

**Integrate the functions in Exercise 1 to 9. :**

1.  $\sqrt{4-x^2}$

**SOLUTION**

$$\begin{aligned} \text{Let } I &= \int \sqrt{4-x^2} dx = \int \sqrt{(2)^2 - x^2} dx = \left[ \frac{x}{2} \sqrt{(2)^2 - x^2} + \frac{4}{2} \sin^{-1} \left( \frac{x}{2} \right) \right] + C \\ &= \frac{x\sqrt{4-x^2}}{2} + \frac{4}{2} \sin^{-1} \left( \frac{x}{2} \right) + C = \frac{x\sqrt{4-x^2}}{2} + 2 \sin^{-1} \left( \frac{x}{2} \right) + C \end{aligned}$$

2.  $\sqrt{1-4x^2}$

**SOLUTION**

$$\begin{aligned} \int \sqrt{1-4x^2} dx &= 2 \int \sqrt{\frac{1}{4} - x^2} dx = 2 \int \sqrt{\left(\frac{1}{2}\right)^2 - x^2} dx = 2 \left[ \frac{x}{2} \sqrt{\frac{1}{4} - x^2} + \frac{1}{8} \sin^{-1} \left( \frac{x}{1/2} \right) \right] + C \\ &= \frac{x\sqrt{1-4x^2}}{2} + \frac{1}{4} \sin^{-1} (2x) + C \end{aligned}$$

3.  $\sqrt{x^2+4x+6}$

**SOLUTION**

$$\begin{aligned} \text{Let } I &= \int \sqrt{x^2+4x+6} dx = \int \sqrt{x^2+4x+4+2} dx = \int \sqrt{(x+2)^2 + (\sqrt{2})^2} dx = \frac{x+2}{2} \sqrt{(x+2)^2 + 2} + \frac{2}{2} \log \left| (x+2) + \sqrt{(x+2)^2 + 2} \right| + C \\ &= \frac{x+2}{2} \sqrt{x^2+4x+6} + \log \left| (x+2) + \sqrt{x^2+4x+6} \right| + C \end{aligned}$$

4.  $\sqrt{x^2+4x+1}$

**SOLUTION**

$$\begin{aligned} \text{Let } I &= \int \sqrt{x^2+4x+1} dx = \int \sqrt{(x^2+4x+4) - 3} dx = \int \sqrt{(x+2)^2 - (\sqrt{3})^2} dx = \frac{x+2}{2} \sqrt{(x+2)^2 - 3} - \frac{3}{2} \log \left| (x+2) + \sqrt{(x+2)^2 - 3} \right| + C \\ &= \frac{x+2}{2} \sqrt{x^2+4x+1} - \frac{3}{2} \log \left| (x+2) + \sqrt{x^2+4x+1} \right| + C \end{aligned}$$

5.  $\sqrt{1-4x-x^2}$

**SOLUTION**

$$\begin{aligned} \text{Let } \int \sqrt{1-4x-x^2} dx &= \int \sqrt{1-(x^2+4x+4)+4} dx = \int \sqrt{5-(x+2)^2} dx = \int \sqrt{(5)^2 - (x+2)^2} dx = \frac{x+2}{2} \sqrt{5-(x+2)^2} + \\ &\frac{5}{2} \sin^{-1} \left( \frac{x+2}{\sqrt{5}} \right) + C \\ &= \frac{x+2}{2} \sqrt{x^2+4x+1} - \frac{3}{2} \log \left| (x+2) + \sqrt{x^2+4x+1} \right| + C = \frac{x+2}{2} \sqrt{1-4x-x^2} + \frac{5}{2} \sin^{-1} \left( \frac{x+2}{\sqrt{5}} \right) + C \end{aligned}$$

6.  $\sqrt{x^2+4x-5}$

**SOLUTION**

$$\text{Let } \int \sqrt{x^2+4x-5} dx = \int \sqrt{(x^2+4x+4) - 9} dx = \int \sqrt{(x+2)^2 - (3)^2} dx = \frac{x+2}{2} \sqrt{(x+2)^2 - (3)^2} - \frac{9}{2} \log \left| (x+2) + \sqrt{(x+2)^2 - (3)^2} \right| + C$$

## INTEGRATION

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$$= \frac{x+2}{2}\sqrt{x^2+4x-5} - \frac{9}{2}\log|x+2+\sqrt{x^2+4x-5}| + C = \frac{x+2}{2}\sqrt{1-4x-x^2} + \frac{5}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{5}}\right) + C$$

7.  $\sqrt{1+3x-x^2}$

**SOLUTION**

$$\begin{aligned} \text{Let } I &= \int \sqrt{1+3x-x^2} dx = \int \sqrt{1-(x^2-3x)} dx = \int \sqrt{1-\left(x^2-3x+\frac{9}{4}\right)+\frac{9}{4}} dx = \int \sqrt{\left(\frac{\sqrt{13}}{2}\right)^2 - \left(x-\frac{3}{2}\right)^2} dx = \left[ \frac{x-\frac{3}{2}}{2} \cdot \sqrt{\frac{13}{4} - \left(x-\frac{3}{2}\right)^2} \right. \\ & \left. + \frac{13}{8} \sin^{-1}\left(\frac{2x-3}{\sqrt{13}}\right) \right] + C \\ &= \frac{2x-3}{4}\sqrt{1+3x-x^2} + \frac{13}{8}\sin^{-1}\left(\frac{2x-3}{\sqrt{13}}\right) + C \end{aligned}$$

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8.  $\int \sqrt{x^2 + 3x} dx$

**SOLUTION**

$$\begin{aligned} \text{Let } I &= \int \sqrt{x^2 + 3x} dx = \int \sqrt{1 - \left(x^2 + 3x + \frac{9}{4}\right) - \frac{9}{4}} dx = \int \sqrt{\left(x + \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2} dx = \frac{\left(x + \frac{3}{2}\right)}{2} \sqrt{\left(x + \frac{3}{2}\right)^2 - \frac{9}{4}} - \frac{9}{8} \log \left| \left(x + \frac{3}{2}\right) + \sqrt{\left(x + \frac{3}{2}\right)^2 - \frac{9}{4}} \right| + C \\ &= \frac{2x + 3}{4} \sqrt{x^2 + 3x} - \frac{9}{8} \log \left| x + \frac{3}{2} + \sqrt{x^2 + 3x} \right| + C \end{aligned}$$

9.  $\int \sqrt{1 + \frac{x^2}{9}} dx$

**SOLUTION**

$$\begin{aligned} \text{Let } I &= \int \sqrt{1 + \frac{x^2}{9}} dx = \frac{1}{3} \int \sqrt{9 + x^2} dx = \frac{1}{3} \int \sqrt{x^2 + 3^2} dx = \frac{1}{3} \left[ \frac{x}{2} \sqrt{x^2 + 9} + \frac{9}{2} \log |x + \sqrt{x^2 + 9}| \right] + C \\ &= \frac{x}{6} \sqrt{x^2 + 9} + \frac{3}{2} \log |x + \sqrt{x^2 + 9}| + C \end{aligned}$$

**Choose the correct answer in Exercises 10 to 11.:**

10.  $\int \sqrt{1 + x^2} dx$  is equal to

- (a)  $\frac{x}{2} \sqrt{1 + x^2} + \frac{1}{2} \log |x + \sqrt{1 + x^2}| + C$
- (b)  $\frac{2}{3} (1 + x^2)^{3/2} + C$
- (c)  $\frac{2}{3} x (1 + x^2)^{3/2} + C$
- (d)  $\frac{x^2}{2} \sqrt{1 + x^2} + \frac{1}{2} x^2 \log |x + \sqrt{1 + x^2}| + C$

**SOLUTION**

(A) Let  $I = \int \sqrt{1 + x^2} dx = \frac{x}{2} \sqrt{1 + x^2} + \frac{1}{2} \log |x + \sqrt{1 + x^2}| + C$

11.  $\int \sqrt{x^2 - 8x + 7} dx$  is equal to

- (a)  $\frac{1}{2} (x - 4) \sqrt{x^2 - 8x + 7} + 9 \log |x - 4 + \sqrt{x^2 - 8x + 7}| + C$
- (b)  $\frac{1}{2} (x + 4) \sqrt{x^2 - 8x + 7} + 9 \log |x + 4 + \sqrt{x^2 - 8x + 7}| + C$
- (c)  $\frac{1}{2} (x - 4) \sqrt{x^2 - 8x + 7} - 3\sqrt{2} \log |x - 4 + \sqrt{x^2 - 8x + 7}| + C$
- (d)  $\frac{1}{2} (x - 4) \sqrt{x^2 - 8x + 7} - \frac{9}{4} \log |x - 4 + \sqrt{x^2 - 8x + 7}| + C$

**SOLUTION**

(D) Let  $I = \int \sqrt{x^2 - 8x + 7} dx = \int \sqrt{(x - 4)^2 - 3^2} dx = \frac{x - 4}{2} \sqrt{x^2 - 8x + 7} - \frac{9}{2} \log |(x - 4) + \sqrt{x^2 - 8x + 7}| + C$



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