NCERT Exercises – 7.8

Evaluate the following definite integrals as limits of sums.:

1. $\int_a^b x\,dx$

SOLUTION

Let $f(x)=x$, then $\int_a^b (f)x\,dx=\int_a^b x\,dx$ Now, $f(a)=a, f(a+h)=a+h$ $f(a+2h)=a+2h$ .... $f(a+(n-1)h)=a+(n-1)h$ Since $\int_a^b f(x)\,dx=\lim_{h\to 0} h[f(a)+f($

Where ,$nh=b-a$ $\therefore I=\int_a^b xdx=\lim_{h\to 0} h[a+(a+h)+(a+2h)+...+(a+(n-1)h)]=\lim_{h\to 0} h[na+h(1+2+....+(n-1))]=$
$\lim_{h\to 0} h\left[na+h\frac{(n-1)(n)}{2}\right]=\lim_{h\to 0}\left[\frac{2nah+nh^2(n-1)}{2}\right]=\lim_{h\to 0}\left[\frac{2nah+nh(nh-h)}{2}\right]=\frac{2a(b-a)+(b-a)(b-a)}{2}=\frac{(2a+b-a)(b-a)}{2}$
$=\frac{(b+a)(b-a)}{2}=\frac{b^2-a^2}{2}$

2. $\int_0^5 (x+1)\,dx$

SOLUTION

Let $f(x)=x+1 \Rightarrow \int_0^5 (f)x\,dx=\int_0^5 (x+1)\,dx$ Where, $a=0, b=5, nh=b-a=5-0=5$ Now, $f(a)=f(0)=0+1=1$
$f(a+h)=f(h)=h+1=1+h$ $f(a+2h)=f(2h)=2h+1=1+2h$ .... $f(a+(n-1)h)=f((n-1)h)=1+(n-1)h=1+(n-1)h$ By definition $\int_a^b f(x)dx=$
$\lim_{h\to 0} h[f(a)+f(a+h)+f(a+2h)+...+f(a+(n-1)h)]$ $\therefore \int_0^5 (x+1)\,dx=\lim_{h\to 0} h[1+(1+h)+(1+2h)+....+(1+(n-1)h)]$
$=\lim_{h\to 0} h[(1+1+1+1.....\text{to}\, n\,\text{terms})+h(1+2+3+...+(n-1))]=\lim_{h\to 0} h\left[n+h\frac{(n-1)n}{2}\right]=\lim_{h\to 0}\left[nh+\frac{(nh-h)nh}{2}\right]=\lim_{h\to 0}\left(5+\frac{(5-h)}{2}\right.$
$5+\frac{25}{2}=\frac{35}{2}$
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3. $\int_2^3 x^2\,dx$

SOLUTION

Let $f(x)=x^2 \Rightarrow \int_2^3 f(x)\,dx=\int_2^3 x^2dx$ Where, $a=2, b=3, nh=1$ Now, $f(a)=f(2)=2^2$ $f(a+h)=f(2+h)=(2+h)^2=4+h^2+4h$ $f(a+2h)=f(2+2h)=(2+2h)^2=4+4h^2+8h$ .... $f(a+(n-1)h)=f[2+(n-1)h]=[2+(n-1)h]^2=4+(n-1)^2h^2+4(n-1)h$ By definition $\int_a^b f(x)dx=\lim_{h\to 0} h[f(a)+f(a+h)+f(a+2h)+...+f(a+(n-1)h)]$ $\therefore \int_2^3 x^2dx$

$$= \lim_{h\to 0} h\left[4n + h^2\left(1+4+9+......+(n-1)^2\right) + 4h\left[1+2+...+(n-1)\right]\right] = \lim_{h\to 0} h\left[4n + \frac{h^2(n-1)n(2n-1)}{6} + 4h\frac{(n-1)n}{2}\right] = \lim_{h\to 0}\left[4nh+\right.$$

$$= 4\times 1 + \frac{1\times 2}{6} + 2\times 1\times 1 = 4 + \frac{1}{3} + 2 = 6 + \frac{1}{3} = \frac{19}{3}$$

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4. $\int_{1}^{4} (x^2 - x)\, dx$

SOLUTION

Let $f(x) = x^2 - x \Rightarrow \int_{1}^{4} f(x)\, dx = \int_{1}^{4} (x^2 - x)\, dx$ Where $a = 1, b = 4, nh = 3$ Now, $f(a) = f(1) = 1 - 1 = 0$ $f(a+h) = f(1+h) = (1+h)^2 - (1+h) = h^2 + h$ $f(a+2h) = f(1+2h) = (1+2h)^2 - (1+2h) = 4h^2 + 2h$ $f(a+3h) = f(1+3h) = (1+3h)^2 - (1+3h) = 9h^2 + 3h$ .... $f(a+(n-1)h) = f(1+(n-1)h) = (1+(n-1)h)^2 - (1+(n-1)h) = (n-1)^2h^2 + (n-1)h$ By definition $\int_{a}^{b} f(x)\, dx = \lim_{h\to 0} h\,[f(a) + f(a$

$\therefore \int_{1}^{4} (x^2 - x)\, dx = \lim_{h\to 0} h\left[0 + (h^2 + h) + (4h^2 + 2h) + (9h^2 + 3h) + \ldots\ldots + \left\{(n-1)^2 + (n-1)h\right\}\right] = \lim_{h\to 0} h\left[h^2\left\{1 + 4 + 9 + \ldots + (n-1)^2\right\}\right.$

$= \lim_{h\to 0} h\left[h^2 \frac{(n-1)n(2n-1)}{6} + \frac{h(n-1)n}{2}\right] = \lim_{h\to 0}\left[\frac{(nh-h)nh(2hn-h)}{6} + \frac{(nh-h)nh}{2}\right] = \lim_{h\to 0}\left[\frac{(3-h)3(6-h)}{6} + \frac{(3-h)3}{2}\right] = \frac{3(3)(6)}{6} + \frac{3(3)}{2} = 9 + \frac{9}{2} = \frac{27}{2}$

5. $\int_{-1}^{1} e^x dx$

SOLUTION

Let $f(x) = e^x \Rightarrow \int_{-1}^{1} f(x)\, dx = \int_{-1}^{1} e^x dx$ Where $a = -1, b = 1, nh = b - a = 1 + 1 = 2$ Now, $f(a) = f(-1) = e^{-1}$ $f(a+h) = f(-1+h) = e^{-1+h}$ $f(a+2h) = f(-1+2h) = e^{-1+2h}$ .... $f(a+(n-1)h) = f(-1+(n-1)h) = e^{-1+(n-1)h}$ By definition $\int_{a}^{b} f(x)\, dx = \lim_{h\to 0} h\,[f(a) + f(a+h) + f(a+2h) + \ldots + f(a+(n-1)h)]$ $\therefore \int_{-1}^{1} e^x dx = \lim_{h\to 0} h\left[e^{-1} + e^{-1+h} + e^{-1+2h}\right.$

$= \lim_{h\to 0} h\left[e^{-1}\left(1 + e^h + e^{2h} + \ldots. + e^{(n-1)h}\right)\right] = \lim_{h\to 0} h \cdot \frac{1}{e} \cdot \frac{1\cdot(e^{nh} - 1)}{e^h - 1} = \lim_{h\to 0}\left(\frac{1}{e}\right)\cdot\frac{1\cdot(e^2 - 1)}{\frac{(e^h - 1)}{h}} = \frac{1}{e}\cdot\frac{e^2 - 1}{\log e} = e - \frac{1}{e}$

6. $I = \int_{0}^{4} (x + e^{2x})\, dx$

SOLUTION

Let $f(x) = x + e^{2x} \Rightarrow \int_{0}^{4} f(x)\, dx = \int_{0}^{4} (x + e^{2x})\, dx$ Where $a = 0, b = 4$and $nh = b - a = 4 - 0 = 4$ By definition $\int_{a}^{b} f(x) = \lim_{h\to 0} h\,[f(a) + f(a+h) + f(a+2h) + \ldots + f(a+(n-1)h)]$ $\therefore \int_{0}^{4} (x + e^{2x})\, dx = \lim_{h\to 0} h\,[f(0) + f(h) + f(2h) + \ldots. + f((n-1)h)]$

$= \lim_{h\to 0} h\left[\left\{0 + e^0\right\} + \left\{h + e^{2h}\right\} + \left\{2h + e^{4h}\right\} + \ldots. + \left\{(n-1)h + e^{2(n-1)h}\right\}\right] = \lim_{h\to 0} h\left[h(1 + 2 + 3 + \ldots + (n-1)) + e^{2h}\left(1 + e^{2h} + e^{4h} + \ldots\right.\right.$

$= \lim_{h\to 0} h\left[h\cdot\frac{(n-1)(1+(n-1))}{2} + e^{2h}\cdot\frac{1\cdot\left(e^{2(n-1)h} - 1\right)}{e^{2h} - 1} + 1\right] = \lim_{h\to 0}\left[\frac{(nh-h)(nh)}{2} + \frac{1}{2}\cdot\frac{e^{2h}\left(e^{2(n-1)h} - 1\right)}{\frac{e^{2h} - 1}{2h}} + h\right] = \lim_{h\to 0}\left[\frac{(4-h)4}{2} + \frac{1}{2}\cdot\ldots\right.$

$= \frac{4\times 4}{2} + \frac{1}{2}\cdot\frac{e^0}{\log e}(e^8 - 1) + 0 = 8 + \frac{1}{2}(e^8 - 1) = \frac{16 + e^8 - 1}{2} = \frac{e^8 + 15}{2}$