NCERT - Exercise 2.1

Find the principal values of the following :

1. $\sin^{-1}\left(-\frac{1}{2}\right)$

SOLUTION

Let $\sin^{-1}\left(-\frac{1}{2}\right) = x \Rightarrow \sin x = -\frac{1}{2}$ We know that the range of the principal value branch of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Then, $\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$, *where* $-\frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Hence, the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$ *is* $-\frac{\pi}{6}$

2. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

SOLUTION

Let $\cos^{-1}\frac{\sqrt{3}}{2} = x \Rightarrow \frac{\sqrt{3}}{2} = \cos x$

We know that the range of the principal value branch of $\cos^{-1}$ is $[0, \pi]$ Then, $\cos x = \frac{\sqrt{3}}{2} = \cos\frac{\pi}{6}$, *where* $\frac{\pi}{6} \in [0, \pi]$

Hence, the principal value of $\cos^{-1}\frac{\sqrt{3}}{2}$ is $\frac{\pi}{6}$.

3. $\text{cosec}^{-1}(2)$

SOLUTION

Let $\text{cosec}^{-1}(2) = x \Rightarrow 2 = \text{cosec x}$

We know that the range of the principal value branch of cosec^{-1} *is* $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$ Then, $2 = \text{cosec x} = \text{cosec}\left(\frac{\pi}{6}\right)$, *where* $\frac{\pi}{6} \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

Hence, the principal value of $\text{cosec}^{-1}(2)$ *is* $\pi/6$.

4. $\tan^{-1}(-\sqrt{3})$

SOLUTION

Let $\tan^{-1}(-\sqrt{3}) = x \Rightarrow -\sqrt{3} = \tan x$ We know that the range of the principal value branch of $\tan^{-1}$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Then, $\tan x = -\sqrt{3} = \tan\left(-\frac{\pi}{3}\right)$, *where* $-\frac{\pi}{3} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Hence, the principal value of $\tan^{-1}(-\sqrt{3})$ is $-\frac{\pi}{3}$.

5. $\cos^{-1}\left(-\frac{1}{2}\right)$

SOLUTION

Let $x = \cos^{-1}\left(-\frac{1}{2}\right) \Rightarrow -\frac{1}{2} = \cos x$

We know that the range of principal value branch of $\cos^{-1}$ *is* $[0, \pi]$ Then, $\left(-\frac{1}{2}\right) = -\cos\frac{\pi}{3} = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right)$,

Where $\frac{2\pi}{3} \in [0, \pi]$ Hence, the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ *is* $\frac{2\pi}{3}$.

6. $\tan^{-1}(-1)$

SOLUTION

Let $\tan^{-1}(-1) = x \Rightarrow -1 = \tan x$ We know that the range of principal value branch of $\tan^{-1}$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Then, $-1 = \tan\left(-\frac{\pi}{4}\right)$ *where* $-\frac{\pi}{4} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ Hence, the principal value of $\tan^{-1}(-1)$ *is* $-\frac{\pi}{4}$.

7. $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

SOLUTION

Let $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = x \Rightarrow \sec x = \frac{2}{\sqrt{3}}$

We know that the range of principal value branch of $\sec^{-1}$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$.

Then, $\frac{2}{\sqrt{3}} = \sec\left(\frac{\pi}{6}\right)$, *where* $\frac{\pi}{6} \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$ Hence, the principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ *is* $\frac{\pi}{6}$.

8. $\cot^{-1}(\sqrt{3})$

SOLUTION

Let $\cot^{-1}(\sqrt{3}) = x \Rightarrow \sqrt{3} = \cot x$

We know that the range of principal value of $\cot^{-1}$ *is* $(0, \pi)$ Then, $\sqrt{3} = \cot\left(\frac{\pi}{6}\right)$, *where* $\frac{\pi}{6} \in (0, \pi)$

Hence, the principal value branch of $\cot^{-1}(\sqrt{3})$ *is* $\frac{\pi}{6}$.

9. $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

SOLUTION

Let $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = x \Rightarrow -\frac{1}{\sqrt{2}} = \cos x$ We know that the range of principal value branch of $\cos^{-1}$*is* $[0, \pi]$ Then, $-\frac{1}{\sqrt{2}} = -\cos\left(\frac{\pi}{4}\right) = \cos\left(\pi - \frac{\pi}{4}\right) = \cos\frac{3\pi}{4}$,

Where $\frac{3\pi}{4} \in [0, \pi]$ Hence, the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is $\frac{3\pi}{4}$.

10. $\text{cosec}^{-1}(-\sqrt{2})$

SOLUTION

Let $\text{cosec}^{-1}(-\sqrt{2}) = x \Rightarrow -\sqrt{2} = \text{cosec x}$

We know that the range of principal value branch of cosec^{-1} is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

Then, $-\sqrt{2} = -\text{cosec}\left(\frac{\pi}{4}\right) = \text{cosec}\left(-\frac{\pi}{4}\right)$,

Where $-\frac{\pi}{4} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$ Hence, the principal value of $\text{cosec}^{-1}(-\sqrt{2})$ *is* $-\frac{\pi}{4}$.

Find the values of the following :

11. $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$

SOLUTION

We know that the range of principal value branch of $\tan^{-1}$, $\cos^{-1}$ *and* $\sin^{-1}$ are $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, $[0, \pi]$ and $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ respectively.

Let $\tan^{-1}(1) = x \Rightarrow 1 = \tan x$ Then, $1 = \tan\left(\frac{\pi}{4}\right)$, *where* $\frac{\pi}{4} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Let $\cos^{-1}\left(-\frac{1}{2}\right) = y \Rightarrow -\frac{1}{2} = \cos y$ Then, $-\frac{1}{2} = -\cos\left(\frac{\pi}{3}\right) = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\frac{2\pi}{3}$,

Where $\frac{2\pi}{3} \in [0,\ \pi]$ Let $\sin^{-1}\left(-\frac{1}{2}\right) = z \Rightarrow -\frac{1}{2} = \sin z$

Then, $-\frac{1}{2} = \sin\left(-\frac{\pi}{6}\right)$, *where* $-\frac{\pi}{6} \in \left[-\frac{\pi}{2},\ \frac{\pi}{2}\right]$

So, $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right) = \left(\frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6}\right) = \frac{3\pi}{4}$.

12. $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$

SOLUTION

We know that the range of principal value branch of $\cos^{-1}$ and $\sin^{-1}$ are $[0,\ \pi]$ and $\left[-\frac{\pi}{2},\ \frac{\pi}{2}\right]$ respectively.

Let $\cos^{-1}\left(\frac{1}{2}\right) = x \Rightarrow \frac{1}{2} = \cos x$

Then, $\frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$, *where* $\frac{\pi}{3} \in [0,\ \pi]$

Let $\sin^{-1}\left(\frac{1}{2}\right) = y \Rightarrow \frac{1}{2} = \sin y$

Then, $\frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$, *where* $\frac{\pi}{6} \in \left[-\frac{\pi}{2},\ \frac{\pi}{2}\right]$

$\therefore \cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} + 2\cdot\frac{\pi}{6} = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$.

13. . If $\sin^{-1}x = y$, then

(A) $0 \le y \le \pi$

(B) $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

(C) $0 < y < \pi$

(D) $-\frac{\pi}{2} < y < \frac{\pi}{2}$

SOLUTION

(B) $\sin^{-1}x = y \Rightarrow x = \sin y$, where the range of principal value branch of $\sin^{-1}$ *is* $\left[-\frac{\pi}{2},\ \frac{\pi}{2}\right]$, *then* $y \in \left[-\frac{\pi}{2},\ \frac{\pi}{2}\right] \Rightarrow -\frac{\pi}{2} \le y \le \frac{\pi}{2}$.

14. $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$ is equal to

(A) π

(B) $-\frac{\pi}{3}$

(C) $\frac{\pi}{3}$

(D) $\frac{2\pi}{3}$

SOLUTION

(B) We know that the range of principal value branch of $\tan^{-1}$ and $\sec^{-1}$ are $\left(-\frac{\pi}{2},\ \frac{\pi}{2}\right)$ and $[0,\ \pi] - \left\{\frac{\pi}{2}\right\}$ respectively.

Let $\tan^{-1}(\sqrt{3}) = x \Rightarrow \sqrt{3} = \tan x$, *then*, $\sqrt{3} = \tan\left(\frac{\pi}{3}\right)$, *where* $\frac{\pi}{3} \in \left(-\frac{\pi}{2},\ \frac{\pi}{2}\right)$

Let $\sec^{-1}(-2) = y \Rightarrow -2 = \sec y$ Then, $-2 = -\sec\left(\frac{\pi}{3}\right) = \sec\left(\pi - \frac{\pi}{3}\right) = \sec\frac{2\pi}{3}$,

Where $\frac{2\pi}{3} \in [0,\ \pi] - \left\{\frac{\pi}{2}\right\}$ $\therefore \tan^{-1}\sqrt{3} - \sec^{-1}(-2) = \frac{\pi}{3} - \frac{2\pi}{3} = \frac{\pi}{3}$.