



Integrate the functions in Exercise 1 to 22.:

1. $x \sin x$ SOLUTION

$$\text{Let } I = \int x \sin x dx = x \int \sin x dx - \int \left[\frac{d}{dx}(x) \int \sin x dx \right] dx = x(-\cos x) - \int 1 \cdot (-\cos x) dx = -x \cos x + \int \cos x dx = -x \cos x + \sin x + C$$

2. $x \sin 3x$

SOLUTION

$$\text{Let } I = \int x \sin 3x dx = x \left(-\frac{\cos 3x}{3} \right) - \int \left[\frac{d}{dx}(x) \left(-\frac{\cos 3x}{3} \right) \right] dx = -\frac{x \cos 3x}{3} + \frac{1}{3} \int \cos 3x dx = -\frac{x \cos 3x}{3} + \frac{1}{3} \cdot \frac{\sin 3x}{3} + C = -\frac{1}{3} x \cos 3x + \frac{1}{9} \sin 3x + C$$

3. $x^2 e^x$

SOLUTION

$$\begin{aligned} \text{Let } I &= \int x^2 e^x dx = x^2 \int e^x dx - \int \left(\frac{d}{dx}(x^2) \int e^x dx \right) dx = x^2 e^x - \left[\int 2x e^x dx \right] + C = x^2 e^x - 2 \int x e^x dx + C = x^2 e^x - 2 \left[x \int e^x dx - \int \left[\frac{d}{dx}(x) \right] e^x dx \right] + C \\ &= x^2 e^x - 2 \left[x e^x - \int e^x dx \right] + C = x^2 e^x - 2x e^x + 2e^x + C = e^x (x^2 - 2x + 2) + C \end{aligned}$$

4. $x \log x$

SOLUTION

$$\begin{aligned} \text{Let } I &= \int x \log x dx = \log x \int x dx - \int \left[\frac{d}{dx}(\log x) \int x dx \right] dx = \log x \left(\frac{x^2}{2} \right) - \int \left(\frac{1}{x} \cdot \frac{x^2}{2} \right) dx = \frac{x^2}{2} \log x - \frac{1}{2} \int x dx + C = \frac{x^2}{2} \log x - \frac{1}{2} \times \frac{x^2}{2} + C = \frac{x^2}{2} \log x - \frac{1}{4} x^2 + C \end{aligned}$$

5. $x \log 2x$

SOLUTION

$$\begin{aligned} \text{Let } \int x \log 2x dx &= (\log 2x) \cdot \frac{x^2}{2} - \int \frac{d}{dx}(\log 2x) \left(\frac{x^2}{2} \right) dx = \log(2x) \cdot \frac{x^2}{2} - \int \frac{2}{2x} \left(\frac{x^2}{2} \right) dx + C = \frac{x^2}{2} \log(2x) - \frac{1}{2} \int x dx + C = \\ &= \frac{x^2}{2} \log(2x) - \frac{1}{2} \cdot \frac{x^2}{2} + C = \frac{x^2}{2} \log(2x) - \frac{x^2}{4} + C \end{aligned}$$

6. $x^2 \log x$

SOLUTION

$$\begin{aligned} \text{Let } I &= \int x^2 \log x dx = \log x \left(\frac{x^3}{3} \right) - \int \left(\left(\frac{d}{dx}(\log x) \right) \left(\frac{x^3}{3} \right) \right) dx = \frac{x^3}{3} \log x - \frac{1}{3} \int x^2 dx = \frac{x^3}{3} \log(x) - \frac{1}{3} \times \frac{x^3}{3} + C = \frac{x^3}{3} \log x - \frac{x^3}{9} + C \end{aligned}$$

7. $x \sin^{-1} x$

SOLUTION

$$\begin{aligned} \text{Let } I &= \int x \sin^{-1} x dx = \int \sin^{-1} x \cdot x dx = \sin^{-1} x \cdot \left(\frac{x^2}{2} \right) - \int \left(\frac{d}{dx}(\sin^{-1} x) \cdot \frac{x^2}{2} \right) dx = \sin^{-1} x \cdot \left(\frac{x^2}{2} \right) - \int \frac{1}{\sqrt{1-x^2}} \cdot \frac{x^2}{2} dx = \frac{x^2}{2} \sin^{-1} x - \\ &= \frac{1}{2} \int \frac{x^2}{\sqrt{1-x^2}} dx = \frac{x^2}{2} \sin^{-1} x - \frac{1}{2} I_1 \Rightarrow I = \frac{x^2}{2} \sin^{-1} x - \frac{1}{2} I_1 \text{ Where, } I_1 = \int \frac{x^2}{\sqrt{1-x^2}} dx \text{ Put } x = \sin \theta \Rightarrow dx = \cos \theta d\theta \therefore I_1 = \end{aligned}$$

INTEGRATION

$$\int \frac{\sin^2 \theta}{\sqrt{1 - \sin^2 \theta}} \cos \theta d\theta = \int \frac{\sin^2 \theta}{\cos \theta} \cos \theta d\theta = \int \sin^2 \theta d\theta = \frac{1}{2} \int (1 - \cos 2\theta) d\theta = \frac{1}{2} \int d\theta - \frac{1}{2} \int \cos 2\theta d\theta = \frac{1}{2} \theta - \frac{1}{2} \frac{\sin 2\theta}{2} + C_1$$

$$= \frac{1}{2} \theta - \frac{1}{2} \sin \theta \cos \theta + C_1 = \frac{1}{2} \sin^{-1} x - \frac{1}{2} x \sqrt{1 - x^2} + C_1 \dots \text{(ii)}$$

From (i) and (ii), we get $I = \frac{x^2}{2} \sin^{-1} x - \frac{1}{2} \left[\frac{1}{2} \sin^{-1} x - \frac{1}{2} x \sqrt{1 - x^2} \right] + C \left[C = \frac{-C_1}{2} \right] = \frac{(2x^2 - 1)}{4} \sin^{-1} x + \frac{x\sqrt{1 - x^2}}{4} + C$

8. $x \tan^{-1} x$

SOLUTION

$$\text{Let } I = \int x \tan^{-1} x dx = \tan^{-1} x \cdot \int x dx - \int \left[\left(\frac{d}{dx} (\tan^{-1} x) \right) \int (x dx) \right] dx = \tan^{-1} x \left(\frac{x^2}{2} \right) - \int \frac{1}{1+x^2} \cdot \frac{x^2}{2} dx = \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \frac{x^2}{x^2+1} dx =$$

$$\frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \frac{x^2 + 1 - 1}{1+x^2} dx = \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \left(1 - \frac{1}{1+x^2} \right) dx = \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} (x - \tan^{-1} x) + C = \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} x + \frac{1}{2} \tan^{-1} x + C$$



www.mathstudy.in

Our Mathematics E-Books

- (a) J.E.E. (Join Entrance Exam)
 - ★ Chapter Tests (Full Syllabus- Fully Solved)
 - ★ Twenty Mock Tests (Full Length - Fully Solved)
- (b) B.I.T.S.A.T. Twenty Mock Tests (Fully Solved)
- (c) C.B.S.E.
 - ★ Work-Book Class XII (Fully Solved)
 - ★ Objective Type Questions Bank C.B.S.E. Class XII (Fully Solved)
 - ★ Chapter Test Papers Class XII (Fully Solved)
 - ★ Past Fifteen Years Topicwise Questions (Fully Solved)
 - ★ Sample Papers Class XII (Twenty Papers Fully Solved- includes 2020 solved paper)
 - ★ Sample Papers Class X (Twenty Papers Fully Solved -includes 2020 solved paper)
- (d) I.C.S.E. & I.S.C.
 - ★ Work-Book Class XII (Fully Solved)
 - ★ Chapter Test Papers Class XII (Fully Solved)
 - ★ Sample Papers Class XII (Twenty Papers Fully Solved -includes 2020 solved paper)
 - ★ Sample Papers Class X (Twenty Papers Fully Solved -includes 2020 solved paper)
- (e) Practice Papers for SAT -I Mathematics (15 Papers - Fully Solved)
- (f) SAT - II Subject Mathematics (15 Papers - Fully Solved)



9. $x \cos^{-1} x$

SOLUTION

$$\begin{aligned} \text{Let } I &= \int x \cos^{-1} x dx = \int \cos^{-1} x \cdot x dx = \cos^{-1} x \cdot \int x dx - \int \left(\frac{d}{dx} (\cos^{-1} x) \int x dx \right) dx = \cos^{-1} x \left(\frac{x^2}{2} \right) - \int \frac{-1}{\sqrt{1-x^2}} \left(\frac{x^2}{2} \right) dx \\ &= \frac{x^2}{2} \cos^{-1} x + \frac{1}{2} \int \frac{x^2}{\sqrt{1-x^2}} dx = \frac{x^2}{2} \cos^{-1} x + \frac{1}{2} I_1 \Rightarrow I = \frac{x^2}{2} \cos^{-1} x + \frac{1}{2} I_1 \dots \text{(i)} \text{ Where } I_1 = \int \frac{x^2}{\sqrt{1-x^2}} dx \text{ Put } x = \cos \theta \Rightarrow dx = \\ &= -\sin \theta d\theta \therefore I_1 = \int \frac{\cos^2 \theta (-\sin \theta)}{\sqrt{1-\cos^2 \theta}} d\theta = - \int \cos^2 \theta d\theta = - \frac{1}{2} \int (1 + \cos 2\theta) d\theta = - \frac{1}{2} \left(\theta + \frac{\sin 2\theta}{2} \right) + C_1 = - \frac{1}{2} \left(\theta + \frac{1}{2} \times 2 \sin \theta \cos \theta \right) + \\ C_1 &= - \frac{1}{2} \left(\theta + \cos \theta \sqrt{1-\cos^2 \theta} \right) + C_1 = - \frac{1}{2} \left(\cos^{-1} x + x \sqrt{1-x^2} \right) + C_1 \dots \text{(ii)} \text{ From (i) and (ii), we get } I = (2x^2 - 1) \frac{\cos^{-1} x}{4} - \\ &\frac{x}{4} \sqrt{1-x^2} + C \left[C = \frac{C_1}{2} \right] \end{aligned}$$

10. $(\sin^{-1} x)^2$

SOLUTION

$$\begin{aligned} \text{Let } I &= \int (\sin^{-1} x)^2 dx \text{ Put } \sin^{-1} x = \theta \Rightarrow x = \sin \theta \Rightarrow dx = \cos \theta d\theta \therefore I = \int \theta^2 \cos \theta d\theta = \theta^2 \int \cos \theta d\theta - \int \left(\frac{d}{d\theta} (\theta^2) \cdot \int \cos \theta d\theta \right) d\theta \\ &= \theta^2 \sin \theta - \int 2\theta \sin \theta d\theta = \theta^2 \sin \theta - 2 \int \theta \sin \theta d\theta + C = \theta^2 \sin \theta - 2 \left[\theta (-\cos \theta) - \int (1) (-\cos \theta) d\theta \right] + C = \theta^2 \sin \theta + 2\theta \cos \theta - \\ &2 \int \cos \theta d\theta + C = \theta^2 \sin \theta + 2\theta \sqrt{1-\sin^2 \theta} - 2 \sin \theta + C = x(\sin^{-1} x)^2 + 2 \sin^{-1} x \sqrt{1-x^2} - 2x + C \end{aligned}$$

11. $\frac{x \cos^{-1} x}{\sqrt{1-x^2}}$

SOLUTION

$$\begin{aligned} \text{Let } I &= \int \frac{x \cos^{-1} x}{\sqrt{1-x^2}} dx \text{ Put } \cos^{-1} x = t \Rightarrow \frac{-1}{\sqrt{1-x^2}} dx = dt \therefore I = - \int t \cos t dt = - \left[t \int \cos t dt - \int \left(\frac{d}{dt} (t) \int \cos t dt \right) dt \right] = \\ &- t \sin t + \int \sin t dt = - t \sin t - \cos t + C = - t \sqrt{1-x^2} - \cos t + C = - \cos^{-1} x \sqrt{1-x^2} - x + C = - \left[\sqrt{1-x^2} \cos^{-1} x + x \right] + C \end{aligned}$$

12. $x \sec^2 x$

SOLUTION

$$\text{Let } I = \int x \sec^2 x dx = x \int \sec^2 x dx - \int \left(\frac{d(x)}{dx} \cdot \int \sec^2 x dx \right) dx = x (\tan x) - \int (1) \tan x dx = x \tan x + \log (\cos x) + C$$

13. $\tan^{-1} x$

SOLUTION

$$\text{Let } I = \int \tan^{-1} x dx = \int \tan^{-1} x \cdot 1 dx = \tan^{-1} x \int (1) dx - \int \left(\frac{d}{dx} (\tan^{-1} x) \cdot \int (1) dx \right) dx = x \tan^{-1} x - \frac{1}{2} \int \frac{2x}{1+x^2} dx + C = x \tan^{-1} x - \frac{1}{2} \log (1+x^2) + C$$

14. $x(\log x)^2$

SOLUTION

$$\begin{aligned} \text{Let } I &= \int x(\log x)^2 dx = \int (\log x)^2 \cdot x dx = (\log x)^2 \int x dx - \int \left(\frac{d}{dx} (\log x)^2 \cdot \int x dx \right) dx = (\log x)^2 \cdot \frac{x^2}{2} - \int \left[(2 \log x) \cdot \frac{1}{x} \right] \left(\frac{x^2}{2} \right) dx + \\ C &= \frac{x^2}{2} (\log x)^2 - \int (\log x) \cdot x dx + C = \frac{x^2}{2} (\log x)^2 - \left[(\log x) \cdot \frac{x^2}{2} - \int \frac{1}{x} \cdot \frac{x^2}{2} dx \right] + C = \frac{x^2}{2} (\log x)^2 - \frac{x^2}{2} \log x + \frac{1}{2} \int x dx + C = \frac{x^2}{2} (\log x)^2 - \\ &\frac{x^2}{2} \log x + \frac{1}{2} \cdot \frac{x^2}{2} + C = \frac{x^2}{2} (\log x)^2 - \frac{x^2}{2} \log x + \frac{1}{4} x^2 + C \end{aligned}$$

INTEGRATION

15. $\int (x^2 + 1) \log x \, dx$

SOLUTION

$$\begin{aligned} \text{Let } I &= \int \log x \cdot (x^2 + 1) \, dx \Rightarrow \log x \cdot \int (x^2 + 1) \, dx - \int \left(\frac{d}{dx}(\log x) \cdot \int (x^2 + 1) \, dx \right) \, dx = \log x \cdot \left(\frac{x^3}{3} + x \right) - \int \frac{1}{x} \left(\frac{x^3}{3} + x \right) \, dx + C \\ &= \left(\frac{x^3}{3} + x \right) \log x - \int \left(\frac{x^2}{3} + 1 \right) \, dx + C = \left(\frac{x^3}{3} + x \right) \log x - \frac{x^3}{9} - x + C \end{aligned}$$

16. $\int e^x (\sin x + \cos x) \, dx$

SOLUTION

$$\text{Let } I = \int e^x (\sin x + \cos x) \, dx \text{ Put } e^x \sin x = t \Rightarrow (e^x \cos x + \sin x e^x) \, dx = dt \Rightarrow e^x (\sin x + \cos x) \, dx = dt \therefore I = \int dt = t + C = e^x \sin x + C$$

17. $\int \frac{x e^x}{(1+x)^2} \, dx$

SOLUTION

$$\text{Let } I = \int \frac{x e^x}{(1+x)^2} \, dx \Rightarrow I = \int e^x \left[\frac{1+x-1}{(1+x)^2} \right] \, dx \Rightarrow I = \int e^x \left[\frac{1}{1+x} - \frac{1}{(1+x)^2} \right] \, dx = \int e^x \left[\frac{1}{1+x} + \left(\frac{d}{dx} \left(\frac{1}{1+x} \right) \right) \right] \, dx = \left[e^x \cdot \frac{1}{1+x} + C \right]$$

18. $\int e^x \left(\frac{1 + \sin x}{1 + \cos x} \right) \, dx$

SOLUTION

$$\begin{aligned} \therefore \text{Let } I &= \int \frac{e^x (1 + \sin x)}{1 + \cos x} \, dx \Rightarrow I = \int e^x \left[\frac{1 + 2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \cos^2 \frac{x}{2}} \right] \, dx = \int e^x \left[\frac{1}{2} \sec^2 \frac{x}{2} + \tan \frac{x}{2} \right] \, dx \Rightarrow I = \int e^x \left[\tan \frac{x}{2} + \frac{1}{2} \sec^2 \frac{x}{2} \right] \, dx = \int e^x \left[\tan \frac{x}{2} + \left(\frac{d}{dx} \left(\frac{1}{2} \sec^2 \frac{x}{2} \right) \right) \right] \, dx \\ &= \int e^x \cdot \tan \left(\frac{x}{2} \right) \, dx + C \end{aligned}$$

19. $\int e^x \left(\frac{1}{x} - \frac{1}{x^2} \right) \, dx$

SOLUTION

$$\text{Let } I = \int e^x \left(\frac{1}{x} - \frac{1}{x^2} \right) \, dx = \int e^x \left(\frac{1}{x} + \left(\frac{d}{dx} \left(\frac{1}{x} \right) \right) \right) \, dx = \left[e^x \times \frac{1}{x} + C \right]$$

20. $\int \frac{(x-3)e^x}{(x-1)^3} \, dx$

SOLUTION

$$\text{Let } I = \int \frac{(x-3)e^x}{(x-1)^3} \, dx = \int \frac{e^x((x-1)-2)}{(x-1)^3} \, dx = \int e^x \left[\frac{1}{(x-1)^2} - \frac{2}{(x-1)^3} \right] \, dx = \int e^x \left[\frac{1}{(x-1)^2} + \frac{d}{dx} \left(\frac{1}{(x-1)^2} \right) \right] \, dx = \frac{e^x}{(x-1)^2} + C$$

21. $\int e^{2x} \sin x \, dx$

SOLUTION

$$\begin{aligned} \text{Let } I &= \int e^{2x} \sin x \, dx = e^{2x} \int \sin x \, dx - \int \left(\frac{d}{dx}(e^{2x}) \cdot \int \sin x \, dx \right) \, dx = e^{2x}(-\cos x) - \int 2e^{2x}(-\cos x) \, dx + C_1 = -e^{2x} \cos x + 2 \int e^{2x} \cos x \, dx + C_1 \\ &= -e^{2x} \cos x + 2 \left[e^{2x} \int \cos x \, dx - \int \left(\frac{d}{dx}(e^{2x}) \cdot \int \cos x \, dx \right) \, dx \right] + C_1 = -e^{2x} \cos x + 2e^{2x} \sin x - 4 \int e^{2x} \sin x \, dx + C_1 = e^{2x} (2 \sin x - \cos x) - 4I + C_1 \\ \therefore 5I &= e^{2x} (2 \sin x - \cos x) + C_1 \Rightarrow I = \frac{e^{2x}}{5} (2 \sin x - \cos x) + C \left[C = \frac{C_1}{5} \right] \end{aligned}$$

22. $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

SOLUTION

$$\begin{aligned} \text{Let } I &= \int \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx \text{ Put } x = \tan t \Rightarrow dx = \sec^2 t dt \therefore I = \int \sin^{-1}\left(\frac{2 \tan t}{1+\tan^2 t}\right) \sec^2 t dt = \int \sin^{-1}(\sin 2t) \sec^2 t dt = \int 2t \sec^2 t dt = \\ &= 2 \int t \sec^2 t dt = 2 \left[t \int \sec^2 t dt - \int \left(\frac{d}{dt}(t) \cdot \int \sec^2 t dt\right) dt \right] = 2 \left[t \tan t - \int 1 \cdot \tan t dt \right] = 2t \tan t + 2 \log |\cos t| + C = 2 \tan^{-1} x \cdot x + \\ &= 2 \log \left| \frac{1}{\sqrt{1+x^2}} \right| + C \\ &= 2x \tan^{-1} x + 2 \log \left[(1+x^2)^{\frac{1}{2}} \right] + C = 2x \tan^{-1} x + 2 \left(-\frac{1}{2} \right) \log |1+x^2| + C = 2x \tan^{-1} x - \log (1+x^2) + C \end{aligned}$$

Choose the correct answer in each of the Exercises 23 and 24. :

23. $\int x^2 e^{x^3} dx$ equals

- (a) $\frac{1}{3}e^{x^3} + C$
- (b) $\frac{1}{3}e^{x^2} + C$
- (c) $\frac{1}{2}e^{x^3} + C$
- (d) $\frac{1}{2}e^{x^2} + C$

SOLUTION

$$\therefore \text{(A) Let } I = \int x^2 e^{x^3} dx \text{ Put } x^3 = t \Rightarrow 3x^2 dx = dt \therefore I = \frac{1}{3} \int e^t dt = \frac{1}{3} e^t + C = \frac{1}{3} e^{x^3} + C$$

24. $\int e^x \sec x (1 + \tan x) dx$ equals

- (a) $e^x \cos x + C$
- (b) $e^x \sec x + C$
- (c) $e^x \sin x + C$
- (d) $e^x \tan x + C$

SOLUTION

$$\therefore \text{(B) } \int e^x (\sec x + \sec x \tan x) dx = e^x \sec x + C$$



www.mathstudy.in

Our Mathematics E-Books

1. J.E.E. (Join Entrance Exam)
 - ★ Chapter Tests (Full Syllabus- Fully Solved)
 - ★ Twenty Mock Tests (Full Length - Fully Solved)
2. B.I.T.S.A.T. Twenty Mock Tests (Fully Solved)
3. C.B.S.E.
 - ★ Work-Book Class XII (Fully Solved)
 - ★ Objective Type Questions Bank C.B.S.E. Class XII (Fully Solved)
 - ★ Chapter Test Papers Class XII (Fully Solved)
 - ★ Past Fifteen Years Topicwise Questions (Fully Solved)
 - ★ Sample Papers Class XII (Twenty Papers Fully Solved- includes 2020 solved paper)
 - ★ Sample Papers Class X (Twenty Papers Fully Solved -includes 2020 solved paper)
4. I.C.S.E. & I.S.C.
 - ★ Work-Book Class XII (Fully Solved)
 - ★ Chapter Test Papers Class XII (Fully Solved)
 - ★ Sample Papers Class XII (Twenty Papers Fully Solved -includes 2020 solved paper)
 - ★ Sample Papers Class X (Twenty Papers Fully Solved -includes 2020 solved paper)
5. Practice Papers for SAT -I Mathematics (15 Papers - Fully Solved)
6. SAT - II Subject Mathematics (15 Papers - Fully Solved)

