NCERT Exercise – 7.6

Integrate the functions in Exercise 1 to 22.:

1. $x\sin x$ SOLUTION

Let $I=\int x\sin x dx = x\int \sin x dx - \int\left[\frac{d}{dx}(x)\int \sin x dx\right]dx = x(-\cos x)-\int 1\cdot(-\cos x)\,dx = -x\cos x+\int \cos x dx = -x\cos x+\sin x+C$

2. $x\sin 3x$

SOLUTION

Let $I=\int x\sin 3x dx = x\left(-\frac{\cos 3x}{3}\right)-\int\left[\frac{d}{dx}(x)\left(\frac{-\cos 3x}{3}\right)\right]dx = -\frac{x\cos 3x}{3}+\frac{1}{3}\int \cos 3x dx = -\frac{x\cos 3x}{3}+\frac{1}{3}\cdot\frac{\sin 3x}{3}+C = -\frac{1}{3}x\cos 3x+\frac{1}{9}\sin 3x+C$

3. x^2e^x

SOLUTION

Let $I=\int x^2e^2dx = x^2\int e^x dx-\int\left(\frac{d}{dx}(x^2)\int e^x dx\right)dx = x^2e^x-\left[\int 2xe^x dx\right]+C = x^2e^x-2\int xe^x dx+C = x^2e^x-2\left[x\int e^x dx-\int\left[\frac{d}{dx}(x)\right.\right.$

$C = x^2e^x-2\left[xe^x-\int e^x dx\right]+C = x^2e^x-2xe^x+2e^x+C = e^x\left(x^2-2x+2\right)+C$

4. $x\log x$

SOLUTION

Let $I=\int x\log x dx = \log x\int x dx-\int\left[\frac{d}{dx}(\log x)\int x dx\right]dx = \log x\left(\frac{x^2}{2}\right)-\int\left(\frac{1}{x}\cdot\frac{x^2}{2}\right)dx = \frac{x^2}{2}\log x-\frac{1}{2}\int x dx+C = \frac{x^2}{2}\log x-\frac{1}{2}\times\frac{x^2}{2}+C = \frac{x^2}{2}\log x-\frac{1}{4}x^2+C$

5. $x\log 2x$

SOLUTION

Let $\int x\log 2x dx = (\log 2x)\cdot\frac{x^2}{2}-\int\frac{d}{dx}(\log 2x)\left(\frac{x^2}{2}\right)dx = \log(2x)\cdot\frac{x^2}{2}-\int\frac{2}{2x}\left(\frac{x^2}{2}\right)dx+C = \frac{x^2}{2}\log(2x)-\frac{1}{2}\int x dx+C = \frac{x^2}{2}\log(2x)-\frac{1}{2}\cdot\frac{x^2}{2}+C = \frac{x^2}{2}\log(2x)-\frac{x^2}{4}+C$

6. $x^2\log x$

SOLUTION

Let $I=\int x^2\log x dx = \log x\left(\frac{x^3}{3}\right)-\int\left(\left(\frac{d}{dx}(\log x)\right)\left(\frac{x^3}{3}\right)\right)dx = \frac{x^3}{3}\log x-\frac{1}{3}\int x^2dx = \frac{x^3}{3}\log(x)-\frac{1}{3}\times\frac{x^3}{3}+C = \frac{x^3}{3}\log x-\frac{x^3}{9}+C$

7. $x\sin^{-1}x$

SOLUTION

Let $I=\int x\sin^{-1}x dx = \int\sin^{-1}x\cdot x dx = \sin^{-1}x\cdot\left(\frac{x^2}{2}\right)-\int\left(\frac{d}{dx}\left(\sin^{-1}x\right)\cdot\frac{x^2}{2}\right)dx = \sin^{-1}x\cdot\left(\frac{x^2}{2}\right)-\int\frac{1}{\sqrt{1-x^2}}\cdot\frac{x^2}{2}dx = \frac{x^2}{2}\sin^{-1}x-\frac{1}{2}\int\frac{x^2}{\sqrt{1-x^2}}dx = \frac{x^2}{2}\sin^{-1}x-\frac{1}{2}I_1 \Rightarrow I = \frac{x^2}{2}\sin^{-1}x-\frac{1}{2}I_1$ Where, $I_1=\int\frac{x^2}{\sqrt{1-x^2}}dx$ Put $x=\sin\theta \Rightarrow dx=\cos\theta d\theta \therefore I_1 =$

$$\int \frac{\sin^2\theta}{\sqrt{1-\sin^2\theta}}\cos\theta d\theta = \int \frac{\sin^2\theta}{\cos\theta}\cos\theta d\theta = \int \sin^2\theta d\theta = \frac{1}{2}\int (1-\cos 2\theta)\, d\theta = \frac{1}{2}\int d\theta - \frac{1}{2}\int \cos 2\theta d\theta = \frac{1}{2}\theta - \frac{1}{2}\frac{\sin 2\theta}{2} + C_1$$

$$= \frac{1}{2}\theta - \frac{1}{2}\sin\theta\cos\theta + C_1 = \frac{1}{2}\sin^{-1}x - \frac{1}{2}x\sqrt{1-x^2} + C_1 \ldots \text{(ii)}$$

From (i) and (ii), we get $I = \frac{x^2}{2}\sin^{-1}x - \frac{1}{2}\left[\frac{1}{2}\sin^{-1}x - \frac{1}{2}x\sqrt{1-x^2}\right] + C \left[C = \frac{-C_1}{2}\right] = \frac{(2x^2-1)}{4}\sin^{-1}x + \frac{x\sqrt{1-x^2}}{4} + C$

8. $x\tan^{-1}x$

SOLUTION

Let $I = \int x\tan^{-1}x dx = \tan^{-1}x \cdot \int x dx - \int \left[\left(\frac{d}{dx}(\tan^{-1}x)\right)\int (x dx)\right] dx = \tan^{-1}x\left(\frac{x^2}{2}\right) - \int \frac{1}{1+x^2}\cdot\frac{x^2}{2}dx = \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int \frac{x^2}{x^2+1}dx =$

$\frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int \frac{x^2+1-1}{1+x^2}dx = \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int \left(1 - \frac{1}{1+x^2}\right) dx = \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}(x - \tan^{-1}x) + C = \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}x + \frac{1}{2}\tan^{-1}x +$

C

9. $x\cos^{-1}x$

SOLUTION

Let $I=\int x\cos^{-1}xdx=\int\cos^{-1}x\cdot xdx=\cos^{-1}x\cdot\int xdx-\int\left(\frac{d}{dx}(\cos^{-1}x)\int xdx\right)dx=\cos^{-1}x\left(\frac{x^2}{2}\right)-\int\frac{-1}{\sqrt{1-x^2}}\left(\frac{x^2}{2}\right)dx$
$=\frac{x^2}{2}\cos^{-1}x+\frac{1}{2}\int\frac{x^2}{\sqrt{1-x^2}}dx=\frac{x^2}{2}\cos^{-1}x+\frac{1}{2}I_1\Rightarrow I=\frac{x^2}{2}\cos^{-1}x+\frac{1}{2}I_1$ ....(i) Where $I_1=\int\frac{x^2}{\sqrt{1-x^2}}dx$ Put $x=\cos\theta\Rightarrow dx=$
$-\sin\theta d\theta\therefore I_1=\int\frac{\cos^2\theta(-\sin\theta)}{\sqrt{1-\cos^2\theta}}d\theta=-\int\cos^2\theta d\theta=-\frac{1}{2}\int(1+\cos2\theta)d\theta=-\frac{1}{2}\left(\theta+\frac{\sin2\theta}{2}\right)+C_1=-\frac{1}{2}\left(\theta+\frac{1}{2}\times2\sin\theta\cos\theta\right)+$
$C_1=-\frac{1}{2}\left(\theta+\cos\theta\sqrt{(1-\cos^2\theta)}\right)+C_1=-\frac{1}{2}\left(\cos^{-1}x+x\sqrt{1-x^2}\right)+C_1$ ....(ii) From (i) and (ii), we get $I=\left(2x^2-1\right)\frac{\cos^{-1}x}{4}-$
$\frac{x}{4}\sqrt{1-x^2}+C\left[C=\frac{C_1}{2}\right]$

10. $\left(\sin^{-1}x\right)^2$

SOLUTION

Let $I=\int\left(\sin^{-1}x\right)^2dx$ Put $\sin^{-1}x=\theta\Rightarrow x=\sin\theta\Rightarrow dx=\cos\theta d\theta\therefore I=\int\theta^2\cos\theta d\theta=\theta^2\int\cos\theta\,d\theta-\int\left(\frac{d}{d\theta}(\theta^2)\cdot\int\cos\theta d\theta\right)d\theta$
$=\theta^2\sin\theta-\int2\theta\sin\theta d\theta=\theta^2\sin\theta-2\int\theta\sin\theta d\theta+C=\theta^2\sin\theta-2\left[\theta(-\cos\theta)-\int(1)(-\cos\theta)d\theta\right]+C=\theta^2\sin\theta+2\theta\cos\theta-$
$2\int\cos\theta\,d\theta+C=\theta^2\sin\theta+2\theta\sqrt{1-\sin^2\theta}-2\sin\theta+C=x\left(\sin^{-1}x\right)^2+2\sin^{-1}x\sqrt{1-x^2}-2x+C$

11. $\frac{x\cos^{-1}x}{\sqrt{1-x^2}}$

SOLUTION

Let $I=\int\frac{x\cos^{-1}x}{\sqrt{1-x^2}}dx$ Put $\cos^{-1}x=t\Rightarrow\frac{-1}{\sqrt{1-x^2}}dx=dt\therefore I=-\int t\cos t dt=-\left[t\int\cos t\,dt-\int\left(\frac{d}{dt}(t)\int\cos t\,dt\right)dt\right]=$
$-t\sin t+\int\sin t\,dt=-t\sin t-\cos t+C=-t\sqrt{1-\cos^2t}-\cos t+C=-\cos^{-1}x\sqrt{1-x^2}-x+C=-\left[\sqrt{1-x^2}\cos^{-1}x+x\right]+C$

12. $x\sec^2x$

SOLUTION

Let $I=\int x\sec^2xdx=x\int\sec^2xdx-\int\left(\frac{d(x)}{dx}\cdot\int\sec^2xdx\right)dx=x(\tan x)-\int(1)\tan xdx=x\tan x+\log(\cos x)+C$

13. $\tan^{-1}x$

SOLUTION

Let $I=\int\tan^{-1}xdx=\int\tan^{-1}x\cdot1dx=\tan^{-1}x\int(1)dx-\int\left(\frac{d}{dx}(\tan^{-1}x)\cdot\int(1)dx\right)dx=x\tan^{-1}x-\frac{1}{2}\int\frac{2x}{1+x^2}dx+C=x\tan^{-1}x-\frac{1}{2}\log($

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14. $x(\log x)^2$

SOLUTION

Let $I=\int x(\log x)^2dx=\int(\log x)^2\cdot xdx=(\log x)^2\int xdx-\int\left(\frac{d}{dx}(\log x)^2\cdot\int xdx\right)dx=(\log x)^2\cdot\frac{x^2}{2}-\int\left[(2\log x)\cdot\frac{1}{x}\right]\left(\frac{x^2}{2}\right)dx+$
$C=\frac{x^2}{2}(\log x)^2-\int(\log x)\cdot xdx+C=\frac{x^2}{2}(\log x)^2-\left[(\log x)\cdot\frac{x^2}{2}-\int\frac{1}{x}\cdot\frac{x^2}{2}dx\right]+C=\frac{x^2}{2}(\log x)^2-\frac{x^2}{2}\log x+\frac{1}{2}\int xdx+C=\frac{x^2}{2}(\log x)^2-$
$\frac{x^2}{2}\log x+\frac{1}{2}\cdot\frac{x^2}{2}+C=\frac{x^2}{2}(\log x)^2-\frac{x^2}{2}\log x+\frac{1}{4}x^2+C$

15. . $(x^2+1)\log x$

SOLUTION

Let $I=\int \log x\cdot(x^2+1)dx \Rightarrow \log x\cdot\int(x^2+1)\,dx-\int\left(\frac{d}{dx}(\log x)\cdot\int(x^2+1)\,dx\right)dx=\log x\cdot\left(\frac{x^3}{3}+x\right)-\int\frac{1}{x}\left(\frac{x^3}{3}+x\right)dx+C$

$=\left(\frac{x^3}{3}+x\right)\log x-\int\left(\frac{x^2}{3}+1\right)dx+C=\left(\frac{x^3}{3}+x\right)\log x-\frac{x^3}{9}-x+C$

16. $e^x(\sin x+\cos x)$

SOLUTION

Let $I=\int e^x(\sin x+\cos x)\,dx$ Put $e^x\sin x=t \Rightarrow (e^x\cos x+\sin xe^x)dx=dt \Rightarrow e^x(\sin x+\cos x)\,dx=dt \therefore I=\int dt=t+C=e^x\sin x+C$

17. $\frac{xe^x}{(1+x)^2}$

SOLUTION

Let$I=\int\frac{xe^x}{(1+x)^2}dx \Rightarrow I=\int e^x\left[\frac{1+x-1}{(1+x)^2}\right]dx \Rightarrow I=\int e^x\left[\frac{1}{1+x}-\frac{1}{(1+x)^2}\right]dx=\int e^x\left[\frac{1}{1+x}+\left(\frac{d}{dx}\left(\frac{1}{1+x}\right)\right)\right]dx=\left[e^x\cdot\frac{1}{1+x}+C\right]$

18. . $e^x\left(\frac{1+\sin x}{1+\cos x}\right)$

SOLUTION

: Let$I=\int\frac{e^x(1+\sin x)}{1+\cos x}dx \Rightarrow I=\int e^x\left[\frac{1+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right]=\int e^x\left[\frac{1}{2}\sec^2\frac{x}{2}+\tan\frac{x}{2}\right]dx \Rightarrow I=\int e^x\left[\tan\frac{x}{2}+\frac{1}{2}\sec^2\frac{x}{2}\right]dx=\int e^x\left[\tan\frac{x}{2}+\left(\frac{d}{d\right.\right.$

$=\int e^x\cdot\tan\left(\frac{x}{2}\right)+C$

19. . $e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)$

SOLUTION

Let $I=\int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx=\int e^x\left(\frac{1}{x}+\left(\frac{d}{dx}\left(\frac{1}{x}\right)\right)\right)dx=\left[e^x\times\frac{1}{x}+C\right]$

20. . $\frac{(x-3)e^x}{(x-1)^3}$

SOLUTION

Let$I=\int\frac{(x-3)e^x}{(x-1)^3}dx=\int\frac{e^x((x-1)-2)}{(x-1)^3}dx=\int e^x\left[\frac{1}{(x-1)^2}-\frac{2}{(x-1)^3}\right]dx=\int e^x\left[\frac{1}{(x-1)^2}+\frac{d}{dx}\left(\frac{1}{(x-1)^2}\right)\right]dx=\frac{e^x}{(x-1)^2}+C$

21. . $e^{2x}\sin x$

SOLUTION

Let$I=\int e^{2x}\sin xdx=e^{2x}\int\sin x\,dx-\int\left(\frac{d}{dx}(e^{2x})\cdot\int\sin x\,dx\right)dx=e^{2x}(-\cos x)-\int 2e^{2x}(-\cos x)\,dx+C_1=-e^{2x}\cos x+2\int e^{2x}\cos x\,dx+$

$=-e^{2x}\cos x+2\left[e^{2x}\int\cos x\,dx-\int\left(\frac{d}{dx}(e^{2x})\cdot\int\cos x\,dx\right)dx\right]+C_1=-e^{2x}\cos x+2e^{2x}\sin x-4\int e^{2x}\sin x\,dx+C_1=e^{2x}(2\sin x-\cos x)-$

$4I+C_1 \therefore 5I=e^{2x}(2\sin x-\cos x)+C_1 \Rightarrow I=\frac{e^{2x}}{5}(2\sin x-\cos x)+C\left[C=\frac{C_1}{5}\right]$

22. $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

SOLUTION

Let $I=\int \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$ Put $x=\tan t \Rightarrow dx=\sec^2 t\,dt \therefore I=\int \sin^{-1}\left(\frac{2\tan t}{1+\tan^2 t}\right)\sec^2 t dt=\int \sin^{-1}(\sin 2t)\sec^2 t dt=\int 2t\sec^2 t\,dt=$

$2\int t\sec^2 t\,dt=2\left[t\int \sec^2 t\,dt-\int\left(\frac{d}{dt}(t)\cdot\int \sec^2 t\,dt\right)dt\right]=2\left[t\tan t-\int 1\cdot\tan t\,dt\right]=2t\tan t+2\log|\cos t|+C=2\tan^{-1}x\cdot x+$

$2\log\left|\frac{1}{\sqrt{1+x^2}}\right|+C$

$=2x\tan^{-1}x+2\log\left[(1+x^2)\frac{1}{2}\right]+C=2x\tan^{-1}x+2\left(-\frac{1}{2}\right)\log|1+x^2|+C=2x\tan^{-1}x-\log(1+x^2)+C$

Choose the correct answer in each of the Exercises 23 and 24. :

23. $\int x^2 e^{x^3}dx$ equals

(a) $\frac{1}{3}e^{x^3}+C$

(b) $\frac{1}{3}e^{x^2}+C$

(c) $\frac{1}{2}e^{x^3}+C$

(d) $\frac{1}{2}e^{x^2}+C$

SOLUTION

:(A) Let $I=\int x^2 e^{x^3}dx$ Put $x^3=t\Rightarrow 3x^2dx=dt\therefore I=\frac{1}{3}\int e^t dt=\frac{1}{3}e^t+C=\frac{1}{3}e^{x^3}+C$

24. $\int e^x\sec x(1+\tan x)dx$ equals

(a) $e^x\cos x+C$

(b) $e^x\sec x+C$

(c) $e^x\sin x+C$

(d) $e^x\tan x+C$

SOLUTION

: (B) $\int e^x(\sec x+\sec x\tan x)dx=e^x\sec x+C$