NCERT Excercise 7.9

Evaluate the following definite integrals in Excercises 1 to 20 .:

1. $\int_{-1}^{1}(x+1)\,dx$

SOLUTION

$$\int_{-1}^{1}(x+1)\,dx=\left[\frac{x^2}{2}+x\right]_{-1}^{1}=\frac{1}{2}\left[(1)^2-(-1)^2+[1-(-1)]\right]=\frac{1}{2}(1-1)+(1+1)=\frac{1}{2}(0)+2=2$$

2. $\int_{2}^{3}\frac{1}{x}dx$ SOLUTION

$$\int_{2}^{3}\frac{1}{x}dx=[\log x]_2^3=\log 3-\log 2=\log\frac{3}{2}$$

3. . $\int_{1}^{2}\left(4x^3-5x^2+6x+9\right)dx$

SOLUTION

$$\int_{1}^{2}\left(4x^3-5x^2+6x+9\right)dx=\left[4\cdot\frac{x^4}{4}-5\cdot\frac{x^3}{4}+6\cdot\frac{x^2}{2}+9x\right]_1^2=\left[x^4-\frac{5}{3}x^3+3x^2+9x\right]_1^2=\left(2^4-1^4\right)-\frac{5}{3}\left(2^3-1^3\right)+3\left(2^2-1^2\right)+$$
$$9\,(2-1)=(16-1)-\frac{5}{3}(8-1)+3\,(4-1)+9\,(1)=15-\frac{35}{3}+9+9=33-\frac{35}{3}=\frac{99-35}{3}=\frac{64}{3}$$

4. $\int_{0}^{\pi/4}\sin 2x\,dx$

SOLUTION

$$\int_{0}^{\pi/4}\sin 2x\,dx=\left[-\frac{1}{2}\cos 2x\right]_0^{\pi/4}=\frac{-1}{2}\left(\cos\frac{\pi}{2}-\cos 0\right)=-\frac{1}{2}(-1)=\frac{1}{2}$$

5. $\int_{0}^{\pi/2}\cos 2x\,dx$

SOLUTION

$$\int_{0}^{\pi/2}\cos 2x\,dx=\left[\frac{1}{2}\sin 2x\right]_0^{\pi/2}=\frac{1}{2}(\sin\pi-\sin 0)=\frac{1}{2}(0-0)=0$$

6. $\int_{4}^{5}e^x dx$

SOLUTION

$$\int_{4}^{5}e^x dx=[e^x]_4^5=e^5-e^4=e^4\,(e-1)$$

7. $\int_0^{\pi/4} \tan x dx$

SOLUTION

: $\int_0^{\pi/4} \tan x dx = [\log \sec x]_0^{\pi/4} = \log\left(\sec\frac{\pi}{4}\right) - \log(\sec\theta) = \log\sqrt{2} - \log 1 = \frac{1}{2}\log 2$

8\. . $\int_{\pi/6}^{\pi/4} \cos ec\, x\, dx$

SOLUTION : $\int_{\pi/6}^{\pi/4} \cos ec\, x\, dx = [\log(\cos ec\, x - \cot x)]_{\pi/6}^{\pi/4} = \log\left(\cos ec\frac{\pi}{4} - \cot\frac{\pi}{4}\right) - \log\left(\cos ec\frac{\pi}{6} - \cot\frac{\pi}{6}\right) = \log\left(\sqrt{2}-1\right) -$ $\log\left(2-\sqrt{3}\right) = \log\left(\frac{\left(\sqrt{2}-1\right)}{2-\sqrt{3}}\right)$

9\. $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

SOLUTION

: $\int_0^1 \frac{dx}{\sqrt{1-x^2}} = \left[\sin^{-1}x\right]_0^1 = \sin^{-1}(1) - \sin^{-1}(0) = \frac{\pi}{2}$

10\. . $\int_0^1 \frac{dx}{1+x^2}$ SOLUTION

: $\int_0^1 \frac{dx}{1+x^2} = \left[\tan^{-1}x\right]_0^1 = \tan^{-1}(1) - \tan^{-1}(0) = \frac{\pi}{4}$

11\. $\int_2^3 \frac{dx}{x^2-1}$

SOLUTION

$\int_2^3 \frac{dx}{x^2-1} = \left[\frac{1}{2}\log\left(\frac{x-1}{x+1}\right)\right]_2^3 = \frac{1}{2}\left[\log\left(\frac{3-1}{3+1}\right) - \log\left(\frac{2-1}{2+1}\right)\right] = \frac{1}{2}\left[\log\left(\frac{2}{4}\right) - \log\left(\frac{1}{3}\right)\right] = \frac{1}{2}\log\left(\frac{2/4}{1/3}\right) = \frac{1}{2}\log\frac{3}{2}$

12. $\int_0^{\pi/2} \cos^2 x dx$

SOLUTION

: $\int_0^{\pi/2} \cos^2 x dx = \int_0^{\pi/2} \frac{1+\cos 2x}{2} dx = \left[\frac{1}{2}\left(x+\frac{\sin 2x}{2}\right)\right]_0^{\pi/2} = \frac{1}{2}\left[\left(\frac{\pi}{2}-0\right)+\left(\frac{\sin x}{2}-\frac{\sin 0}{2}\right)\right] = \frac{\pi}{4}$

13. . $\int_2^3 \frac{x\,dx}{x^2+1}$

SOLUTION

: $\int_2^3 \frac{x}{x^2+1} dx = \frac{1}{2}\int_2^3 \frac{2x}{x^2+1} dx = \frac{1}{2}\left[\log\left(x^2+1\right)\right]_2^3 = \frac{1}{2}[\log 10 - \log 5] = \frac{1}{2}\log\frac{10}{5} = \frac{1}{2}\log 2$

14. $\int_0^1 \frac{2x+3}{5x^2+1} dx$

SOLUTION

: $\int_0^1 \frac{2x+3}{5x^2+1} dx = \int_0^1 \left(\frac{2x}{5x^2+1}+\frac{3}{5x^2+1}\right) dx = \frac{1}{5}\int_0^1 \frac{10x}{5x^2+1} dx + \frac{3}{5}\int_0^1 \frac{dx}{x^2+\left(\frac{1}{\sqrt{5}}\right)^2} = \frac{1}{5}\left[\log\left(5x^2+1\right)\right]_0^1 + \frac{3}{5}\times\frac{1}{\frac{1}{\sqrt{5}}}\left[\tan^{-1}\left(\frac{x}{\frac{1}{\sqrt{5}}}\right)\right]_0^1$

$= \frac{1}{5}(\log 6 - \log 1) + \frac{3}{\sqrt{5}}\left(\tan^{-1}\sqrt{5} - 0\right) = \frac{1}{5}\log 6 + \frac{3}{\sqrt{5}}\tan^{-1}\sqrt{5}$

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15. $\int_0^1 xe^{x^2} dx$

SOLUTION

Let $I = \int_0^1 xe^{x^2} dx = \frac{1}{2}\int_0^1 2xe^{x^2} dx$ Put $x^2 = t \Rightarrow 2xdx = dt$ When $x=0, t=0$ and when $x=1, t=1$ $\therefore I = \frac{1}{2}\int_0^1 e^t dt = \left[\frac{1}{2}e^t\right]_0^1 = \frac{1}{2}\left[e^1 - e^0\right] = \frac{1}{2}[e-1]$

16. $\int_1^2 \frac{5x^2}{x^2+4x+3}$

SOLUTION

Let $I = \int_1^2 \frac{5x^2}{x^2+4x+3} dx$ Since the degree of numerator denominator and is same so the, fraction is improper. To mark it proper, we have to divide $5x^2$ by x^2+4x+3 . $x^2+4x+3 \overline{\big)\, 5x^2 \,}$ quotient 5; $5x^2+20x+15$; $-\;-\;-$; $-20x-15$ $\therefore I = \int_1^2 \left(5+\frac{-20x-15}{x^2+4x+3}\right) dx = \int_1^2 \left(5-\frac{20x+15}{x^2+4x+3}\right) dx$

$\frac{20x+15}{x^2+4x+3} = \frac{20x+15}{(x+1)(x+3)} = \frac{A}{x+1}+\frac{B}{x+3} \Rightarrow 20x+15 = A(x+3)+B(x+1)$ ....(i) Putting $x=-1$ in (i), we get $-20+15 = A(-1+3) \Rightarrow -5 = 2A \Rightarrow A = \frac{-5}{2}$ Putting $x=-3$ in (i), we get $-60+15 = B(-3+1) \Rightarrow -45 = -2B \Rightarrow B = \frac{45}{2}$

$\therefore I=\int_1^2\left(5+\frac{5}{2(x+1)}-\frac{45}{2(x+3)}\right)dx=\left[5x+\frac{5}{2}\log(x+1)-\frac{45}{2}\log(x+3)\right]_1^2=5(2-1)+\frac{5}{2}[\log 3-\log 2]-\frac{45}{2}[\log 5-log 4]$

$=5+\frac{5}{2}\log\frac{3}{2}-\frac{45}{2}\log\frac{5}{4}=5-\frac{5}{2}\left(9\log\frac{5}{4}-\log\frac{3}{2}\right)$

17. $\int_0^{\pi/4}\left(2\sec^2 x+x^3+2\right)dx$

SOLUTION

: Let $I=\int_0^{\pi/4}\left(2\sec^2 x+x^3+2\right)dx=\left[2\tan x+\frac{x^4}{4}+2x\right]_0^{\pi/4}=2\left(\tan\frac{\pi}{4}-\tan 0\right)+\frac{1}{4}\left(\frac{\pi^4}{256}-0\right)+2\left(\frac{\pi}{4}-0\right)=2(1-0)+\frac{\pi^4}{1024}+\frac{\pi}{2}=\frac{\pi^4}{1024}+\frac{\pi}{2}+2$

18. $\int_0^{\pi}\left(\sin^2\frac{x}{2}-\cos^2\frac{x}{2}\right)dx$

SOLUTION

: Let $\int_0^{\pi}\left(\sin^2\frac{x}{2}-\cos^2\frac{x}{2}\right)dx=-\int_0^{\pi}\cos x dx$

$=-[\sin x]_0^{\pi}=-(\sin\pi-\sin 0)=-(0-0)=0$

19. $\int_0^2\frac{6x+3}{x^2+4}dx$

SOLUTION

: Let $I=\int_0^2\frac{6x+3}{x^2+4}dx=\int_0^2\frac{6x}{x^2+4}dx+\int_0^2\frac{3}{x^2+4}dx=3\int_0^2\frac{2x}{x^2+4}dx+\left[3\times\frac{1}{2}\tan^{-1}\frac{x}{2}\right]_0^2$ Let $I_1=3\int_0^2\frac{2x}{x^2+4}dx$ Put$x^2+4=t\Rightarrow 2x dx=dt$ When $x=0, t=4$ and when $x=2, t=8$ $\therefore I_1=3\int_4^8\frac{dt}{t}=[3\log t]_4^8=3(\log 8-\log 4)=3\log 2\Rightarrow I=3\log 2+\frac{3}{2}\left[\tan^{-1}1-\tan^{-1}0\right]=3\log 2+\frac{3}{2}\times\frac{\pi}{4}=3\log 2+\frac{3\pi}{8}$

20. $\int_0^1\left(xe^x+\sin\frac{\pi x}{4}\right)dx$

SOLUTION

: Let $I=\int_0^1\left[xe^x+\sin\left(\frac{\pi x}{4}\right)\right]dx=\int_0^1 xe^x dx+\int_0^1\sin\frac{\pi x}{4}dx=\left[xe^x-\int\left(\frac{d}{dx}(x)\cdot\int e^x dx\right)dx\right]_0^1-\left[\frac{\cos\frac{\pi x}{4}}{\frac{\pi}{4}}\right]_0^1=\left[xe^x-\int e^x dx\right]_0^1-\frac{4}{\pi}\left[\cos\frac{\pi x}{4}\right]_0^1=[xe^x-e^x]_0^1-\frac{4}{\pi}\left(\cos\frac{\pi}{4}-\cos 0\right)=(e^1-0)-(e^1-e^0)-\frac{4}{\pi}\left(\frac{1}{\sqrt{2}}-1\right)=e-e+1-\frac{4}{\pi\sqrt{2}}+\frac{4}{\pi}=1+\frac{4}{\pi}-\frac{2\sqrt{2}}{\pi}$

Choose the correct answer in Exercises 21 and 22. :

21. $\int_1^{\sqrt{3}}\frac{dx}{1+x^2}$ equals

(a) $\frac{\pi}{3}$

(b) $\frac{2\pi}{3}$

(c) $\frac{\pi}{6}$

(d) $\frac{\pi}{12}$

SOLUTION

: (D) : Let $I = \int_{1}^{\sqrt{3}} \frac{dx}{1+x^2} = \left[\tan^{-1}x\right]_{1}^{\sqrt{3}} = \tan^{-1}\sqrt{3} - \tan^{-1}(1) = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$

22. $\int_{0}^{2/3} \frac{dx}{4+9x^2}$ equals

(a) $\frac{\pi}{6}$

(b) $\frac{\pi}{12}$

(c) $\frac{\pi}{24}$

(d) $\frac{\pi}{4}$

SOLUTION

(C) : Let $I = \int_{0}^{2/3} \frac{dx}{4+9x^2} = \frac{1}{9}\int_{0}^{2/3} \frac{dx}{\left(\frac{2}{3}\right)^2 + x^2} = \frac{1}{9} \times \frac{1}{\frac{2}{3}}\left[\tan^{-1}\left(\frac{3x}{2}\right)\right]_{0}^{2/3} = \frac{1}{6}\left[\tan^{-1}\left(\frac{3x}{2}\right)\right]_{0}^{2/3} = \frac{1}{6}\left[\tan^{-1}(1) - \tan^{-1}(0)\right] = \frac{1}{6} \times$

$\frac{\pi}{4} = \frac{\pi}{24}$