


**NCERT - Exercise 2.1**

Find the principal values of the following :

1.  $\sin^{-1}\left(-\frac{1}{2}\right)$

**SOLUTION**

Let  $\sin^{-1}\left(-\frac{1}{2}\right) = x \Rightarrow \sin x = -\frac{1}{2}$  We know that the range of the principal value branch of  $\sin^{-1}$  is  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ .

Then,  $\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$ , where  $-\frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Hence, the principal value of  $\sin^{-1}\left(-\frac{1}{2}\right)$  is  $-\frac{\pi}{6}$

2.  $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

**SOLUTION**

Let  $\cos^{-1}\frac{\sqrt{3}}{2} = x \Rightarrow \frac{\sqrt{3}}{2} = \cos x$

We know that the range of the principal value branch of  $\cos^{-1}$  is  $[0, \pi]$  Then,  $\cos x = \frac{\sqrt{3}}{2} = \cos \frac{\pi}{6}$ , where  $\frac{\pi}{6} \in [0, \pi]$

Hence, the principal value of  $\cos^{-1}\frac{\sqrt{3}}{2}$  is  $\frac{\pi}{6}$ .

3.  $\operatorname{cosec}^{-1}(2)$

**SOLUTION**

Let  $\operatorname{cosec}^{-1}(2) = x \Rightarrow 2 = \operatorname{cosec} x$

We know that the range of the principal value branch of  $\operatorname{cosec}^{-1}$  is  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$  Then,  $2 = \operatorname{cosec} x = \operatorname{cosec}\left(\frac{\pi}{6}\right)$ , where  $\frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

Hence, the principal value of  $\operatorname{cosec}^{-1}(2)$  is  $\pi/6$ .

4.  $\tan^{-1}(-\sqrt{3})$

**SOLUTION**

Let  $\tan^{-1}(-\sqrt{3}) = x \Rightarrow -\sqrt{3} = \tan x$  We know that the range of the principal value branch of  $\tan^{-1}$  is  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Then,  $\tan x = -\sqrt{3} = \tan\left(-\frac{\pi}{3}\right)$ , where  $-\frac{\pi}{3} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Hence, the principal value of  $\tan^{-1}(-\sqrt{3})$  is  $-\frac{\pi}{3}$ .

5.  $\cos^{-1}\left(-\frac{1}{2}\right)$

**SOLUTION**

Let  $x = \cos^{-1}\left(-\frac{1}{2}\right) \Rightarrow -\frac{1}{2} = \cos x$

We know that the range of principal value branch of  $\cos^{-1}$  is  $[0, \pi]$  Then,  $\left(-\frac{1}{2}\right) = -\cos \frac{\pi}{3} = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right)$ ,

Where  $\frac{2\pi}{3} \in [0, \pi]$  Hence, the principal value of  $\cos^{-1}\left(-\frac{1}{2}\right)$  is  $\frac{2\pi}{3}$ .

6.  $\tan^{-1}(-1)$

**SOLUTION**

Let  $\tan^{-1}(-1) = x \Rightarrow -1 = \tan x$  We know that the range of principal value branch of  $\tan^{-1}$  is  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Then,  $-1 = \tan\left(-\frac{\pi}{4}\right)$  where  $-\frac{\pi}{4} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$  Hence, the principal value of  $\tan^{-1}(-1)$  is  $-\frac{\pi}{4}$ .

7.  $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

**SOLUTION**

Let  $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = x \Rightarrow \sec x = \frac{2}{\sqrt{3}}$

We know that the range of principal value branch of  $\sec^{-1}$  is  $[0, \pi] - \left\{\frac{\pi}{2}\right\}$ .

Then,  $\frac{2}{\sqrt{3}} = \sec\left(\frac{\pi}{6}\right)$ , where  $\frac{\pi}{6} \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$  Hence, the principal value of  $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$  is  $\frac{\pi}{6}$ .

8.  $\cot^{-1}(\sqrt{3})$

**SOLUTION**

Let  $\cot^{-1}(\sqrt{3}) = x \Rightarrow \sqrt{3} = \cot x$

We know that the range of principal value of  $\cot^{-1}$  is  $(0, \pi)$  Then,  $\sqrt{3} = \cot\left(\frac{\pi}{6}\right)$ , where  $\frac{\pi}{6} \in (0, \pi)$

Hence, the principal value branch of  $\cot^{-1}(\sqrt{3})$  is  $\frac{\pi}{6}$ .

9.  $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

**SOLUTION**

Let  $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = x \Rightarrow -\frac{1}{\sqrt{2}} = \cos x$  We know that the range of principal value branch of  $\cos^{-1}$  is  $[0, \pi]$  Then,  $-\frac{1}{\sqrt{2}} = -\cos\left(\frac{\pi}{4}\right) = \cos\left(\pi - \frac{\pi}{4}\right) = \cos\frac{3\pi}{4}$ ,

Where  $\frac{3\pi}{4} \in [0, \pi]$  Hence, the principal value of  $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$  is  $\frac{3\pi}{4}$ .

10.  $\operatorname{cosec}^{-1}(-\sqrt{2})$

**SOLUTION**

Let  $\operatorname{cosec}^{-1}(-\sqrt{2}) = x \Rightarrow -\sqrt{2} = \operatorname{cosec} x$

We know that the range of principal value branch of  $\operatorname{cosec}^{-1}$  is  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

Then,  $-\sqrt{2} = -\operatorname{cosec}\left(\frac{\pi}{4}\right) = \operatorname{cosec}\left(-\frac{\pi}{4}\right)$ ,

Where  $-\frac{\pi}{4} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$  Hence, the principal value of  $\operatorname{cosec}^{-1}(-\sqrt{2})$  is  $-\frac{\pi}{4}$ .

**Find the values of the following :**

11.  $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$

**SOLUTION**

We know that the range of principal value branch of  $\tan^{-1}$ ,  $\cos^{-1}$  and  $\sin^{-1}$  are  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ ,  $[0, \pi]$  and  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$  respectively.

Let  $\tan^{-1}(1) = x \Rightarrow 1 = \tan x$  Then,  $1 = \tan\left(\frac{\pi}{4}\right)$ , where  $\frac{\pi}{4} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$$\text{Let } \cos^{-1}\left(-\frac{1}{2}\right) = y \Rightarrow -\frac{1}{2} = \cos y \text{ Then, } -\frac{1}{2} = -\cos\left(\frac{\pi}{3}\right) = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\frac{2\pi}{3},$$

$$\text{Where } \frac{2\pi}{3} \in [0, \pi] \text{ Let } \sin^{-1}\left(-\frac{1}{2}\right) = z \Rightarrow -\frac{1}{2} = \sin z$$

$$\text{Then, } -\frac{1}{2} = \sin\left(-\frac{\pi}{6}\right), \text{ where } -\frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\text{So, } \tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right) = \left(\frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6}\right) = \frac{3\pi}{4}.$$

12.  $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$

**SOLUTION**

We know that the range of principal value branch of  $\cos^{-1}$  and  $\sin^{-1}$  are  $[0, \pi]$  and  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$  respectively.

$$\text{Let } \cos^{-1}\left(\frac{1}{2}\right) = x \Rightarrow \frac{1}{2} = \cos x$$

$$\text{Then, } \frac{1}{2} = \cos\left(\frac{\pi}{3}\right), \text{ where } \frac{\pi}{3} \in [0, \pi]$$

$$\text{Let } \sin^{-1}\left(\frac{1}{2}\right) = y \Rightarrow \frac{1}{2} = \sin y$$

$$\text{Then, } \frac{1}{2} = \sin\left(\frac{\pi}{6}\right), \text{ where } \frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\therefore \cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} + 2 \cdot \frac{\pi}{6} = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}.$$

13. . If  $\sin^{-1}x = y$ , then

(A)  $0 \leq y \leq \pi$

(B)  $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

(C)  $0 < y < \pi$

(D)  $-\frac{\pi}{2} < y < \frac{\pi}{2}$

**SOLUTION**

(B)  $\sin^{-1}x = y \Rightarrow x = \sin y$ , where the range of principal value branch of  $\sin^{-1}$  is  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ , then  $y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \Rightarrow -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$ .

14.  $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$  is equal to

(A)  $\pi$

(B)  $-\frac{\pi}{3}$

(C)  $\frac{\pi}{3}$

(D)  $\frac{2\pi}{3}$

**SOLUTION**

(B) We know that the range of principal value branch of  $\tan^{-1}$  and  $\sec^{-1}$  are  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$  and  $[0, \pi] - \left\{\frac{\pi}{2}\right\}$  respectively.

$$\text{Let } \tan^{-1}(\sqrt{3}) = x \Rightarrow \sqrt{3} = \tan x, \text{ then, } \sqrt{3} = \tan\left(\frac{\pi}{3}\right), \text{ where } \frac{\pi}{3} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$\text{Let } \sec^{-1}(-2) = y \Rightarrow -2 = \sec y \text{ Then, } -2 = -\sec\left(\frac{\pi}{3}\right) = \sec\left(\pi - \frac{\pi}{3}\right) = \sec\frac{2\pi}{3},$$

$$\text{Where } \frac{2\pi}{3} \in [0, \pi] - \left\{\frac{\pi}{2}\right\} \therefore \tan^{-1}\sqrt{3} - \sec^{-1}(-2) = \frac{\pi}{3} - \frac{2\pi}{3} = -\frac{\pi}{3}.$$



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