



## NCERT Exercise 7.9

**Evaluate the following definite integrals in Exercises 1 to 20 .:**

1.  $\int_{-1}^1 (x+1) dx$

**SOLUTION**

$$\int_{-1}^1 (x+1) dx = \left[ \frac{x^2}{2} + x \right]_{-1}^1 = \frac{1}{2} [(1)^2 - (-1)^2 + [1 - (-1)]] = \frac{1}{2} (1 - 1) + (1 + 1) = \frac{1}{2} (0) + 2 = 2$$

2.  $\int_2^3 \frac{1}{x} dx$  **SOLUTION**

$$\int_2^3 \frac{1}{x} dx = [\log x]_2^3 = \log 3 - \log 2 = \log \frac{3}{2}$$

3.  $\int_1^2 (4x^3 - 5x^2 + 6x + 9) dx$

**SOLUTION**

$$\begin{aligned} \int_1^2 (4x^3 - 5x^2 + 6x + 9) dx &= \left[ 4 \cdot \frac{x^4}{4} - 5 \cdot \frac{x^3}{3} + 6 \cdot \frac{x^2}{2} + 9x \right]_1^2 = \left[ x^4 - \frac{5}{3}x^3 + 3x^2 + 9x \right]_1^2 \\ &= (2^4 - 1^4) - \frac{5}{3}(2^3 - 1^3) + 3(2^2 - 1^2) + 9(2 - 1) \\ &= (16 - 1) - \frac{5}{3}(8 - 1) + 3(4 - 1) + 9(1) = 15 - \frac{35}{3} + 9 + 9 = 33 - \frac{35}{3} = \frac{99 - 35}{3} = \frac{64}{3} \end{aligned}$$

4.  $\int_0^{\pi/4} \sin 2x dx$

**SOLUTION**

$$\int_0^{\pi/4} \sin 2x dx = \left[ -\frac{1}{2} \cos 2x \right]_0^{\pi/4} = -\frac{1}{2} \left( \cos \frac{\pi}{2} - \cos 0 \right) = -\frac{1}{2} (-1) = \frac{1}{2}$$

5.  $\int_0^{\pi/2} \cos 2x dx$

**SOLUTION**

$$\int_0^{\pi/2} \cos 2x dx = \left[ \frac{1}{2} \sin 2x \right]_0^{\pi/2} = \frac{1}{2} (\sin \pi - \sin 0) = \frac{1}{2} (0 - 0) = 0$$

6.  $\int_4^5 e^x dx$

**SOLUTION**

$$\int_4^5 e^x dx = [e^x]_4^5 = e^5 - e^4 = e^4 (e - 1)$$

7.  $\int_0^{\pi/4} \tan x dx$

**SOLUTION**

$$: \int_0^{\pi/4} \tan x dx = [\log \sec x]_0^{\pi/4} = \log \left( \sec \frac{\pi}{4} \right) - \log (\sec \theta) = \log \sqrt{2} - \log 1 = \frac{1}{2} \log 2$$

8.  $\int_{\pi/6}^{\pi/4} \operatorname{cosec} x dx$

**SOLUTION** :  $\int_{\pi/6}^{\pi/4} \operatorname{cosec} x dx = [\log (\operatorname{cosec} x - \cot x)]_{\pi/6}^{\pi/4} = \log \left( \operatorname{cosec} \frac{\pi}{4} - \cot \frac{\pi}{4} \right) - \log \left( \operatorname{cosec} \frac{\pi}{6} - \cot \frac{\pi}{6} \right) = \log (\sqrt{2} - 1) - \log (2 - \sqrt{3}) = \log \left( \frac{(\sqrt{2} - 1)}{2 - \sqrt{3}} \right)$

9.  $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

**SOLUTION**

$$: \int_0^1 \frac{dx}{\sqrt{1-x^2}} = [\sin^{-1} x]_0^1 = \sin^{-1} (1) - \sin^{-1} (0) = \frac{\pi}{2}$$

10.  $\int_0^1 \frac{dx}{1+x^2}$  **SOLUTION**

$$: \int_0^1 \frac{dx}{1+x^2} = [\tan^{-1} x]_0^1 = \tan^{-1} (1) - \tan^{-1} (0) = \frac{\pi}{4}$$

11.  $\int_2^3 \frac{dx}{x^2-1}$

**SOLUTION**

$$\int_2^3 \frac{dx}{x^2-1} = \left[ \frac{1}{2} \log \left( \frac{x-1}{x+1} \right) \right]_2^3 = \frac{1}{2} \left[ \log \left( \frac{3-1}{3+1} \right) - \log \left( \frac{2-1}{2+1} \right) \right] = \frac{1}{2} \left[ \log \left( \frac{2}{4} \right) - \log \left( \frac{1}{3} \right) \right] = \frac{1}{2} \log \left( \frac{2/4}{1/3} \right) = \frac{1}{2} \log \frac{3}{2}$$



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12.  $\int_0^{\pi/2} \cos^2 x dx$

**SOLUTION**

$$\therefore \int_0^{\pi/2} \cos^2 x dx = \int_0^{\pi/2} \frac{1 + \cos 2x}{2} dx = \left[ \frac{1}{2} \left( x + \frac{\sin 2x}{2} \right) \right]_0^{\pi/2} = \frac{1}{2} \left[ \left( \frac{\pi}{2} - 0 \right) + \left( \frac{\sin \pi}{2} - \frac{\sin 0}{2} \right) \right] = \frac{\pi}{4}$$

13.  $\int_2^3 \frac{x dx}{x^2 + 1}$

**SOLUTION**

$$\therefore \int_2^3 \frac{x}{x^2 + 1} dx = \frac{1}{2} \int_2^3 \frac{2x}{x^2 + 1} dx = \frac{1}{2} [\log(x^2 + 1)]_2^3 = \frac{1}{2} [\log 10 - \log 5] = \frac{1}{2} \log \frac{10}{5} = \frac{1}{2} \log 2$$

14.  $\int_0^1 \frac{2x+3}{5x^2+1} dx$

**SOLUTION**

$$\therefore \int_0^1 \frac{2x+3}{5x^2+1} dx = \int_0^1 \left( \frac{2x}{5x^2+1} + \frac{3}{5x^2+1} \right) dx = \frac{1}{5} \int_0^1 \frac{10x}{5x^2+1} dx + \frac{3}{5} \int_0^1 \frac{dx}{x^2 + \left( \frac{1}{\sqrt{5}} \right)^2} = \frac{1}{5} [\log(5x^2+1)]_0^1 + \frac{3}{5} \times \frac{1}{\frac{1}{\sqrt{5}}} \left[ \tan^{-1} \left( \frac{x}{\frac{1}{\sqrt{5}}} \right) \right]_0^1$$

$$= \frac{1}{5} (\log 6 - \log 1) + \frac{3}{\sqrt{5}} (\tan^{-1} \sqrt{5} - 0) = \frac{1}{5} \log 6 + \frac{3}{\sqrt{5}} \tan^{-1} \sqrt{5}$$

\*\*\*\*\*

15.  $\int_0^1 x e^{x^2} dx$

**SOLUTION**

Let  $I = \int_0^1 x e^{x^2} dx = \frac{1}{2} \int_0^1 2x e^{x^2} dx$  Put  $x^2 = t \Rightarrow 2x dx = dt$  When  $x = 0, t = 0$  and when  $x = 1, t = 1 \therefore I = \frac{1}{2} \int_0^1 e^t dt = \left[ \frac{1}{2} e^t \right]_0^1 = \frac{1}{2} [e^1 - e^0] = \frac{1}{2} [e - 1]$

16.  $\int_1^2 \frac{5x^2}{x^2 + 4x + 3} dx$

**SOLUTION**

Let  $I = \int_1^2 \frac{5x^2}{x^2 + 4x + 3} dx$  Since the degree of numerator denominator and is same so the, fraction is improper. To mark it proper,

we have to divide  $5x^2$  by  $x^2 + 4x + 3$ .  $x^2 + 4x + 3 \left) \begin{array}{r} 5 \\ 5x^2 + 20x + 15 \\ \hline -20x - 15 \end{array} \right. \therefore I = \int_1^2 \left( 5 + \frac{-20x - 15}{x^2 + 4x + 3} \right) dx = \int_1^2 \left( 5 - \frac{20x + 15}{x^2 + 4x + 3} \right) dx$

$\frac{20x + 15}{x^2 + 4x + 3} = \frac{20x + 15}{(x+1)(x+3)} = \frac{A}{x+1} + \frac{B}{x+3} \Rightarrow 20x + 15 = A(x+3) + B(x+1) \dots (i)$  Putting  $x = -1$  in (i), we get  $-20 + 15 = A(-1+3) \Rightarrow -5 = 2A \Rightarrow A = -\frac{5}{2}$  Putting  $x = -3$  in (i), we get  $-60 + 15 = B(-3+1) \Rightarrow -45 = -2B \Rightarrow B = \frac{45}{2}$

$$\therefore I = \int_1^2 \left( 5 + \frac{5}{2(x+1)} - \frac{45}{2(x+3)} \right) dx = \left[ 5x + \frac{5}{2} \log(x+1) - \frac{45}{2} \log(x+3) \right]_1^2 = 5(2-1) + \frac{5}{2} [\log 3 - \log 2] - \frac{45}{2} [\log 5 - \log 4]$$

$$= 5 + \frac{5}{2} \log \frac{3}{2} - \frac{45}{2} \log \frac{5}{4} = 5 - \frac{5}{2} \left( 9 \log \frac{5}{4} - \log \frac{3}{2} \right)$$

17.  $\int_0^{\pi/4} (2\sec^2 x + x^3 + 2) dx$

**SOLUTION**

$$\therefore \text{Let } I = \int_0^{\pi/4} (2\sec^2 x + x^3 + 2) dx = \left[ 2 \tan x + \frac{x^4}{4} + 2x \right]_0^{\pi/4} = 2 \left( \tan \frac{\pi}{4} - \tan 0 \right) + \frac{1}{4} \left( \frac{\pi^4}{256} - 0 \right) + 2 \left( \frac{\pi}{4} - 0 \right) = 2(1-0) + \frac{\pi^4}{1024} + \frac{\pi}{2}$$

$$\frac{\pi}{2} = \frac{\pi^4}{1024} + \frac{\pi}{2} + 2$$

18.  $\int_0^{\pi} \left( \sin^2 \frac{x}{2} - \cos^2 \frac{x}{2} \right) dx$

**SOLUTION**

$$\therefore \text{Let } \int_0^{\pi} \left( \sin^2 \frac{x}{2} - \cos^2 \frac{x}{2} \right) dx = - \int_0^{\pi} \cos x dx$$

$$= - [\sin x]_0^{\pi} = -(\sin \pi - \sin 0) = -(0-0) = 0$$

19.  $\int_0^2 \frac{6x+3}{x^2+4} dx$

**SOLUTION**

$$\therefore \text{Let } I = \int_0^2 \frac{6x+3}{x^2+4} dx = \int_0^2 \frac{6x}{x^2+4} dx + \int_0^2 \frac{3}{x^2+4} dx = 3 \int_0^2 \frac{2x}{x^2+4} dx + \left[ 3 \times \frac{1}{2} \tan^{-1} \frac{x}{2} \right]_0^2$$

Let  $I_1 = 3 \int_0^2 \frac{2x}{x^2+4} dx$  Put  $x^2+4 = t \Rightarrow 2x dx = dt$  When  $x = 0, t = 4$  and when  $x = 2, t = 8 \therefore I_1 = 3 \int_4^8 \frac{dt}{t} = [3 \log t]_4^8 = 3(\log 8 - \log 4) = 3 \log 2 \Rightarrow I = 3 \log 2 + \frac{3}{2} [\tan^{-1} 1 - \tan^{-1} 0] = 3 \log 2 + \frac{3}{2} \times \frac{\pi}{4} = 3 \log 2 + \frac{3\pi}{8}$

20.  $\int_0^1 \left( xe^x + \sin \frac{\pi x}{4} \right) dx$

**SOLUTION**

$$\therefore \text{Let } I = \int_0^1 \left[ xe^x + \sin \left( \frac{\pi x}{4} \right) \right] dx = \int_0^1 xe^x dx + \int_0^1 \sin \frac{\pi x}{4} dx = \left[ xe^x - \int \left( \frac{d}{dx}(x) \cdot \int e^x dx \right) dx \right]_0^1 - \left[ \frac{\cos \frac{\pi x}{4}}{\frac{\pi}{4}} \right]_0^1 = \left[ xe^x - \int e^x dx \right]_0^1 - \left[ \frac{\cos \frac{\pi x}{4}}{\frac{\pi}{4}} \right]_0^1$$

$$= \left[ xe^x - e^x \right]_0^1 - \frac{4}{\pi} \left( \cos \frac{\pi}{4} - \cos 0 \right) = (e^1 - 0) - (e^1 - e^0) - \frac{4}{\pi} \left( \frac{1}{\sqrt{2}} - 1 \right) = e - e + 1 - \frac{4}{\pi\sqrt{2}} + \frac{4}{\pi} = 1 + \frac{4}{\pi} - \frac{2\sqrt{2}}{\pi}$$

**Choose the correct answer in Exercises 21 and 22. :**

21.  $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$  equals

- (a)  $\frac{\pi}{3}$
- (b)  $\frac{2\pi}{3}$
- (c)  $\frac{\pi}{6}$
- (d)  $\frac{\pi}{12}$

## SOLUTION

$$: (D) : \text{Let } I = \int_1^{\sqrt{3}} \frac{dx}{1+x^2} = [\tan^{-1}x]_1^{\sqrt{3}} = \tan^{-1}\sqrt{3} - \tan^{-1}(1) = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$$

22.  $\int_0^{2/3} \frac{dx}{4+9x^2}$  equals

- (a)  $\frac{\pi}{6}$
- (b)  $\frac{\pi}{12}$
- (c)  $\frac{\pi}{24}$
- (d)  $\frac{\pi}{4}$

## SOLUTION

$$(C) : \text{Let } I = \int_0^{2/3} \frac{dx}{4+9x^2} = \frac{1}{9} \int_0^{2/3} \frac{dx}{\left(\frac{2}{3}\right)^2 + x^2} = \frac{1}{9} \times \frac{1}{\frac{2}{3}} \left[ \tan^{-1} \left( \frac{3x}{2} \right) \right]_0^{2/3} = \frac{1}{6} \left[ \tan^{-1} \left( \frac{3x}{2} \right) \right]_0^{2/3} = \frac{1}{6} [\tan^{-1}(1) - \tan^{-1}(0)] = \frac{1}{6} \times$$

$$\frac{\pi}{4} = \frac{\pi}{24}$$



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