(x-1)e^x+1\Rightarrow f(x)=(x-1)e^x+e^x\Rightarrow f(x)=xe^x>0$ for all $x>0$ Thus, $f(x)>0$ for all $x>0$ Hence, $f(x)$ is a strictly increasing function for all $x>0$

23. Let $I=\int\frac{\sqrt{\tan x}}{\sin x\cos x}dx$.......(1) Rationalize $\tan x$ in the numerator, we get, $=\int\frac{\sqrt{\tan x}}{\sin x\cos x}\times\frac{\sqrt{\tan x}}{\sqrt{\tan x}}dx=\int\frac{\tan x}{(\sqrt{\tan x})\cdot\sin x\cos x}dx=\int\frac{\frac{\sin x}{\cos x}}{(\sqrt{\tan x})\cdot\sin x\cos x}dx$
$=\int\frac{1}{(\sqrt{\tan x})\cdot\cos^2 x}dx=\int\frac{\sec^2 x}{\sqrt{\tan x}}dx$ Let $\tan x=t\Rightarrow\sec^2 xdx=dt$ $I=\int\frac{1}{\sqrt{t}}dt=2\sqrt{t}+C=2\sqrt{\tan x}+C$

24. We have, $\frac{dy}{dx}=e^{ex}\cdot\frac{d}{dx}[\cos(bx+c)]+\cos(bx+c)\cdot\frac{d}{dx}(e^{ax})=e^{ax}\cdot\{-b\sin(bx+c)\}+\cos(bx+c)\cdot ae^{ax}=e^{ax}\cdot\{a\cos(bx+c)-b\sin(bx+c)\}$

25. $\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)=\theta$ $\cos\theta=\frac{-\sqrt{3}}{2}$ $\cos\theta=-\cos\left(\frac{\pi}{6}\right)$ $\pi+\theta$ goes in 4th quadrant and $\pi-\theta$ goes in 2nd quadrant. In both quadrant cosine is negative". $\cos\theta=\cos\left(\pi-\frac{\pi}{6}\right)$ $\cos\theta=\cos\frac{5\pi}{6}$ $\theta=\frac{5\pi}{6}$ Now, $\cos\left\{\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)+\frac{\pi}{6}\right\}=\cos\left\{\frac{5\pi}{6}+\frac{\pi}{6}\right\}=\cos\pi=-1$

OR

$\tan^{-1}\frac{1-x}{1+x}=\frac{1}{2}\tan^{-1}x(x>0)$ $\tan^{-1}(A-B)=\tan^{-1}\left(\frac{A-B}{1+AB}\right)$ Here, $A=1$ and $B=x$ Apply formula in (1) Therefore (1) becomes, $\tan^{-1}1-\tan^{-1}x=\frac{1}{2}\tan^{-1}x(x>0)$ $\frac{\pi}{4}-\tan^{-1}x=\frac{1}{2}\tan^{-1}x$ $\frac{\pi}{4}=\frac{1}{2}\tan^{-1}x+\tan^{-1}x$ $\frac{\pi}{4}=\left(\frac{1}{2}+1\right)\tan^{-1}x$ $\frac{\pi}{4}=\frac{3}{2}\tan^{-1}x$ $\tan\frac{\pi}{6}=x$ $\frac{1}{\sqrt{3}}=x$

26. Let $I=\int_0^{\pi/2}\frac{\sin^2 x}{\sin x+\cos x}dx$.(i) $\Rightarrow I=\int_0^{\pi/2}\frac{\sin\left(\frac{\pi}{2}-x\right)dx}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}=\int_0^{\frac{\pi}{2}}\frac{\cos^2 x}{\cos x+\sin x}dx$ (ii) Adding (i) and (ii) we get $2I=\int_0^{\pi/2}\frac{\sin^2 x+\cos^2 x}{\sin x+\cos x}dx=$ $\int_0^{\frac{\pi}{2}}\frac{dx}{\sin x+\cos x}\Rightarrow 2I=\int_0^{\frac{\pi}{2}}\frac{dx}{\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x\right)}\Rightarrow 2I=\frac{1}{\sqrt{2}}\int_0^{\frac{\pi}{2}}\frac{dx}{\sin x\cos\frac{\pi}{4}+\cos x\sin\frac{\pi}{4}}=\frac{1}{\sqrt{2}}\int_0^{\frac{\pi}{2}}\frac{dx}{\sin\left(x+\frac{\pi}{4}\right)}\Rightarrow 2I=\frac{1}{\sqrt{2}}\int_0^{\frac{\pi}{2}}\operatorname{cosec}\left(x+\frac{\pi}{4}\right)dx$

$\Rightarrow 2I=\frac{1}{\sqrt{2}}\left[\log\left|\operatorname{cosec}\left(x+\frac{\pi}{4}\right)-\cot\left(x+\frac{\pi}{4}\right)\right|\right]_0^{\frac{\pi}{2}}\Rightarrow 2I=\frac{1}{\sqrt{2}}\left[\log\left|\operatorname{cosec}\left(\frac{x}{2}+\frac{\pi}{4}\right)-\cot\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|-\log\left|\operatorname{cosec}\frac{\pi}{4}-\cot\frac{\pi}{4}\right|\right]=\frac{1}{\sqrt{2}}\left[\log\left|\operatorname{cosec}\frac{3\pi}{4}-\right.\right.$
$=\frac{1}{\sqrt{2}}\left[\log\left|\sqrt{2}+1\right|-\log\left|\sqrt{2}-1\right|\right]=\frac{1}{\sqrt{2}}\log\left|\frac{\sqrt{2}+1}{\sqrt{2}-1}\right|=\frac{1}{\sqrt{2}}\log\left(\sqrt{2}+1\right)^2\Rightarrow 2I=\frac{2}{\sqrt{2}}\log\left(\sqrt{2}+1\right)\Rightarrow I=\frac{1}{\sqrt{2}}\log\left(\sqrt{2}+1\right)$

27. Given: $f(x)=\left(x^2-3x+2\right)$.........(1) We have, $f:f(x)=f\left[f(x)=f\left(x^2-3x+2\right)=f(y)\right.$ where, $y=\left(x^2-3x+2\right)$ Put $y=\left(x^2-3x+2\right)$ in (1) $=\left(y^2-3y+2\right)=\left(x^2-3x+2\right)^2-3\left(x^2-3x+2\right)+2=\left(x^4-6x^3+10x^2-3x\right)$

28. $\int\frac{dx}{(x^2+6x+13)}=\int\frac{dx}{(x^2+6x+9)+4}=\int\frac{dx}{(x+3)^2+2^2}$ Let $(x+3)=t$ $\int\frac{dx}{(x+3)^2+2^2}=\int\frac{dx}{t^2+2^2}=\frac{1}{2}\tan^{-1}\frac{t}{2}+C=\frac{1}{2}\tan^{-1}\frac{(x+3)}{2}+C$

SECTION -IV (Three Mark Questions)

29. We have, $I=\int\left[\sqrt{\tan x}+\sqrt{\cot x}\right]dx$. Then $I=\int\left(\sqrt{\frac{\sin x}{\cos x}}+\sqrt{\frac{\cos x}{\sin x}}\right)dx\Rightarrow I=\int\frac{\sin x+\cos x}{\sqrt{\sin x+\cos x}}dx\Rightarrow I=\sqrt{2}\int\frac{\sin x+\cos x}{\sqrt{2\sin x\cos x}}dx\Rightarrow I=$ $\sqrt{2}\int\frac{\sin x+\cos x}{\sqrt{\sin 2x+1-1}}dx\Rightarrow I=\sqrt{2}\int\frac{\sin x+\cos x}{\sqrt{1-(1-\sin 2x)}}dx\Rightarrow I=\sqrt{2}\int\frac{\sin x+\cos x}{\sqrt{1-(\sin x-\cos x)^2}}dx$ Put $\sin x-\cos x=t\Rightarrow(\cos x+\sin x)dx=dt$

$\therefore I=\sqrt{2}\int\frac{dt}{\sqrt{1-t^2}}=\sqrt{2}\sin^{-1}t+C=I=\sqrt{2}\sin^{-1}(\sin x-\cos x)+C$

30. Let $\cos^{-1}\frac{a}{b}=\theta$ then $\frac{a}{b}=\cos\theta$ LHS $=\tan\left\{\frac{\pi}{4}+\frac{1}{2}\theta\right\}+\tan\left\{\frac{\pi}{4}-\frac{1}{2}\theta\right\}$ Apply $\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$ We know that $\tan\left\{\frac{\pi}{4}+\frac{1}{2}\theta\right\}+$ $\tan\left\{\frac{\pi}{4}-\frac{1}{2}\theta\right\}=\frac{\tan\frac{\pi}{4}+\tan\left(\frac{\theta}{2}\right)}{1-\tan\frac{\pi}{4}\tan\left(\frac{\theta}{2}\right)}+\frac{\tan\frac{\pi}{4}-\tan\left(\frac{\theta}{2}\right)}{1+\tan\frac{\pi}{4}\tan\left(\frac{\theta}{2}\right)}=\frac{1+\tan\left(\frac{\theta}{2}\right)}{1-\tan\left(\frac{\theta}{2}\right)}+\frac{1-\tan\left(\frac{\theta}{2}\right)}{1+\tan\left(\frac{\theta}{2}\right)}$ Taking L.C.M. and solve it. $=\frac{\left\{1+\tan\left(\frac{\theta}{2}\right)\right\}^2+\left\{1-\tan\left(\right.\right.}{1-\tan^2\left(\frac{\theta}{2}\right)}$

$=2\left\{\frac{1+\tan^2\left(\frac{\theta}{2}\right)}{1-\tan^2\left(\frac{\theta}{2}\right)}\right\}$ Put $\tan\frac{\theta}{2}=\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}$, we get $\tan\left\{\frac{\pi}{4}+\frac{1}{2}\theta\right\}+\tan\left\{\frac{\pi}{4}-\frac{1}{2}\theta\right\}=\frac{2\left\{\sin^2\frac{\theta}{2}+\cos^2\frac{\theta}{2}\right\}}{\sin^2\frac{\theta}{2}-\cos^2\frac{\theta}{2}}=\frac{2}{\sin^2\frac{\theta}{2}-\cos^2\frac{\theta}{2}}\left[\sin^2\theta-\cos^2\theta=\cos 2\theta\right]$

$=\frac{2}{\cos 2\frac{\theta}{2}}=\frac{2}{\cos\theta}=\frac{2b}{a}=$ RHS L.H.S. = R.H.S. (Hence proved)

31. $f(x)=\left(\sin^4 x+\cos^4 x\right)$ $f'(x)=4\sin^3 x\cos x-4\cos^3 x\sin x=-4\cos x\sin x\left(\cos^2 x-\sin^2 x\right)=-2\sin 2x\cos 2x=-\sin 4x$ And $f''(x)=-4\cos 4x$ Now, reason to find the local minima we get, $f(x)=0$ $f'(x)=0-\sin 4x=0$ $4x=\pi$ $x=\frac{\pi}{4}$ $x=\frac{\pi}{4}$ is a point of local maximum or local minimum.

Now, $f''(x)=-4\cos\pi=4>0$ Local minimum value $=f\left(\frac{\pi}{4}\right)=\frac{1}{2}$

OR

Let r be the radius and h the height of the cylinder. Then, $\pi r^2 h=2156h=\left(\frac{2156}{\pi r^2}\right)$ Let S be the total surface area. $S=\left(2\pi r^2+2\pi rh\right)=$ $\left(2\pi r^2+2\pi r\times\frac{2156}{\pi r^2}\right)=\left(2\pi r^2+\frac{4312}{r}\right)\frac{dS}{dr}=\left(4\pi r-\frac{4312}{r^2}\right)\frac{d^2S}{dr^2}=\left(4\pi-\frac{8624}{r^3}\right)$ Now, $\frac{dS}{dr}=0\Rightarrow\left(4\pi r-\frac{4312}{r^2}\right)=0$ And $\left[\frac{d^2S}{dr^2}\right]_{r=7}=$

$\left(4\pi+\frac{8624}{343}\right)>0$ $4\pi r^3=4312$ $r=\left(\frac{4312}{4\pi}\right)^{\frac{1}{3}}$ =7

32. The coordinates of the given points are A$(-2,3,5)$, B$(1,2,3)$ and C$(7,0,-1)$ The equation of line AB are, $\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}\Rightarrow\frac{x+2}{1+2}=\frac{y-3}{2-3}=\frac{z-5}{3-5}\Rightarrow\frac{x+2}{3}=\frac{y-3}{-1}=\frac{z-5}{-2}$ Putting$x=7$, $y=0$ and $z=-1$ in the above equation. $\frac{7+2}{3}=\frac{0-3}{-1}=\frac{-1-5}{-2}$ Add reason for this step: $3=3=3$ Which is clearly true Thus, the point C$(7,0,-1)$ satisfies the equation of lineAB. $\therefore$ C lies on line AB. Hence the given pointA, B, C are collinear.

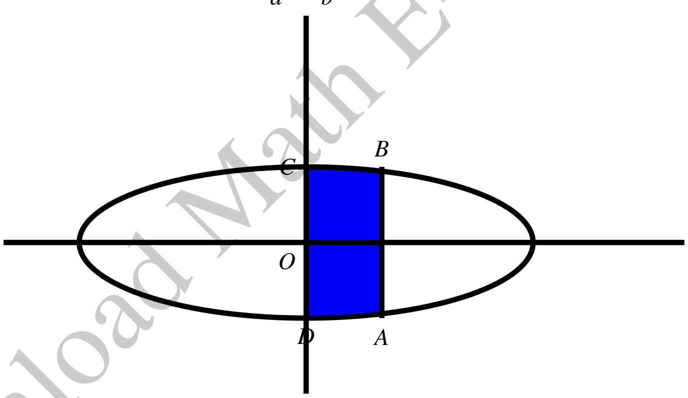
33. Clearly, the sample space is $S=\{1,2,3,4,5,6,7,8,9,10\}$ Let A = Event that the number on the drawn card is even B = Event that the number on the drawn card is more than 3 Then A $=\{2,4,6,8,10\}$, B $=\{4,5,6,7,8,9,10\}$ And A $\cap$ B $=\{4,6,8,10\}$ P(A) $=\frac{n(\text{A})}{n(\text{S})}=\frac{5}{10}=\frac{1}{2}=\frac{5}{10}=\frac{1}{2}=\frac{7}{10}$ P(A$\cap$B) $=\frac{n(\text{A}\cap\text{B})}{n(\text{S})}=\frac{4}{10}=\frac{2}{5}$ Suppose B has already occurred and then A occurs. Now, $P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{\frac{2}{5}}{\frac{7}{10}}=\left(\frac{2}{5}\times\frac{10}{7}\right)=\frac{4}{7}$

34. Given equation of the form: $\sqrt{a+x}\frac{dy}{dx}+x=0$ Divide both sides by $\sqrt{a+x}$, we get, $\frac{dy}{dx}+\frac{x}{\sqrt{a+x}}=0$ $\frac{dy}{dx}=\frac{-x}{\sqrt{a+x}}$ $dy=\frac{-x}{\sqrt{a+x}}dx$ Integrate both sides $\int dy=\int\frac{-x}{\sqrt{a+x}}dx$ Add a and subtract a in left hand side of numerator, we get, $y=-\int\frac{[(a+x)-a]}{\sqrt{a+x}}dx$ $y=-\int\left(\sqrt{a+x}-\frac{a}{\sqrt{a+x}}\right)dx$ $y=-\int\sqrt{a+x}dx+a\int(a+x)^{-\frac{1}{2}}dx$ $y=-\frac{2}{3}(a+x)^{\frac{3}{2}}+2a\sqrt{a+x}+\text{C}$

35. Let $y=e^{\tan^{-1}\sqrt{x}}$ Substitute $\sqrt{x}=t$ and $\tan^{-1}\sqrt{x}=\tan^{-1}t=u$ $y=e^u$ where $u=\tan^{-1}t$ and $t=\sqrt{x}$ Now $y=e^u\Rightarrow\frac{dy}{du}=e^u$ $u=\tan^{-1}t\Rightarrow\frac{du}{dt}=\frac{1}{1+t^2}$ And, $t=\sqrt{x}\Rightarrow\frac{dt}{dx}=\frac{1}{2}x^{\frac{-1}{2}}=\frac{1}{2\sqrt{x}}$ $\frac{dy}{dx}=\left(\frac{dy}{du}\times\frac{du}{dt}\times\frac{dt}{dx}\right)=e^u\left(\frac{1}{1+t^2}\right)\left(\frac{1}{2\sqrt{x}}\right)=e^{\tan^{-1}t}\left(\frac{1}{1+t^2}\right)\left(\frac{1}{2\sqrt{x}}\right)=\frac{e^{\tan^{-1}\sqrt{x}}}{2\sqrt{x}(1+x)}$ Add reason in this step: (Put the value of t) Hence, $\frac{dy}{dx}=\frac{e^{\tan^{-1}\sqrt{x}}}{2\sqrt{x}(1+x)}$

SECTION -V (Five Mark Questions)

36. The rough sketch of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the line $x=0$ and $x=ae$ is shown in the figure.



The required area of shaded region ABCD $=2\int_0^{ae}ydx$ where $y=\frac{b}{a}\sqrt{a^2-x^2}$ $\therefore A=\frac{2b}{a}\int_0^{ae}\sqrt{a^2-x^2}dx=\frac{2b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^{ae}=\frac{2b}{2a}\left[ae\sqrt{a^2-a^2e^2}+a^2\sin^{-1}\frac{ae}{a}\right]-0=\frac{b}{a}\left[ae\sqrt{a^2(1-e^2)}+a^2\sin^{-1}e\right]=ab\left[e\sqrt{1-e^2}+\sin^{-1}e\right]$ sq. units

37. Let h and r be height and radius of the cylinder. Then, its volume $V=\pi r^2h\Rightarrow h=\frac{V}{\pi r^2}$

Now, total surface area $S=2\pi rh+2\pi r^2\Rightarrow S=2\pi r\frac{V}{\pi r^2}+2\pi r^2=\frac{2V}{r}+2\pi r^2$

$\Rightarrow\frac{dS}{dr}=\frac{-2V}{r^2}+4\pi r$

For maxima/minima, $\frac{dS}{dr}=0 \Rightarrow 4\pi r=\frac{2V}{r^2} \Rightarrow r^3=\frac{2V}{4\pi}=\frac{V}{2\pi}$

Also, $\frac{d^2S}{dr^2}=\frac{4V}{r^3}+4\pi$

and $\left(\frac{d^2S}{dr^2}1\right)_{r^3=\frac{V}{2\pi}}=8\pi+4\pi=16\pi>0.$

$\therefore$ Total surface area is minimum when $r^3=\frac{V}{2\pi}$

$\Rightarrow r^3=\frac{\pi r^2 h}{2\pi} \Rightarrow h=2r.$

38. The given planes are $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=1$ $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})-1=0$.......(1) $\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})+4=0$.......(2) The equation of any plane passing through the line of intersection of these planes is $[\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})-1]+\lambda[\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})+4]=0$ $\vec{r}\cdot[(1+2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(1-\lambda)\hat{k}]+(4\lambda-1)=0$.........(3) Let $a_1=1+2\lambda$ $b_1=1+3\lambda$ $c_1=1-\lambda$ $d=4\lambda-1$ The required plane is parallel to x -axis are $<1,0,0>$ The direction ratios of x -axis are $<1,0,0>$ $a_2=1$, $b_2=0$, $c_2=0$ $1(1+2\lambda)+0(1+3\lambda)+0(1-\lambda)=0 \Rightarrow 1+2\lambda=0 \Rightarrow \lambda=\frac{-1}{2}$ Substitute $\lambda=\frac{-1}{2}$ in equation (3), we get $\vec{r}\cdot\left[\left(1+2\left(\frac{-1}{2}\right)\right)i+\left(1+3\left(\frac{-1}{2}\right)\right)\hat{j}+\left(1-\left(\frac{-1}{2}\right)\right)\hat{k}\right]+\left(4\left(\frac{-1}{2}\right)-1\right)=0$ $\vec{r}\cdot\left[(1-1)\hat{i}+\left(1-\frac{3}{2}\right)\hat{j}+\left(1+\frac{1}{2}\right)\hat{k}\right]+(-2-1)=0$ $\vec{r}\cdot\left[0\hat{i}+\frac{-1}{2}\hat{j}+\frac{3}{2}\hat{k}\right]-3=0$ Multiply by -2 on both sides, we get, $\vec{r}\cdot[\hat{j}-3\hat{k}]+6=0$ Put $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ we get $(x\hat{i}+y\hat{j}+z\hat{k})\cdot(0\hat{i}+\hat{j}-3\hat{k})+6=0$ $y-3z+6=0$

OR

As the line passing through the point $(1,2,-4)$ Let the position vector of this point denoted by $\vec{a}=\hat{i}+2\hat{j}-4\hat{k}$

Let the equation of line passing through given point and parallel to vector $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ be $\vec{r}=\vec{a}+\lambda\vec{b}$

$=(\hat{i}+2\hat{j}-4\hat{k})+\lambda\left(b_1\hat{i}+b_2\hat{j}+b_3\hat{k}\right)$...........(1) Given two lines are $\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7}$......(2) And $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$.........(3) Line (1) and (2) are perpendicular to each other. $3b_1-16b_2+7b_3=0$.....(4) Also Line (1) and (3) are perpendicular to each other. $3b_1+8b_2-5b_3=0$......(5) From equation (4) and (5), we get, $\frac{b_1}{(-16)(-5)-8\times 7}=\frac{b_2}{7\times 3-3(-5)}=\frac{b_3}{3\times 8-3(-16)}$ $\frac{b_1}{24}=\frac{b_2}{36}=\frac{b_3}{72}$ $\frac{b_1}{2}=\frac{b_2}{3}=\frac{b_3}{6}$ Direction ratios of $\vec{b}=2\hat{i}+3\hat{j}+6\hat{k}$ are 2,3 and 6 Putting $\vec{b}=2\hat{i}+3\hat{j}+6\hat{k}$ in equation (1), we get $\vec{r}=(i+2\hat{j}-4\hat{k})+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$

SOLUTION- SAMPLE PAPER- XIV

PART -A

SECTION -I (One Mark Questions)

1. Matrix of order 8 can have following possible ordered pairs $1\times8=8$ $8\times1=8$ $2\times4=8$ Or $4\times2=8$ contain 8 elements, a matrix containing 8 elements can have any one of the following orders: $1\times8, 8\times1, 2\times4 or 4\times2$

2. we have $I=\int_{-1}^{1} e^{|x|}dx$ Here, $f(x)=e^{|x|}$ and $f(-x)=e^{|-x|}=e^{|x|}=f(x)$ $\therefore f(x)$ is an even function $\therefore I=\int_{-1}^{1} e^{|x|dx}=2\int_{0}^{1} e^{x}dx=2\left[e^{x}\right]_0^1=2(e-1)$

3. Given a pack of 52 cards, there are 26 red cards. In a pack of 52 cards, there are 26 red cards. If A is the event that a red card is drawn, then $P(A)=\frac{26}{52}=\frac{1}{2}$ After drawing a red card, there are still 25 left in the pack. Let B be the event of drawing a red in the second draw, then $\mathrm{P(B)}=\frac{25}{51}$ since A and B are independent events, $\mathrm{P(A\cap B)}=\mathrm{P(A)P(B)}$ Required probability is $\frac{1}{2}\times\frac{25}{51}=\frac{25}{102}$

4. $P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{\frac{4}{6}}{1}=\frac{4}{6}$

5. $P=\begin{bmatrix}2&1&2\\1&6&-3\end{bmatrix}$ and $Q=\begin{bmatrix}-2&0&4\\5&-3&2\end{bmatrix}$ To find 3P multiply both sides of matrix P by 3 we get, $3P=\begin{bmatrix}6&3&6\\3&18&-9\end{bmatrix}$ Now subtracting Q on both sides, we get, $3P-Q=\begin{bmatrix}6&3&6\\3&18&-9\end{bmatrix}-Q=\begin{bmatrix}6&3&6\\3&18&-9\end{bmatrix}-\begin{bmatrix}-2&0&4\\5&-3&2\end{bmatrix}=\begin{bmatrix}6+2&3-0&6-4\\3-5&18+3&-9-2\end{bmatrix}$ $3P-Q=\begin{bmatrix}8&3&2\\-2&21&-11\end{bmatrix}$

6. Given A and B are mutually independent events. Therefore, $A\cap B=0$ Therefore, $\mathrm{P(A\cap B)}=0$ We know that $\mathrm{P(A)+P(B)-P(A\cap B)}=\mathrm{P(A\cup B)}(1)$ Substitute the values, 0.5 for P(A), x for P(B) and 0.9 for $\mathrm{P(A\cup B)}$ we get (1) becomes $0.5+x-0=0.9$ $x=0.9-0.5=0.4$ $x=0.4$

7. If $A=\begin{bmatrix}1&-2&1\\-2&3&1\\1&1&5\end{bmatrix}$ $|A|=1(15-1)+2(-10-1)+1(-2-3)$ Solve right hand side, we get, $|A|=-13$Now, $\mathrm{A}_{11}=14$, $\mathrm{A}_{12}=11$, $\mathrm{A}_{13}=-5$ $\mathrm{A}_{21}=11$, $\mathrm{A}_{22}=4$, $\mathrm{A}_{23}=-3$ $\mathrm{A}_{31}=-5$, $\mathrm{A}_{32}=-3$, $\mathrm{A}_{33}=-1$ $\mathrm{AdjA}=\begin{bmatrix}14&11&-5\\11&4&-3\\-5&-3&-1\end{bmatrix}$

8. We know that Projection of the vector $\vec{a}=\hat{i}+2\hat{j}+4\hat{k}$ on the vector $\vec{b}=2\hat{i}-\hat{j}+2\hat{k}$ is: $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}=\frac{(\hat{i}+2\hat{j}+4\hat{k})\cdot(2\hat{i}-\hat{j}2\hat{k})}{\sqrt{(2)^2+(-1)^2+(2)}}=\frac{2-2+8}{\sqrt{9}}=\frac{8}{3}$

9. Given: $f(x)=2x^2-3x$, $f(x)=4x-3$..(1) Now $4x-3=0$ This implies, $x=\frac{3}{4}$ Therefore, we have two intervals $\left(-\infty,\frac{3}{4}\right)$ and $\left(\frac{3}{4},\infty\right)$ For interval $\left(\frac{3}{4},\infty\right)$ taking $x=1$ (say), then from eq. (1), $f(x)>0$ Therefore, f is strictly increasing in $\left(\frac{3}{4},\infty\right)$

10. $\sqrt[3]{x^6}=\sqrt[3]{x^{2\times3}}=\sqrt[3]{(x^2)^3}=\left(\left(x^2\right)^3\right)^{\frac{1}{3}}=(x)^2$ Now, substitute $(x)^2$ for $\sqrt[3]{x^6}$ we get $\int\sqrt[3]{x^6}dx=\int x^2dx=\frac{x^3}{3}+c=\frac{1}{3}x^3+c$

11. Direction ratios of $\frac{x-1}{4}=\frac{y-3}{4}=\frac{z-2}{1}$ be $<a_1,b_1,c_1>$ that are $<4,4,1>$ Direction ratios of $\frac{x-5}{4}=\frac{y-2}{1}=\frac{z-3}{8}$ be $<a_2,b_2,c_2>$ that are $<4,1,8>$ Let θ be the angle between the two given lines $\cos\theta=\left|\frac{a_1a_2+b_1b_2+c_1c_2}{(\sqrt{a_1^2+b_1^2+c_1^2})\cdot(\sqrt{a_2^2+b_2^2+c_2^2})}\right|$ Substitute the values of $<4,4,1>$ for $<a_1,b_1,c_1>$ and $<4,1,8>$ for $<a_2,b_2,c_2>$ in (1) we get, $\cos\theta=\left|\frac{4\times4+4\times1+1\times8}{(\sqrt{4^2+4^2+1^2})\cdot(\sqrt{4^2+1^2+8^2})}\right|=\left|\frac{4\times4+4\times1+1\times8}{(\sqrt{4^2+4^2+1^2})\cdot(\sqrt{4^2+1^2+8^2})}\right|$ $=|\frac{28}{\sqrt{33}\sqrt{81}|}=\frac{28}{9\sqrt{33}}$ $\cos\theta=\frac{28}{9\sqrt{33}}$ $\theta=\cos^{-1}\left(\frac{28}{9\sqrt{33}}\right)$

12. The slope of the tangent at $x=2$ is given by $\left[\frac{dy}{dx}\right]_{x=2}=3x^2-1=3(2)^2-1=11$

13. order of differential equation is highest order of diffentiation that is 2 (here) degree of differential equation is highest power of highest order of diffentiation that is 1 Sum of order and degree of differential equation is $2+1=3$

14. Putting $x=\tan\theta$, we get $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)=\tan^{-1}\left[\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right]=\tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right)=\tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)=\tan^{-1}\left[\frac{2\sin^2(\theta/2)}{2\sin(\theta/2)\cos(\theta/2)}\right]=$ $\tan^{-1}\left(\tan\left(\frac{\theta}{2}\right)\right)=\frac{\theta}{2}=\frac{1}{2}\tan^{-1}x \therefore \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)=\frac{1}{2}\tan^{-1}x$

15. $\vec{a}=2\hat{i}-4\hat{j}+5\hat{k} \therefore \lambda\vec{a}=2\lambda\hat{i}-4\lambda\hat{j}+5\lambda\hat{k}$ $|\lambda\vec{a}|=\sqrt{(2\lambda)^2+(4\lambda)^2+(5\lambda)^2}=\sqrt{4\lambda^2+16\lambda^2+25\lambda^2}=\sqrt{45\lambda^2}$ magnitude of unit vector is 1 $1=3\lambda\sqrt{5}\lambda=\frac{1}{3\sqrt{5}}$

16. $\int_2^4 3xdx=\left[\frac{3x^2}{2}\right]_2^4=24-6=18$

SECTION -II (CASE STUDY)

CASE STUDY -I

17. (a) $|A|=15-14=1$

(b) $C_{11}=5; C_{12}=-7; C_{21}=-2; C_{22}=3$

Adj A = $\begin{pmatrix}5 & -2\\ -7 & 3\end{pmatrix}$

(c) $C_{11}=9; C_{12}=-8; C_{21}=-7; C_{22}=6$

Adj B = $\begin{pmatrix}9 & -7\\ -8 & 6\end{pmatrix}$

(d) adjAB = adjB.adjA = $\begin{pmatrix}9 & -7\\ -8 & 6\end{pmatrix}\begin{pmatrix}5 & -2\\ -7 & 3\end{pmatrix}$

= $\begin{pmatrix}94 & -39\\ -82 & 34\end{pmatrix}$

(e) $AB^{-1}=\frac{1}{|AB|}adj(AB)=\frac{-1}{2}\begin{pmatrix}94 & -39\\ -82 & 34\end{pmatrix}$

CASE STUDY : II

18. (a) $y-5=\frac{7-5}{4-2}(x-2)=x-y+3=0$

(b) $y-7=\frac{2-7}{6-4}(x-4)\Rightarrow 5x+2y-34=0$

(c) $y-5=\frac{2-5}{6-2}(x-2)\Rightarrow 3x+4y-26=0$

(d) Required area is = $\int_4^2(x+3)dx+\int_6^4\frac{5x-34}{2}dx-\int_6^2\frac{3x-26}{4}dx$

= 7 sq units

(e) False, we can find the y coordinate of the third vertex as well with the given data.

PART -B

SECTION-III(Each question carries 2 Marks)

19. $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ $\vec{b}=2\hat{i}+4\hat{j}+9\hat{k}$ $\therefore \vec{a}+\vec{b}=(i+2j-3\hat{k})+(2\hat{i}+4\hat{j}+9\hat{k})=3\hat{i}+6\hat{j}+6\hat{k}$ $\therefore$ unit vector parallel to $(\vec{a}+\vec{b})=\frac{3\hat{i}+6\hat{j}+6\hat{k}}{\sqrt{3^2+6^2+6^2}}$ $=\pm\frac{1}{9}(3\hat{i}+6\hat{j}+6\hat{k})=\pm\frac{1}{3}(i+2\hat{j}+2\hat{k})$

20. $2\begin{bmatrix}2&4\\5&x\end{bmatrix}+\begin{bmatrix}0&y\\2&1\end{bmatrix}=\begin{bmatrix}4&0\\12&5\end{bmatrix}$ On solving the matrix on left hand side we get, $\begin{bmatrix}4&8+y\\12&2x+1\end{bmatrix}=\begin{bmatrix}4&0\\12&5\end{bmatrix}$ On equating the elements of both side elements, we get, $y=-8$ $x=2$

21. $\int_0^1\frac{3x^2dx}{1+x^3}$ Put $1+x^3=t$ $3x^2dx=dt$ For $x\rightarrow 1$ then $t\rightarrow 2$ For $x\rightarrow 0$ then $t\rightarrow 1$ $\int_1^2\frac{1}{t}dt=\log t]_1^2=\log 2-\log 1=\log 2-0=\log 2$

OR

$\Rightarrow\int\left[1+\frac{1}{(1+x^2)}-\frac{2}{\sqrt{1-x^2}}+\frac{5}{x\sqrt{x^2-1}}+a^x\right]dx\Rightarrow\int 1dx+\int\frac{1}{(1+x^2)}dx-\int\frac{2}{\sqrt{1-x^2}}dx+\int\frac{5}{x\sqrt{x^2-1}}dx+\int a^xdx\Rightarrow x+\tan^{-1}x-2\sin^{-1}x+5\sec^{-1}x+\frac{a^x}{\ln a}+c$

22. Given points are $A(2,3,4),B(-1,-2,1)$ and $C(5,8,7)$ As we know that the direction cosines of points Direction ratios of $AB(-3,-5,-3)$ Direction ratios of $BC(6,10,6)$ Therefore, Direction ratio of AB = 2X direction ratios of BC This implies they are proportional. Hence, Since point B is common to both ABandBC, point$A$, B and C are collinear.

23. Equation of parabola having vertex at origin and axis along positive x-axis is $y^2=4ax$ (i) Differentiating (i) with respect to x, we get $2yy'=4a$ (ii) Substitute the value of$4a$in equation (i) from equation (ii), we get $y^2=2yy'x\Rightarrow 2xy'-y=0$

24. L.H.S. $=\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{2}{11}=\tan^{-1}\left(\frac{\frac{1}{2}+\frac{2}{11}}{1-\frac{1}{11}}\right)=\tan^{-1}\frac{3}{10}=$ R.H.S. Hence proved

OR

$\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$ $\tan^{-1}\left(\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}-2\sin\frac{x}{2}\cos\frac{x}{2}}\right)$

Divide numerator and denominator by $\cos\frac{x}{2}$ $\tan^{-1}\left(\frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}\right)=\tan^{-1}\left(\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right)$

$=\tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)=\frac{\pi}{4}+\frac{x}{2}$

25. Function $f(x)=\frac{x^2-25}{x-5}, x\neq -5$ For any real number $k\neq -5$ we get $\underset{x\to k}{Lt}f(x)=\underset{x\to k}{Lt}\frac{x^2-25}{x-5}=\underset{x\to k}{Lt}\frac{(x-5)(x+5)}{x-5}=k-5$

And $f(k)=\frac{(k-5)(k+5)}{k-5}=k-5$ since $\underset{x\to k}{\mathrm{Lt}}f(x)=f(k)$

Therefore, $f(x)$ is continuous at every point of domain of f and it is a continuous function.

26. Let $I=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\cos 2x\log\sin x\,dx$ Integrating by parts, we get $=\left[\log\sin x\left(\frac{\sin 2x}{2}\right)\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}-\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{\cos x}{\sin x}\times\frac{\sin 2x}{2}dx$

$$=\frac{1}{2}\left[\log\sin\frac{\pi}{2}(\sin\pi)-\log\sin\frac{\pi}{4}\left(\sin\frac{\pi}{2}\right)\right]-\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{\cos x\cdot 2\sin x\cdot\cos x}{2\sin x}dx$$

$$=\frac{1}{2}\left[0-\log\frac{1}{\sqrt{2}}\right]-\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\cos^2xdx=\frac{1}{4}\log 2-\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\left(\frac{1+\cos 2x}{2}\right)dx=\frac{1}{4}\log 2-\frac{1}{2}\left[x+\frac{\sin 2x}{2}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$=\frac{1}{4}\log 2+\frac{1}{2}\left[\left(\frac{\pi}{2}+0\right)+\left(\frac{\pi}{4}+\frac{1}{2}\right)\right]=\frac{1}{4}\log 2-\frac{\pi}{8}+\frac{1}{4}$$

27. Since marginal revenue is the rate of change of total revenue with respect to the number of units sold, we have Marginal revenue $(MR.) = \frac{dR}{dx} = 6x+36$ When $x=5$, $M.R. = 6(5)+36 = 30+36 = 66$. Hence the required marginal revenue is$Rs.66$.

28. If two dice are thrown, then total number of cases $=36$ Number of cases of total of 9 or 11 $\{(3,6),(4,5),(6,3),(5,4),(6,5),(5,6)\}$ P(total 9 or 11)$=\frac{6}{36}=\frac{1}{6}$ P(neither total of 9 or 11)$=1-P$(total 9 or11) $=1-\frac{1}{6}=\frac{5}{6}$

SECTION -IV (Three Mark Questions)

29. Let $\vec{a}=\vec{\alpha}+\vec{\beta}$ where $\vec{\alpha}$ is parallel to $\vec{b}$ and $\vec{\beta}$ is perpendicular to $\vec{b}$ $\therefore$ $\vec{\alpha}=\lambda\vec{b},\lambda$ is a scalar i.e,. $\vec{\alpha}=\lambda(2\hat{i}+4\hat{j}-2\hat{k})=2\lambda\hat{i}+4\lambda\hat{j}-2\lambda\hat{k}$ and $\vec{\beta}=\vec{a}-\vec{\alpha}=(2\hat{i}-\hat{j}+3\hat{k})-(2\lambda\hat{i}+4\lambda\hat{j}-2\lambda\hat{k}) \Rightarrow \vec{\beta}=(2-2\lambda)\hat{i}-(1+4\lambda)\hat{j}+(3+2\lambda)\hat{k}$ $\vec{\beta}$ is perpendicular to $\vec{b}$ $\therefore$ $\vec{\beta}\cdot\vec{b}=0$
$\Rightarrow ((2-2\lambda)\hat{i}-(1+4\lambda)\hat{j}+(3+2\lambda)\hat{k})\cdot(2\hat{i}+4\hat{j}-2\hat{k})=0 \Rightarrow 2(2-2\lambda)-4(1+4\lambda)-2(3+2\lambda)=0 \Rightarrow 4-4\lambda-4-16\lambda-6-4\lambda.=0$
$\Rightarrow 24\lambda=-6 \Rightarrow \lambda=\frac{-1}{4}$ $\therefore$ $\vec{\alpha}=\frac{-1}{4}(2\hat{i}+4\hat{j}-2\hat{k})=\frac{-1}{2}\hat{i}-\hat{j}+\frac{1}{2}\hat{k}$ and $\vec{\beta}=\left(2+\frac{1}{2}\right)\hat{i}-(1-1)\hat{j}+\left(3-\frac{1}{2}\right)\hat{k}=\frac{5}{2}\hat{i}+\frac{5}{2}\hat{k}=\frac{5}{2}(\hat{i}+\hat{k})$

30. R is reflexive if $(\mathrm{A},\mathrm{A})\varepsilon\mathrm{R}$ For any $\mathrm{A}\varepsilon\mathrm{R}$ $d(\mathrm{A},\mathrm{A})=0$, which is less than 3 units. This implies, $(\mathrm{A},\mathrm{A})\varepsilon\mathrm{R}$
Thus, R is reflexive. R is symmetric if $(\mathrm{A},\mathrm{B})\varepsilon\mathrm{R}$ then this implies $(\mathrm{B},\mathrm{A})\varepsilon\mathrm{R}$ $d(\mathrm{A},\mathrm{B})<3$ units $d(\mathrm{B},\mathrm{A})<3$ units this implies, $(\mathrm{B},\mathrm{A})\varepsilon\mathrm{R}$
Thus, R is symmetric. R is transitive if $(\mathrm{A},\mathrm{B})\varepsilon\mathrm{R}$ and $(\mathrm{B},\mathrm{C})\varepsilon\mathrm{R} \Rightarrow (\mathrm{A},\mathrm{C})\varepsilon\mathrm{R}$
Consider points $\mathrm{A}(0,0)$, $\mathrm{B}(1.5,0)$ and $\mathrm{C}(3.2,0)$ $d(\mathrm{A},\mathrm{B})=1.5$ units <3 units and $d(\mathrm{B},\mathrm{C})=1.7$ units <3 units and $d(\mathrm{A},\mathrm{C})=3.2$ units 3 this implies, (A,B) e R and $(\mathrm{B},\mathrm{C})\varepsilon\mathrm{R}$, this implies, $(\mathrm{A},\mathrm{C})\notin\mathrm{R}$
Thus, R is not transitive. Thus, R is reflexive, symmetric but not transitive.

31. Given $f(x)=\begin{cases} kx^2, \text{if} x\le 2 \\ 3, \text{if} x>2 \end{cases}$ At $x=2$...........(1)
Given that f is defined at $x=2$ and $f(2)=k(2)^2=4k$ Also, given that $f(x)$ is continuous at $x=2$ This implies, $\lim_{x\to 2^-} f(x)=\lim_{x\to 2^+} f(x)=f(2)$....(2) Put the values in (2) from defined function $f(x)$(1), we get, $\Rightarrow \lim_{x\to 2^-}\left(kx^2\right)=\lim_{x\to 2^+}(3)=4k \Rightarrow k\times 2^2=3=4k \Rightarrow 4k=3=4k$ or$4k=3$
$k=\frac{3}{4}$

OR

Let $u=x^{x^2-3}$ and $v=(x-3)^{x^2}$ Therefore, $y=u+v$ Differentiating w.r.t x we get, $\frac{dy}{dx}=\frac{du}{dx}+\frac{dv}{dx}$.(1) $u=x^{x^2-3}$ $\log u=x^2-3\log x$
Differentiating w.r.t x we get, $\frac{1}{u}\frac{du}{dx}=\frac{x^2-3}{x}+2x\log x\frac{du}{dx}=u\left(\frac{x^2-3}{x}+2x\log x\right)$ $\frac{du}{dx}=x^{x^2-3}\left(\frac{x^2-3}{x}+2x\log x\right)$
Also, $v=(x-3)^{x^2}$ $\log v=\log(x-3)^{x^2}$ $\log v=x^2\log(x-3)$ Differentiating w.r.t x we get, $\frac{1}{v}\frac{dv}{dx}=2x\log(x-3)+\frac{1}{x-3}\left(x^2\right)\frac{dv}{dx}=(x-3)^{x^2}\left(2x\log(x-3)+\right.$
Now, $\frac{dy}{dx}=\frac{du}{dx}+\frac{dv}{dx}$ Substitute the values of $\frac{du}{dx}$ and $\frac{dv}{dx}$ in (1) and solve it $\frac{dy}{dx}=x^{x^2-3}\left(\frac{x^2-3}{x}+2x\log x\right)+(x-3)^{x^2}\left(2x\log(x-3)+\frac{x^2}{x-3}\right)$

32. $\int\frac{1+\sin 2x}{1+\cos 2x}e^{2x}dx=\frac{1}{2}\int\frac{1+\sin t}{1+\cos t}e^t dt$ (put $2x=t$)
$\frac{1}{2}\int\frac{1}{2\cos^2\frac{t}{2}}+\frac{2\sin\frac{t}{2}\cos\frac{t}{2}}{2\cos^2\frac{t}{2}}e^t dt$ $\frac{1}{2}\int\left(\frac{1}{2}\sec^2\frac{t}{2}+\tan\frac{t}{2}\right)e^t dt$ $\tan\frac{t}{2}=f(t)$ then
$\sec^2\frac{t}{2}=f'(t)$ Using, $\int\left(f(t)+f'(t)\right)e^t dt=f(t)e^t+c$ $I=\frac{1}{2}\tan\frac{t}{2}e^t+C=\frac{1}{2}\tan xe^{2x}+C$

33. We have, $y+x\sin\left(\frac{y}{x}\right)=x\frac{dy}{dx} \Rightarrow \frac{dy}{dx}=\frac{y}{x}+\sin\left(\frac{y}{x}\right)$
Put $y=vx$ and $\frac{dy}{dx}=v+x\frac{dv}{dx}$ $\therefore$ $v+x\frac{dv}{dx}=v+\sin v \Rightarrow x\frac{dv}{dx}=\sin v$
$\Rightarrow \frac{dv}{\sin v}=\frac{dx}{x} \Rightarrow \int\frac{dv}{\sin v}=\int\frac{dx}{x}$
$\Rightarrow \int \cos ec\, v\, dv=\int\frac{dx}{x}$
$\Rightarrow \log|cosec\, v-\cot v|=1og\, x+\log C \Rightarrow cosec\frac{y}{x}-\cot\frac{y}{x}=Cx$

34. Let $I = \int_0^{\frac{\pi}{2}} \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx$

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{2\sin x cos x dx}{\sin^4 x + \cos^4 x}$

Dividing numerator and denominator by $\cos^4 x$, we get $I = \int_0^{\frac{\pi}{2}} \frac{2\tan x \sec^2 x dx}{\tan^4 x + 1}$

Put $\tan^2 x = t \Rightarrow 2\tan x sec^2 x dx = dt$

When $x = 0, t = 0$ and when $x = \frac{\pi}{2}, t = \infty \therefore I = \int_0^{\infty} \frac{dt}{t^2+1} = \left[\tan^{-1} t\right]_0^{\infty} = \frac{\pi}{2} - 0 = \frac{\pi}{2}$

35. Equation of a line passing through the points $(x_1, y_1, z_1)\,(x_2, y_2, z_2)$ is

$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$

Equation of $Y - Z$ plane is $x = 0$

Here, the line passing through the points A$(9,5,4)$ and B$(3,1,6)$ is given by, Substituting for $(x_1, y_1, z_1)\,(x_2, y_2, z_2)$ as A$(9,5,4)$ and B$(3,1,6)$ we get, $\frac{x-9}{3-9} = \frac{y-5}{1-5} = \frac{z-4}{6-4}$........(1)

Equate (1) to k $\frac{x-9}{-6} = \frac{y-5}{-4} = \frac{z-4}{+2} = k$ We will get $x = 9 - 6k$; $y = -4k + 5$; $z = 4 + 2k$ Now, in $Y - Z$ plane $x = 0$ $9 - 6k = 0 \Rightarrow k = \frac{9}{6} = \frac{3}{2}$

Substituting the value of k in $(9 - 6k, 5 - 4k, 4 + 2k)$ to find the coordinates. $\left(9 - 6\left(\frac{3}{2}\right), 5 - 4\left(\frac{3}{2}\right), 4 + 2\left(\frac{3}{2}\right)\right) = (9 - 9, 5 - 6, 4 + 3)$ $= (0, -1, 7)$

SECTION -V (Five Mark Questions)

36. Let a cylinder be inscribed in a cone of radius R and height h. Let the cylinder's radius be r and its height be h_1.

In the given figure,

DB =R, DE =r, Height of cone = h, Height of cylinder = h_1

$\therefore \frac{AI}{AD} = \frac{GI}{BD} \Rightarrow \frac{h - h_1}{h} = \frac{r}{R}$

$\Rightarrow r = \frac{R}{h}(h - h_1)$

Volume (V) of the cylinder $= \pi r^2 h_1$

$\Rightarrow V = \pi \frac{R^2}{h^2}(h - h_7)^2 h_1 \Rightarrow V = \pi \frac{R^2}{h^2}(h^2 + h^2 - 2hh_\eta)h_1$

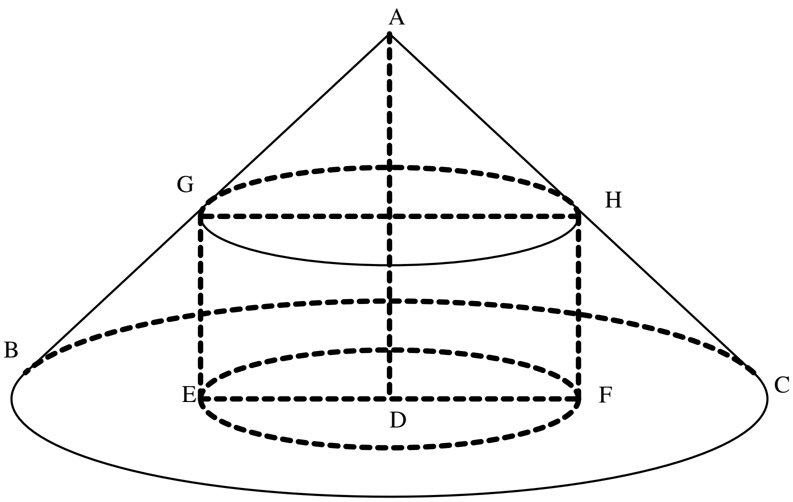
$\frac{dV}{dh_7} = \frac{\pi R^2}{h^2}\left[(0 + 2h_1 - 2h)h_1 + (h^2 + h_1^2 - 2hh_1)\right]$

$= \frac{\pi R^2}{h^2}\left[2h_1^2 - 2hh_1 + k^2 + h_1^2 - 2hh_1\right] = \frac{\pi R^2}{h^2} - [h^2 + 3h_{12} - 4hh_1]$

For maxima/minima, $\frac{dV}{dh_1} = 0$

$\Rightarrow 3h_1(h_1 - h\rangle - h(h_1 - h) = 0 \Rightarrow (h_1 - h)(3h_1 - h) = 0 \Rightarrow h_1 = h, h_1 = \frac{h}{3}$

It can be noted that if $h_1 = h$, then the cylinder cannot be inscribed in the cone.



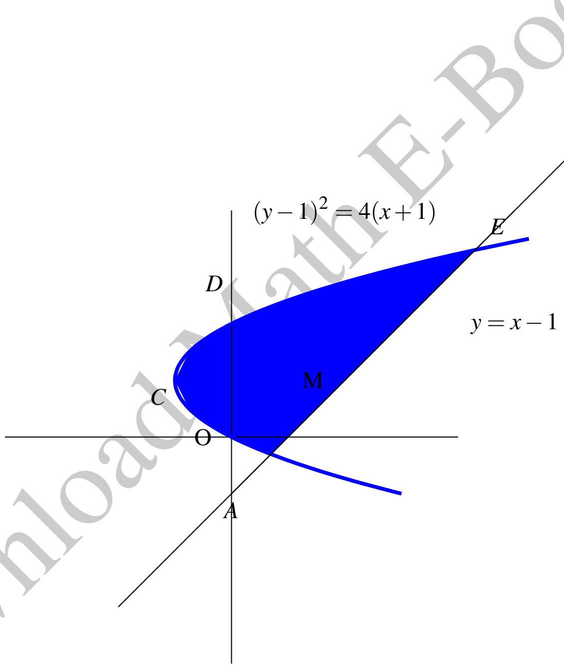
$$\frac{d^2V}{dh_1^2} = \frac{\pi R^2}{h^2}[0+6h_1-4h] = \frac{\pi R^2}{h^2}[6h-4h]$$

$$\frac{d^2V}{dh_1^2}\left(_{h_1=\frac{h}{3}}\right) = \frac{\pi R^2}{h^2}\left[\frac{6h}{3}-4h\right] = \frac{-2\pi R^2}{h} < 0$$

So, the volume of the cylinder is maximum when $h_1 = \frac{h}{3}$. Hence, the height of the cylinder of the maximum, volume that can be inscribed in a cone of height h is $\frac{1}{3}h$.

37. We have, $y = x-1$ (i) and $(y-1)^2 = 4(x+1)$ (ii) Equation (ii) represents a parabola whose vertex is $(-1,1)$ and intersects x-axis at $\left(\frac{-3}{4},0\right)$ and y-axis at $(0,3)$ and $(0,\ -1)$. Solving (i) & (ii) , we get their points of intersection as $(0,\ -1)$ and $(8,\ 7)$ The rough sketch of the parabola and line is shown in the figure.

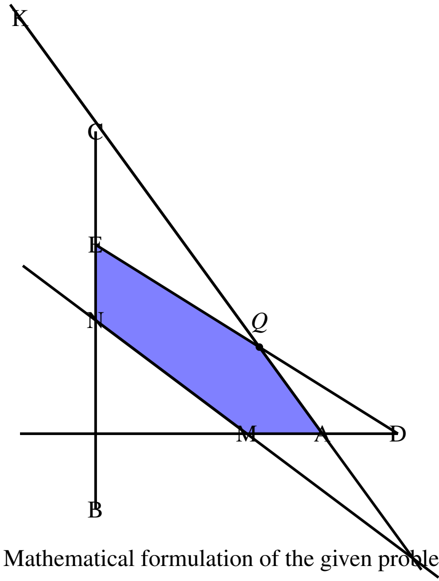
The required area of shaded region



$= \int_{-1}^{7} (x_I - x_2)dy$ [Taking horizontal strips] where $x_1 = y+1$ and $x_2 = \frac{(y-1)^2}{4} - 1 = \int_{-1}^{7}\left(y - \frac{(y-1)^2}{4} + 2\right)dy \Rightarrow A = \left[\frac{y^2}{2} - \frac{(y-1)^3}{12} + 2y\right]_{-1}^{7}$

$= \left[\frac{49}{2} - \frac{216}{12} + 14\right] - \left[\frac{1}{2} + \frac{8}{12} - 2\right] = \frac{64}{3}$ sq. units

38. Let x and y be the number items of M and N respectively. Total profit on the production $= Rs.(600x+400y)$



Mathematical formulation of the given problem is as follows : Maximize $Z = 600x + 400y$ subject to the constraints : $x + 2y \leq 12$ (constraint on Machine I) ...(i) $2x + y \leq 12$ (constraint on Machine II) ...(ii) $x + \frac{5}{4}y \geq 5$ (constraint on Machine III) ...(iii) $x \geq 0, y \geq 0$ ...(iv)

Let us draw the graph of constraints (i) to (iv). AQENM is the feasible region (shaded) as shown in figure determined by the constraints (i) to (iv). Observe that the feasible region is bounded and coordinates of the comer points M,A, Q , E, N are (5, 0), (6, 0), (4, 4), (0, 6) and (0, 4) respectively. Let us evaluate $Z = 600x + 400y$ at these comer points.

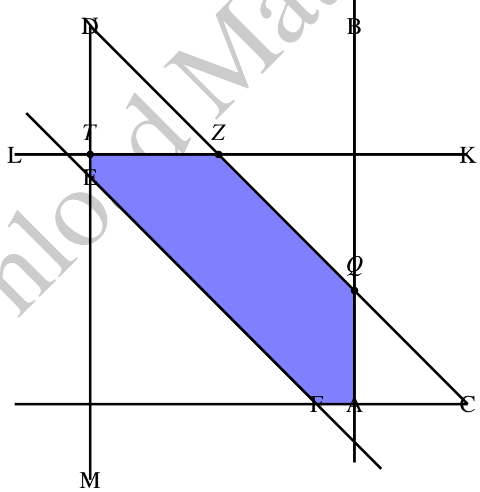
Corner point	$Z = 600x + 400y$
(5, 0)	3000
(6, 0)	3600
(4, 4)	4000 $\rightarrow$ maximum
(0, 6)	2400
(0, 4)	1600

We see that the point (4, 4) is giving the maximum value of Z. Hence, the manufacturer has to produce 4 units of each item to get the maximum profit of Rs.4000.

OR

The problem can be explained diagrammatically as follows (figure).

Let x units and y units of the commodity be transported from the factory at P to the depots at A and B respectively. Then, $(8 - x - y)$ units will be transported to depot at C. Hence, we have $x \geq 0, y \geq 0$ and $8 - x - y \geq 0$ i.e. $x \geq 0, y \geq 0$ and $x + y \leq 8$ Now, the weekly requirement of the depot at A is 5 units of the commodity. Since x units are transported from the factory at P, the remaining $(5 - x)$ units need to be transported from the factory at Q. Obviously, $5 - x \geq 0$, i.e., $x \leq 5$. Similarly, $(5 - y)$ and $6 - (5 - x + 5 - y) = x + y - 4$units are to be transported from the factory Q to the depots at B and C respectively. Thus, $5 - y \geq 0, x + y - 4 \geq 0$ i.e. $y \leq 5$, $x + y \geq 4$ Total transportation cost Z is given by $Z = 160x + 100y + 100(5 - x) + 120(5 - y) + 100x(x + y - 4) + 150(8 - x - y) = 10(x - 7y + 190)$ Therefore, the problem reduces to Minimize $Z = 10(x - 7y + 190)$ subject to the constraints : $x \geq 0, y \geq 0$.(i) $x + y \leq 8$ ...(ii) $x \leq 5$ ...(iii) $y \leq 5$ ...(iv) and $x + y \geq 4$ ...(v)



The shaded region AQZTEF represented by the constraints (i) to (v) is the feasible region (figure).

Observe that the feasible region is bounded. The coordinates of the comer points of the feasible region are (0, 4), (0, 5), (3, 5), (5, 3), (5, 0) and (4, 0). Let us evaluate Z at these points.

Corner Point	$Z = 10(x - 7y + 190)$
(0, 4)	1620
(0, 5)	1550 $\rightarrow$ minimum
(3, 5)	1580
(5, 3)	1740
(5, 0)	1950
(4, 0)	1940

From the table, we see that the minimum value of Z is 1550 at the point (0, 5). Hence, the optimal transportation strategy will be to deliver 0, 5 and 3 units from the factory at P and 5, 0 and 1 units from the factory at Q to the depots at A,B and C respectively. Corresponding to this strategy, the transportation cost would be minimum, i.e. Rs. 1550.

SOLUTION- SAMPLE PAPER- XV

PART -A

SECTION -I (One Mark Questions)

1. We know that the range of principal value branch of $\tan^{-1}$ and $\sec^{-1}$ are $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $[0, \pi]-\left\{\frac{\pi}{2}\right\}$ respectively.
   Let $\tan^{-1}(\sqrt{3}) = x \Rightarrow \sqrt{3} = \tan x$, *then*, $\sqrt{3} = \tan\left(\frac{\pi}{3}\right)$, *where* $\frac{\pi}{3} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
   Let $\sec^{-1}(-2) = y \Rightarrow -2 = \sec y$ Then, $-2 = -\sec\left(\frac{\pi}{3}\right) = \sec\left(\pi - \frac{\pi}{3}\right) = \sec\frac{2\pi}{3}$,
   Where $\frac{2\pi}{3} \in [0, \pi]-\left\{\frac{\pi}{2}\right\} \therefore \tan^{-1}\sqrt{3} - \sec^{-1}(-2) = \frac{\pi}{3} - \frac{2\pi}{3} = \frac{\pi}{3}$.
2. we have, $I = \int_0^2 e^{3-4x}dx = \left[\frac{e^{3-4x}}{-4}\right]_0^2 = \frac{1}{4}\left[e^{3-8} - e^{3-0}\right] = \frac{-1}{4}\left[e^{-5} - e^3\right]$
3. The equations of the line passing through $A(-1,3,2)$ and $B(-4,2,-2)$ are $\frac{x+1}{-4+1} = \frac{y-3}{2-3} = \frac{z-2}{-2-2}$ $\frac{x+1}{-3} = \frac{y-3}{-1} = \frac{z-2}{-4}$ $\frac{x+1}{3} = \frac{y-3}{1} = \frac{z-2}{4}$........(1)
   If the points$A(-1,3,2)$, $B(-4,2,-2)$ and $C(5,5,\alpha)$ are collinear, then the coordinates of C must satisfy equation (1). Therefore, $\frac{5+1}{3} = \frac{5-3}{1} = \frac{\alpha-2}{4}$ $2 = 2 = \frac{\alpha-2}{4}$ $\frac{\alpha-2}{4} = 2$ $\alpha = 10$
4. Given, the expression is $\begin{vmatrix} \cos 70^\circ & \sin 20^\circ \\ \sin 70^\circ & \cos 20^\circ \end{vmatrix}$ Now,
   $\begin{vmatrix} \cos 70^\circ & \sin 20^\circ \\ \sin 70^\circ & \cos 20^\circ \end{vmatrix} = \cos 70^\circ \cos 20^\circ - \sin 70^\circ \sin 20^\circ = \cos(70^\circ + 20^\circ)$
   $= \cos 90^\circ = 0$
5. Given, the equation of the line is $\frac{x-2}{3} = \frac{2y+1}{2} = \frac{5-z}{1}$
   These equations can be re-written as $\frac{x-2}{3} = \frac{y+\frac{1}{2}}{1} = \frac{z-5}{-1}$ Clearly, direction ratios of this line are proportional to $3,1,-1$ So, the direction ratios of the parallel line are also proportional to $3,1,-1$
   The required line passes through $(1,-1,0)$ and its direction ratios are proportional to $3,1,-1$ So, its equations are: $\frac{x-1}{3} = \frac{y+1}{1} = \frac{z-0}{-1}$
6. We have, $\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}, \vec{b} = 2\hat{i} - 4\hat{j} - 3\hat{k}$ and $\vec{c} = \hat{i} + 2\hat{j} - \hat{k}$
   Now, $3\vec{a} - 2\vec{b} + 4\vec{c} = 3(3\hat{i} - 4\hat{j} - 4\hat{k}) - 2(2\hat{i} - 4\hat{j} - 3\hat{k}) + 4(\hat{i} + 2\hat{j} - \hat{k})$
   $\Rightarrow 3\vec{a} - 2\vec{b} + 4\vec{c} = 9\hat{i} + 4\hat{j} - 10\hat{k}$
   or $|3\vec{a} - 2\vec{b} + 4\vec{c}| = \sqrt{9^2 + 4^2 + (-10)^2} = \sqrt{81 + 16 + 100} = \sqrt{197}$
7. Given, $P(A) = 0.8$, $P(B) = 0.5$ and $P\left(\frac{B}{A}\right) = 0.4$
   We have, $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$ $P(A \cap B) = P\left(\frac{B}{A}\right)P(A)$ $P(A \cap B) = 0.4 \times 0.8$ $P(A \cap B) = 0.32$
   Now, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$ $P\left(\frac{A}{B}\right) = \frac{0.32}{0.5}$ $P\left(\frac{A}{B}\right) = 0.64$
8. Given, the expression is $\int_0^2 \sqrt{6x+4}dx$ Let, $6x+4 = u$ $\frac{d}{dx}(6x+4) = \frac{du}{dx}$ $6dx = du$ When, $x = 0, u = 4$ $x = 2, u = 16$ $\int_0^2 \sqrt{6x+4}dx = \frac{1}{6}\int_4^{16} \sqrt{u}du$
   $= \frac{1}{6}\left[\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right]_4^{16} = \frac{1}{6} \times \frac{2}{3}\left[16^{\frac{3}{2}} - 4^{\frac{3}{2}}\right]$
   $= \frac{56}{9}$

9. The equation of the plane parallel to the plane $\vec{r}\cdot(2\hat{i}-\hat{j}+2\hat{k})-5=0$ is $\vec{r}\cdot(2\hat{i}-\hat{j}+2\hat{k})=d$......(1)
If it passes through $\hat{i}+\hat{j}+\hat{k}$ then $(\hat{i}+\hat{j}+\hat{k})\cdot(2\hat{i}-\hat{j}+2\hat{k})=d2-1+2=d$ d = 3
Putting d = 3 in equation (1) $\vec{r}\cdot(2\hat{i}-\hat{j}+2\hat{k})=3\vec{r}\cdot(2\hat{i}-\hat{j}+2\hat{k})-3=0$

10. let, the events be $A=$ The bulb produced is red B $=$ The bulb produced is defective.

Given, $P(A)=\frac{10}{100}=\frac{1}{10}$ And $\mathrm{P(A\cap B)}=\frac{2}{100}=\frac{1}{50}$ Therefore, required probability $=\mathrm{P}\left(\frac{\mathrm{B}}{\mathrm{A}}\right)=\frac{P(A\cap B)}{P(A)}=\frac{\frac{1}{50}}{\frac{1}{20}}=\frac{1}{5}$

11. we have, $I=\int_{-\pi}^{\pi}x^{10}\sin^{7}x\,dx$ Let $f(x)=x^{10}\sin^{7}x$

And $f(-x)=(-x)^{10}[\sin(-x)]^{7}=-x^{10}\sin^{7}x=-f(x)\therefore f(x)$ is an odd function. $\therefore I=\int_{-\pi}^{\pi}x^{10}\sin^{7}x\,dx=0$

12. Given function $f(x)=\begin{cases}\left(x^{3}-x^{2}+2x-2\right), & \text{if} x\neq 1\\ 4, & \text{if} x=0\end{cases}$ Left hand limit at $x=1$:
$\lim_{x\to1^{-}}f(x)=\lim_{x\to1^{-}}\left(x^{3}-x^{2}+2x-2\right)=\lim_{h\to0}\left\{(1-h)^{3}-(1-h)^{2}+2(1-h)-2\right\}$
$=\lim_{h\to0}(1-h)^{3}-\lim_{h\to0}(1-h)^{2}+2\lim_{h\to0}(1-h)-2=1-1+2-2=0$ Right hand limit at $x=1$: $\lim_{x\to1^{+}}f(x)=\lim_{x\to1^{+}}\left(x^{3}-x^{2}+2x-2\right)=$ $\lim_{h\to0}\left\{(1+h)^{3}-(1+h)^{2}+2(1+h)-2\right\}=\lim_{h\to0}(1+h)^{3}-\lim_{h\to0}(1+h)^{2}+2\lim_{h\to0}(1+h)-2=1-1+2-2=0$ but $f(0)$ is not equal to zero, so the point of discontinuity is zero.

13. Given that, 5A $=\begin{bmatrix}5&10&-15\\2&3&4\\1&0&-5\end{bmatrix}$ 5A $=\begin{bmatrix}5&10&-15\\2&3&4\\1&0&-5\end{bmatrix}$ $A=\begin{bmatrix}\frac{5}{5}&\frac{10}{5}&\frac{-15}{5}\\\frac{2}{5}&\frac{3}{5}&\frac{4}{5}\\\frac{1}{5}&\frac{0}{5}&\frac{-5}{5}\end{bmatrix}$

$A=\begin{bmatrix}1&2&-3\\\frac{2}{5}&\frac{3}{5}&\frac{4}{5}\\\frac{1}{5}&0&-1\end{bmatrix}$

14. $adjAB=adjB\cdot adjA\Rightarrow adj(AB)=\begin{bmatrix}1&-2\\-3&1\end{bmatrix}\begin{bmatrix}2&3\\4&-1\end{bmatrix}\Rightarrow adj(AB)=\begin{bmatrix}-6&5\\-2&-10\end{bmatrix}$

15. We have, $\vec{a}=2\hat{i}+2\hat{j}-\hat{k}$ and $\vec{b}=6\hat{i}-3\hat{j}+2\hat{k}$ Now, $\vec{a}\cdot\vec{b}=(2\hat{i}+2\hat{j}-\hat{k})\cdot(6\hat{i}-3\hat{j}+2\hat{k})=12-6-2=4$

16. $\begin{vmatrix}x+1&x\\x&x+1\end{vmatrix}=(x+1)^{2}-x^{2}=x^{2}+2x+1-x^{2}=2x+1$

SECTION -II (CASE STUDY)

CASE STUDY -I

17. (a) $\int\frac{2^{x}+6^{x}}{5^{x}}dx=\int(\frac{2}{5})^{x}+(\frac{6}{5})^{x}dx$
$=\frac{(2/5)^{x}}{log(2/5)}+\frac{(6/5)^{x}}{log(6/5)}+C$

(b) $\int cos^{2}xsin^{3}xdx=\int cos^{2}x(1-cos^{2}x)sinxdx$
let cosx =t $\Rightarrow -sinxdx=dt$
$=\int t^{2}(1-t^{2})dt$
$\frac{t^{3}}{3}-\frac{t^{5}}{5}+C$

(c) logx = t $\Rightarrow\frac{1}{x}dx=dt$
$\int t^{3}dt=\frac{t^{4}}{4}+C=\frac{(logx)^{4}}{4}+C$

(d) tanx = t $\Rightarrow sec^2xdx = dt$

$\int tan^3xsec^2xdx = \int t^3dt$

$= \frac{t^4}{4} + C$

$= \frac{tan^4x}{4} + C$

(e) $\int cos^4xsin^3xdx = \int cos^4x(1-cos^2x)sinxdx$

let $cosx = t \Rightarrow -sinxdx = dt$

$= -\int t^4(1-t^2)dt$

$= -\frac{t^5}{5} + \frac{t^7}{7} + C$

$= -\frac{cos^5x}{5} + \frac{cos^7x}{7} + C$

CASE STUDY : II

18. (a) Value of $a_{23} = 3$

(b) Sum = $1+4+4 = 9$

(c) $C_{11} = 7; C_{12} = -1; C_{13} = -1$

$C_{21} = -3; C_{22} = 1; C_{23} = 0 C_{31} = -3; C_{32} = 0; C_{33} = 1$

therefore cofactor matrix = $\begin{pmatrix} 7 & -1 & -1 \\ -3 & 1 & 0 \\ -3 & 0 & 1 \end{pmatrix}$

(d) We know that adjoint of matrix = $[cofactormatrix]^T$

= $\begin{pmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$

(e) Inverse of the matrix = $A^{-1} = \frac{1}{|A|}adj.A$

= $\begin{pmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$

PART -B

SECTION-III(Each question carries 2 Marks)

19. Let, θ be the angle between $\vec{a}$ and $\vec{b}$ We have, $|\vec{a}| = \sqrt{3}, |\vec{b}| = 2$ and $\vec{a}\cdot\vec{b} = \sqrt{6}$ Therefore, $\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|}$ $\cos\theta = \frac{\sqrt{6}}{\sqrt{3}\cdot 2}$ $\cos\theta = \frac{1}{\sqrt{2}}$ $\theta = \frac{\pi}{4}$

20. Given curve is $y^2 = 4ax$ Differentiate $y^2 = 4ax$ with respect to x $2y\frac{dy}{dx} = 4a$ mat $\left(\frac{a}{m^2}, \frac{2a}{m}\right) = m$

Tangent: $y - b = m(x-a)$ Substitute $\frac{a}{m^2}$ for a and $\frac{2a}{m}$ for b in the above equation. $y - \frac{2a}{m} = m\left(x - \frac{a}{m^2}\right)$ $m^2x - my + a = 0$

Normal: $y - b = \frac{-1}{m}(x-a)$ Substitute $\frac{a}{m^2}$ for a and $\frac{2a}{m}$ for b in the above equation. $y - \frac{2a}{m} = \frac{-1}{m}\left(x - \frac{a}{m^2}\right)$ $m^3y + m^2x - 2am^2 - a = 0$

21. Let, $I = \int_1^2 \frac{3x}{9x^2-1}dx$ Let, $t = 9x^2 - 1 \Rightarrow 18xdx = dt$ When $x = 1, t = 8$ and for $x = 2, t = 35$ So, $I = \int_8^{35} \frac{3}{18t}dt$

$= \frac{1}{6}\int_8^{35}\frac{1}{t}dt = \frac{1}{6}|\log_e t|_8^{35} = \frac{1}{6}(\log_e 35 - \log_e 8)$

22. Let, I $= \int \tan^{-1}\sqrt{\frac{1-\sin x}{1+\sin x}}dx$ $I = \int \tan^{-1}\sqrt{\frac{(1-\sin x)^2}{1-\sin^2 x}}dx = \int \tan^{-1}\sqrt{\frac{(1-\sin x)^2}{\cos^2 x}}dx$

$$= \int \tan^{-1}\left(\frac{1-\sin x}{\cos x}\right) dx = \int \tan^{-1}\left(\frac{1-\cos\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)}\right) dx$$

$$I = \int \tan^{-1}\left(\frac{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right) dx$$

$$= \int \tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right) dx = \int \left(\frac{\pi}{4}-\frac{x}{2}\right) dx = \frac{\pi}{4}x - \frac{x^2}{4} + c$$

OR

Let, I $= \int (2x+9)^5 dx$ Assume $2x+9=t \Rightarrow 2dx = dt$ $I = \int t^5 \frac{dt}{2} = \frac{1}{2}\int t^5 dt = \frac{1}{2}\frac{t^6}{6} + c = \frac{(2x+9)^6}{12} + c$

23. Let, $I = \int \left(9\sin x - 7\cos x - \frac{6}{\cos^2 x} + \frac{2}{\sin^2 x} + \cot^2 x\right) dx$ $I = \int \left(9\sin x - 7\cos x - \frac{6}{\cos^2 x} + \frac{2}{\sin^2 x} + \cot^2 x\right) dx$

$= \int 9\sin x dx - \int 7\cos x dx - \int \frac{6}{\cos^2 x} dx + \int \frac{2}{\sin^2 x} dx + \int \cot^2 x dx = -9\cos x - 7\sin x - \int 6\sec^2 x dx + \int 2\csc^2 x dx + \int \left(\csc^2 x - 1\right) dx$

$= -9\cos x - 7\sin x - 6\tan x - 3\cot x - x + c$

24. $\frac{dy}{dx} = 1 - x + y - xy$

Given that, $\frac{dy}{dx} = 1 - x + y - xy$ $\frac{dy}{dx} = 1 - x + y - xy = 1 - x + y(1-x) = (1+y)(1-x)$

By rearranging the terms we get :

$\frac{dy}{1+y} = (1-x)dx$

Integrate both sides of the above equation. $\int \frac{dy}{1+y} = \int (1-x)dx + c$ $\log|1+y| = x - \frac{x^2}{2} + c_o$

25. $= \sin\left\{\frac{\pi}{2} - \left(-\frac{\pi}{3}\right)\right\} = \sin\left\{\frac{\pi}{2} + \frac{\pi}{3}\right\} = \sin\left(\frac{5\pi}{6}\right) = \sin\left(\pi - \frac{\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

OR

Given that, $\sin^{-1}(2x\sqrt{1-x^2}) = 2\sin^{-1}x$ LHS $= \sin^{-1}(2x\sqrt{1-x^2})$..........(1) Let, $x = \sin\mathrm{A}$ Substitute sin A for x in the equation (1) LHS $=$ $\sin^{-1}(2\sin\mathrm{A}\sqrt{1-(\sin\mathrm{A})^2}) = \sin^{-1}(2\sin A \times \cos A) = \sin^{-1}(\sin 2A) = 2\mathrm{A}$

From equation (2)$\mathrm{A} = \sin^{-1}x$ $2\mathrm{A} = 2\sin^{-1}x$=R.H.S.

26. $\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$

We have, $\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right) = \tan^{-1}\left(\frac{\sin x}{1+\cos x}\right) = \tan^{-1}\left(\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right) = \tan^{-1}\left(\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}\right)$

$= \tan^{-1}\left(\tan\frac{x}{2}\right) = \frac{x}{2}$

Differentiate with respect to x. $\Rightarrow \frac{d\left(\frac{x}{2}\right)}{dx} \Rightarrow \frac{1}{2}\frac{dx}{dx} \Rightarrow \frac{1}{2}$

27. Given that, $y = x^3 - 2x + 7$ $\frac{dy}{dx} = \frac{d}{dx}\left(x^3 - 2x + 7\right) = 3x^2 - 2$ m at $(1,6) = 1$

Tangent: $y - b = m(x-a)$ $y - 6 = 1(x-1)$ $x - y + 5 = 0$

Normal: $y - b = \frac{-1}{m}(x-a)$ $y - 6 = -1(x-1)$ $x + y - 7 = 0$

OR

$x=1-a\sin\theta$ Differentiate x with respect to θ $\frac{dx}{d\theta}=0-a\cos\theta=-a\cos\theta$

$y=b\cos^2\theta$ differentiate y with respect to θ $\frac{dy}{d\theta}=2b\cos\theta(-\sin\theta)=-2b\cos\theta\sin\theta$

Dividing equations (2) by (1) $\frac{dy}{dx}=\frac{-2b\cos\theta\sin\theta}{-a\cos\theta}=\frac{2b\sin\theta}{a}$

At $\theta=\pi/2$, $\frac{dy}{dx}=\frac{2b\sin\frac{\pi}{2}}{a}=\frac{2b}{a}$

Now, slope of normal $=\frac{-1}{\text{slopeoftangent}}=\frac{-1}{\frac{2b}{a}}=\frac{-a}{2b}$

28. since $\sin(0)=\sin(\pi)$ but $0\neq\pi$ Therefore, f is not one-one
Now, Range of $f=[-1,1]\neq$ R Therefore, f is not onto.

SECTION -IV (Three Mark Questions)

29. We have, $I=\int\frac{1+x\cos x}{x\left(1-x^2e^{2\sin x}\right)}dx$ Put $xe^{\sin x}=t\Rightarrow e^{\sin x}(1+x\cos x)\,dx=dt\ \therefore I=\int\frac{dt}{t\left(1-t^2\right)}=\int\frac{dt}{t(1+t)(1-t)}$

We write, $\frac{1}{t(1+t)(1-t)}=\frac{A}{t}+\frac{B}{1+t}+\frac{C}{1-t}\Rightarrow 1=A\left(1-t^2\right)+B\left(t-t^2\right)+C\left(t+t^2\right)$

Equating the coefficients of t^2, t and constant terms, we get $A+B-C=0, B+C=0$ and $A=1$ Solving these equations, we get $A=1, B=\frac{-1}{2}$ and $C=\frac{1}{2}\therefore I=\int\frac{dt}{t\left(1-t^2\right)}=\int\frac{dt}{t}-\frac{1}{2}\int\frac{dt}{1+t}+\frac{1}{2}\int\frac{dt}{1-t}$

$=\log t-\frac{1}{2}\log|1+t|-\frac{1}{2}\log|1-t|+C=\log t-\frac{1}{2}\log\left|1-t^2\right|+C$

$=\log\left|xe^{\sin x}\right|-\frac{1}{2}\log\left|1-x^2e^{2\sin x}\right|+C$

30. Let, $f(x)=xe^x$ Then, $f(x+h)=(x+h)e^{(x+h)}$ $\frac{d}{dx}(f(x))=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$ $\frac{d}{dx}(f(x))=\lim_{h\to0}\frac{(x+h)e^{x+h}-xe^x}{h}$

$\frac{d}{dx}(f(x))=\lim_{h\to0}\frac{\left(xe^{x+h}-xe^x\right)+he^{x+h}}{h}$ $\frac{d}{dx}(f(x))=\lim_{h\to0}\left\{xe^x\left(\frac{e^h-1}{h}\right)+e^{x+h}\right\}$

$\frac{d}{dx}(f(x))=xe^x\lim_{h\to0}\left(\frac{e^h-1}{h}\right)+\lim_{h\to0}e^{x+h}=xe^x+e^x=(x+1)e^x$

OR

We have, $f(x)=\sqrt{x^2+1}$, $g(x)=\frac{x+1}{x^2+1}$ and $h(x)=2x-3$

Therefore, $f'(x)=\frac{x}{\sqrt{x^2+1}}$ $g'(x)=\frac{1-2x-x^2}{\left(x^2+1\right)^2}$

And $h'(x)=2$ for all $x\in R$ Now, $h'(x)=2$ for all $x\in R$ $h'\left(g'(x)\right)=2$ for all $x\in R$

$f'\left(h'\left(g'(x)\right)\right)=f'(2)$ for all $x\in R$ since $f'(x)=\frac{x}{\sqrt{x^2+1}}$ $f'\left(h'\left(g'(x)\right)\right)=\frac{2}{\sqrt{2^2+1}}=\frac{2}{\sqrt5}$ for all $x\in R$ therefore, $f'(2)=\frac{2}{\sqrt5}$

31. The equation of the family of curves is $y^2=a\left(b^2-x^2\right)$.(i)
Clearly, there are two arbitrary constants in this equation. So, we shall differentiate it two times to get a differential equation of second order.
Differentiating (i) with respect to x, we get $2y\frac{dy}{dx}=-2ax$ $y\frac{dy}{dx}=-ax$..(ii)
Differentiating (ii) with respect to x, we get $y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=-a$ $a=-\left\{y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right\}$.....(iii)
Substituting the value of a obtained from (iii) and (ii), we get $x\left\{y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right\}=y\frac{dy}{dx}$, which is the required differential equation.

32. 6 can be through with a pair of dice in. the following ways: (1, 5),(5, 1),(4, 2), (2, 4),(3, 3)

So, probability of throwing a $'6' = \frac{5}{36}$ and probability of not throwing a $'6' = 1 - \frac{5}{36} = \frac{31}{36}$.

Now, 7 can be thrown with a pair of dice in 6 ways, viz. $(1,6)$, $(6,1)$, $(2,5)$, $(5,2)$, $(4,3)$, $(3,4)$

So, probability of throwing a $'7' = \frac{6}{36} = \frac{1}{6}$ and probability of not throwing a $'7' = 1 - \frac{1}{6} = \frac{5}{6}$ Let E and F be two events defined as: E =throwing a in a single throw of a pair of dice and F =throwing a in a single throw of a pair of dice

Then, $P(E) = \frac{5}{36'} P(\bar{E}) = \frac{31}{36'} P(F) = \frac{1}{6}$ and $P(\bar{F}) = \frac{5}{6}$ A wins if he throws in 1st or 3rd or 5th. throws.

Probability of A throwing a 6 in first throw $= P(E) = \frac{5}{36}$ A will get 6 in third if he fails in first and B fails in second throw. $\therefore$ Probability of A throwing a 6 in third throw

$P(\bar{E} \cap \bar{F} \cap E) = P(\bar{E})P(\bar{F})P(E) = \frac{31}{36} \times \frac{5}{6} \times \frac{5}{36}$

Similarly, probability of A throwing a 6 in fifth throw $= P(\bar{E} \cap \bar{F} \cap \bar{E} \cap \bar{F} \cap E) = P(\bar{E})P(\bar{F})P(\bar{E})P(\bar{F})P(E) = \left(\frac{31}{36}\right)^2 \times \left(\frac{5}{6}\right)^2 \times \frac{5}{36}$ and so no

Hence, probability of winning of $A = P((E) \cup (\bar{E} \cap \bar{F} \cap E) \cup (\bar{E} \cap \bar{F} \cap \bar{E} \cap \bar{F} \cap E) \cup \ldots) = P(E) + P(\bar{E} \cap \bar{F} \cap E) + P(\bar{E} \cap \bar{F} \cap \bar{E} \cap \bar{F} \cap E) + \ldots$

$= \frac{5}{36} + \left(\frac{31}{36} \times \frac{5}{6}\right) \times \frac{5}{36} + \left(\frac{31}{36} \times \frac{5}{6}\right)^2 \times \frac{5}{36} + \ldots = \frac{5/36}{1 - (31/36) \times (5/6)} = \frac{30}{61}$

Thus, probability of winning of $B = 1 - \frac{30}{61} = \frac{31}{61}$.

33. Let $I = \int_0^{\pi/2} \frac{\cos x}{(1 + \sin x)(2 + \sin x)} dx$ Put $\sin x = t \Rightarrow \cos x \, dx = dt$

When $x = 0, t = 0$ and when $x = \frac{\pi}{2}, t = 1 \therefore I = \int_0^1 \frac{dt}{(1+t)(2+t)}$

We write, $\frac{1}{(1+t)(2+t)} = \frac{A}{1+t} + \frac{B}{2+t} \Rightarrow 1 = A(2+t) + B(1+t)$.(i)

Putting $t = -1$ and -2 in (i), we get $A = 1$ and $B = -1 \therefore I = \int_0^1 \frac{dt}{1+t} - \int_0^1 \frac{dt}{2+t} = [\log(1+t)]_0^1 - [\log(2+t)]_0^1 = \log 2^2 - \log 3 = \log \frac{4}{3}$

34. Given that, $\vec{a} = (\hat{i} - 2\hat{j} + 3\hat{k})$ and $\vec{b} = (\hat{i} + 2\hat{j} - \hat{k})$ Here, $a_1 = 1$, $a_2 = -2$, $a_3 = 3$ and $b_1 = 1$, $b_2 = 2$, $b_3 = -1$

Know that, $\vec{a} \times \vec{b} = (a_2 b_3 - b_2 a_3)\hat{i} + (a_3 b_1 - b_3 a_1)\hat{j} + (a_1 b_2 - b_1 a_2)\hat{k}$

Substitute the values of $a_1, a_2, a_3, b_1, b_2, b_3$ $\vec{a} \times \vec{b} = \{(-2) \times (-1) - 2 \times 3)\hat{i} + (3 \times 1 - (-1) \times 1)\hat{j} + (1 \times 2 - (-2) \times 1)\hat{k}$

$|\vec{a} \times \vec{b}| = \sqrt{(-4)^2 + (4)^2 + (4)^2} = 4\sqrt{3}$

$\vec{a} \times \vec{b} = \frac{-4\hat{i} + 4\hat{j} + 4\hat{k}}{4\sqrt{3}} \vec{r} = \pm \frac{-\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$

35. We have, $f(x) = kx^3 - 9kx^2 + 9x + 3 \Rightarrow f'(x) = 3kx^2 - 18kx + 9 = 3(kx^2 - 6kx + 3)$

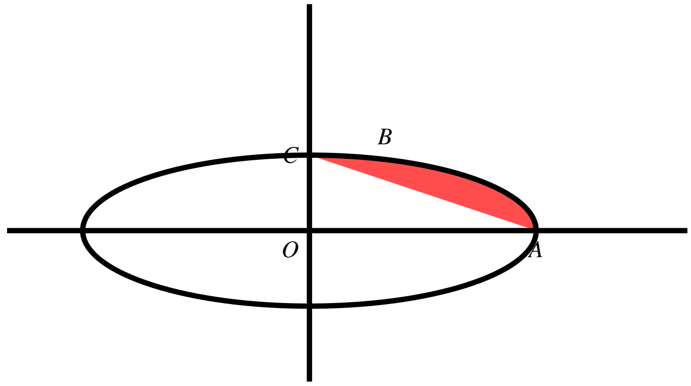
Since $f(x)$ is increasing for all real values of x, therefore $f'(x) > 0 \, \forall x \in R \Rightarrow 3(kx^2 - 6kx + 3) > 0 \, \forall x \in R$ $kx^2 - 6b + 3 > 0 \, \forall x \in R \Rightarrow k > 0$ and $36k^2 - 12k < 0$

$[ax^2 + bx + c > 0 \, \forall x \in R \Rightarrow a > 0$ and $b^2 - 4ac < 0] \Rightarrow k > 0$ and $12k(3k-1) < 0$

$\Rightarrow k > 0$and $3k - 1 < 0 k > 0 \Rightarrow k > 0$ and $k < \frac{1}{3} \Rightarrow k \in \left(0, \frac{1}{3}\right)$ Hence, $f(x)$ is increasing in R, if $k \in \left(0, \frac{1}{3}\right)$

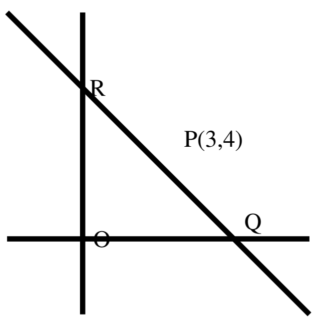
SECTION -V (Five Mark Questions)

36. The rough sketch of ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and the line $\frac{x}{4} + \frac{y}{3} = 1$ is shown in the figure



The required area of shaded region $= \int_0^4 (y_1 - y_2)dx$ where $y_1 = \frac{3}{4}\sqrt{16-x^2}$ and $y_2 = \frac{3}{4}(4-,x)$ $\therefore A = \int_0^4 \left[\frac{3}{4}\sqrt{16-x^2} - \frac{3}{4}(4-x)\right] dx = \frac{3}{4}\int_0^4 \left(\sqrt{16-x^2} - 4 + x\right) dx \Rightarrow A = \frac{3}{4}\left[\frac{x}{2}\sqrt{16-x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4} - 4x + \frac{x^2}{2}\right]_0^4 = \frac{3}{4}\left[0 + 8\sin^{-1}1 - 16 + 8 - 0\right] = \frac{3}{4}\left[8 \times \frac{\pi}{2} - 8\right] = \frac{3}{4}[4\pi - 8] = 3(\pi - 2)$ sq. units

37. Equation of line passing through $(3,4)$ and whose slope is m (say) is given by $y - 4 = m(x-3)$



This line cuts x-axis i.e., $y = 0$ at $Q\left(3 - \frac{4}{m}, 0\right)$ and y-axis i.e., $x = 0$ at $R(0, 4-3m)$

Area of $\Delta OQR = \frac{1}{2} \times OQ \times OR \therefore A = \frac{1}{2}\left(3 - \frac{4}{m}\right)(4-3m) = \frac{1}{2}\left(12 - 9m - \frac{16}{m} + 12\right)$

$\Rightarrow A = \frac{1}{2}\left(24 - 9m - \frac{16}{m}\right) \Rightarrow \frac{dA}{dm} = \frac{1}{2}\left(-9 + \frac{16}{m^2}\right)$

For maxima or minima, $\frac{dA}{dm} = 0$

$\Rightarrow \frac{1}{2}\left(-9 + \frac{16}{m^2}\right) = 0 \Rightarrow m^2 = \frac{16}{9} \Rightarrow m = \pm\frac{4}{3}$

Also, $\frac{d^2A}{dm^2} = \frac{-16}{m^3}$

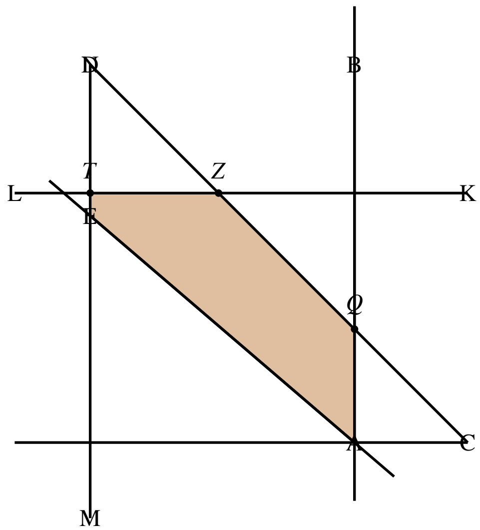
So, $\left(\frac{d^2A}{dm^2}\right)_{m=\frac{4}{3}} = \frac{-16}{\left(\frac{4}{3}\right)^3} = \frac{-27}{4} < 0$

Area is maximum; so $m = \frac{4}{3}$ is neglected. and $\left(\frac{d^2A}{dm^2}\right)_{m=\frac{-4}{3}} = \frac{27}{4} > 0$

$\therefore$ Area of ΔOQR is minimum, when $m = \frac{-4}{3}$

Thus, Equation of line $y - 4 = \frac{-4}{3}(x-3) \Rightarrow 4x + 3y = 24$

38. Let the depot A transport x thousand bricks to builder P and y thousand bricks to builder Q. Then, the above LPP can be stated mathematically as follows.



Minimize $Z = 30x - 30y + 1800$ subject to $x + y \leq 30; x \leq 15; y \leq 20; x + y \geq 15$ and $x \geq 0, y \geq 0$ To solve this LPP graphically, we first convert inequations into equations and then draw the corresponding lines. The feasible region of the LPP is shaded in figure. The coordinates of the corner points of the feasible region are A (15, 0), Q(15, 15), Z(10, 20), T (0, 20) and E(0, 15). These points have been obtained by solving the corresponding intersecting lines simultaneously.

The values of the objective function at the comer points of the feasible region are given in the following table.

Point (x, y)	Value of the objective function $Z = 30x - 30y + 1800$
$(15, 0)$	$Z = 30 \times 15 - 30 \times 0 + 1800 = 2250$
$(15, 15)$	$Z = 30 \times 15 - 30 \times 15 + 1800 = 1800$
$(10, 20)$	$Z = 30 \times 10 - 30 \times 20 + 1800 = 1500$
$(0, 20)$	$Z = 30 \times 0 - 30 \times 20 + 1800 = 1200$
$(0, 15)$	$Z = 30 \times 0 - 30 \times 15 + 1800 = 1350$

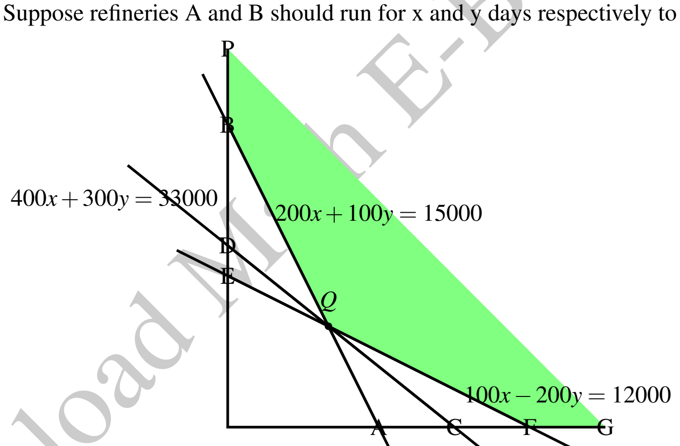
Clearly, Z is minimum at $x = 0, y = 20$ and the minimum value of Z is 1200. 0, 20 and 10 thousand bricks from depot A and 15,0 and 5 thousand bricks. Hence, the manufacturer have to deliver from depot B respectively. In this case, the minimum transportation cost will be Rs. 1200.

OR

The given data may be put in the following tabular form :

Refinery	High Grade	Medium Grade	Low Grade	Cost Per Day
A	100	300	200	Rs. 400
B	200	400	100	Rs. 300
Minimum Requirement	12000	20000	15000	

Suppose refineries A and B should run for x and y days respectively to minimize the total cost.



The mathematical form of the above LPP is Minimize $Z = 400x + 300y$ subject to $100x + 200y \geq 12,000$ $300x + 400y \geq 20,000$ $200x + 100y \geq 15000$ *and* $x, y \geq 0$ The feasible region of the above LPP is represented by the shaded region in figure.

The corner points of the feasible region are$F\,(120, 0)$, Q(60, 30) and $B(0, 150)$. The value of the objective function at these points are given in the following table:

Point (x, y)	Value of the objective function
$(120, 0)$	$Z = 400 \times 120 + 300 \times 0 = 48000$
$(60, 30)$	$Z = 400 \times 60 + 300 \times 30 = 33000$
$(0, 150)$	$Z = 400 \times 0 + 300 \times 50 = 45000$

Clearly, Z is minimum when$x = 60, y = 30$. The feasible region is unbounded. So, we find the half-plane represented by$400x + 300y < 33000$. Clearly, the half-plane does not have points common with the feasible region. So, Z is minimum at $x = 60, y = 30$. Hence, the refinery A should run for 60 days and the refinery5should run for 30 days to minimize the cost while satisfying the constraints.