NCERT Exercises – 7.7

Integrate the functions in Exercise 1 to 9. :

1. $\sqrt{4-x^2}$

SOLUTION

Let $I=\int\sqrt{4-x^2}dx=\int\sqrt{(2)^2-x^2}dx=\left[\frac{x}{2}\sqrt{(2)^2-x^2}+\frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]+C$

$=\frac{x\sqrt{4-x^2}}{2}+\frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)+C=\frac{x\sqrt{4-x^2}}{2}+2\sin^{-1}\left(\frac{x}{2}\right)+C$

2. $\sqrt{1-4x^2}$

SOLUTION

$\int\sqrt{1-4x^2}dx=2\int\sqrt{\frac{1}{4}-x^2}dx=2\int\sqrt{\left(\frac{1}{2}\right)^2-x^2}dx=2\left[\frac{x}{2}\sqrt{\frac{1}{4}-x^2}+\frac{1}{8}\sin^{-1}\left(\frac{x}{1/2}\right)\right]+C$

$=\frac{x\sqrt{1-4x^2}}{2}+\frac{1}{4}\sin^{-1}(2x)+C$

3. $\sqrt{x^2+4x+6}$

SOLUTION

Let $I=\int\sqrt{x^2+4x+6}dx=\int\sqrt{x^2+4x+4+2}dx=\int\sqrt{(x+2)^2+\left(\sqrt{2}\right)^2}dx=\frac{x+2}{2}\sqrt{(x+2)^2+2}+\frac{2}{2}\log\left|(x+2)+\sqrt{(x+2)^2+2}\right|+C$

$=\frac{x+2}{2}\sqrt{x^2+4x+6}+\log\left|(x+2)+\sqrt{x^2+4x+6}\right|+C$

4. $\sqrt{x^2+4x+1}$

SOLUTION

Let $I=\int\sqrt{x^2+4x+1}dx=\int\sqrt{(x^2+4x+4)-3}dx=\int\sqrt{(x+2)^2-\left(\sqrt{3}\right)^2}dx=\frac{x+2}{2}\sqrt{(x+2)^2-3}-\frac{3}{2}\log\left|(x+2)+\sqrt{(x+2)^2-3}\right|+C$

$=\frac{x+2}{2}\sqrt{x^2+4x+1}-\frac{3}{2}\log\left|(x+2)+\sqrt{x^2+4x+1}\right|+C$

5. $\sqrt{1-4x-x^2}$

SOLUTION

Let $\int\sqrt{1-4x-x^2}dx=\int\sqrt{1-(x^2+4x+4)+4}dx=\int\sqrt{5-(x+2)^2}dx=\int\sqrt{(5)^2-(x+2)^2}dx=\frac{x+2}{2}\sqrt{5-(x+2)^2}+\frac{5}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{5}}\right)+C$

$=\frac{x+2}{2}\sqrt{x^2+4x+1}-\frac{3}{2}\log\left|(x+2)+\sqrt{x^2+4x+1}\right|+C=\frac{x+2}{2}\sqrt{1-4x-x^2}+\frac{5}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{5}}\right)+C$

6. $\sqrt{x^2+4x-5}$

SOLUTION

Let $\int\sqrt{x^2+4x-5}dx=\int\sqrt{(x^2+4x+4)-9}dx=\int\sqrt{(x+2)^2-(3)^2}dx=\frac{x+2}{2}\sqrt{(x+2)^2-(3)^2}-\frac{9}{2}\log\left|(x+2)+\sqrt{(x+2)^2-(3)^2}\right|+C$

$$= \frac{x+2}{2}\sqrt{x^2+4x-5} - \frac{9}{2}\log\left|x+2+\sqrt{x^2+4x-5}\right| + C = \frac{x+2}{2}\sqrt{1-4x-x^2} + \frac{5}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{5}}\right) + C$$

7. $\sqrt{1+3x-x^2}$

SOLUTION

Let $I = \int \sqrt{1+3x-x^2}dx = \int \sqrt{1-(x^2-3x)}dx = \int \sqrt{1-\left(x^2-3x+\frac{9}{4}\right)+\frac{9}{4}}dx = \int \sqrt{\left(\frac{\sqrt{13}}{2}\right)^2-\left(x-\frac{3}{2}\right)^2}dx = \left[\frac{x-\frac{3}{2}}{2}\cdot\sqrt{\frac{13}{4}-\left(\right.\right.\right.$

C

$$= \frac{2x-3}{4}\sqrt{1+3x-x^2} + \frac{13}{8}\sin^{-1}\left(\frac{2x-3}{\sqrt{13}}\right) + C$$

8. $\sqrt{x^2+3x}$

SOLUTION

Let $I=\int\sqrt{x^2+3x}dx=\int\sqrt{1-\left(x^2+3x+\frac{9}{4}\right)-\frac{9}{4}}dx=\int\sqrt{\left(x+\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2}dx=\frac{\left(x+\frac{3}{2}\right)}{2}\sqrt{\left(x+\frac{3}{2}\right)^2-\frac{9}{4}}-\frac{9}{8}\log\left|\left(x+\frac{3}{2}\right)+\sqrt{\right.}$

C

$=\frac{2x+3}{4}\sqrt{x^2+3x}-\frac{9}{8}\log\left|x+\frac{3}{2}+\sqrt{x^2+3x}\right|+C$

9. $\sqrt{1+\frac{x^2}{9}}$

SOLUTION

Let $I=\int\sqrt{1+\frac{x^2}{9}}dx=\frac{1}{3}\int\sqrt{9+x^2}dx=\frac{1}{3}\int\sqrt{x^2+3^2}dx=\frac{1}{3}\left[\frac{x}{2}\sqrt{x^2+9}+\frac{9}{2}\log\left|x+\sqrt{x^2+9}\right|\right]+C$

$=\frac{x}{6}\sqrt{x+9}+\frac{3}{2}\log\left|x+\sqrt{x^2+9}\right|+C$

Choose the correct answer in Exercises 10 to 11.:

10. $\int\sqrt{1+x^2}dx$ is equal to

(a) $\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\log\left|\left(x+\sqrt{1+x^2}\right)\right|+C$

(b) $\frac{2}{3}\left(1+x^2\right)^{3/2}+C$

(c) $\frac{2}{3}x\left(1+x^2\right)^{3/2}+C$

(d) $\frac{x^2}{2}\sqrt{1+x^2}+\frac{1}{2}x^2\log\left|x+\sqrt{1+x^2}\right|+C$

SOLUTION

(A) Let $I=\int\sqrt{1+x^2}dx=\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\log\left|x+\sqrt{1+x^2}\right|+C$

11. $\int\sqrt{x^2-8x+7}dx$ is equal to

(a) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}+9\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

(b) $\frac{1}{2}(x+4)\sqrt{x^2-8x+7}+9\log\left|x+4+\sqrt{x^2-8x+7}\right|+C$

(c) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-3\sqrt{2}\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

(d) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-\frac{9}{4}\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

SOLUTION

(D) Let $I=\int\sqrt{x^2-8x+7}dx=\int\sqrt{(x-4)^2-3^2}dx=\frac{x-4}{2}\sqrt{x^2-8x+7}-\frac{9}{2}\log\left|(x-4)+\sqrt{x^2-8x+7}\right|+C$