NCERT Exercise -7.4

Integrate the rational functions in Exercise 1 to 23. :

1. $\dfrac{3x^2}{x^6+1}$

SOLUTION

: Let $I = \int \dfrac{3x^2}{x^6+1}dx$ Put $x^3 = t \Rightarrow 3x^2dx = dt \therefore I = \int \dfrac{dt}{t^2+1} = \tan^{-1}t + C = \tan^{-1}\left(x^3\right) + C$

2. $\dfrac{1}{\sqrt{1+4x^2}}$

SOLUTION

: Let $I = \int \dfrac{1}{1+4x^2}dx = \dfrac{1}{2}\int \dfrac{dx}{\sqrt{\dfrac{1}{4}+x^2}} = \dfrac{1}{2}\int \dfrac{dx}{\sqrt{\left(\dfrac{1}{2}\right)^2+x^2}} = \dfrac{1}{2}\log\left|x+\sqrt{\dfrac{1}{4}+x^2}\right| + C_1 = \dfrac{1}{2}\log\left|\dfrac{2x+\sqrt{1+4x^2}}{2}\right| + C_1 = \dfrac{1}{2}\log\left|2x+\sqrt{1+4.\right.$

$\dfrac{1}{2}\log 2 + C_1 = \dfrac{1}{2}\log\left|2x+\sqrt{1+4x^2}\right| + C\ \left[C = \dfrac{-1}{2}\log 2 + C_1\right]$ Alternative solution : Let $I = \int \dfrac{1}{\sqrt{1+4x^2}}dx = \int \dfrac{dx}{\sqrt{1+(2x)^2}}$ Put

$2x = \tan\theta \Rightarrow 2dx = \sec^2\theta d\theta \therefore I = \dfrac{1}{2}\int \dfrac{\sec^2\theta}{\sqrt{1+\tan^2\theta}}d\theta = \dfrac{1}{2}\int \dfrac{\sec^2\theta}{\sec\theta}d\theta = \dfrac{1}{2}\int \sec\theta d\theta = \dfrac{1}{2}\log|\sec\theta+\tan\theta| + C = \dfrac{1}{2}\log\left|\tan\theta+\sqrt{1+\tan^2\right.$

$C = \dfrac{1}{2}\log\left|2x+\sqrt{1+(2x)^2}\right| + C = \dfrac{1}{2}\log\left|2x+\sqrt{1+4x^2}\right| + C$

3. $\dfrac{1}{\sqrt{(2-x)^2+1}}$

SOLUTION

: Let $I = \int \dfrac{dx}{\sqrt{(2-x)^2+1}}$ Put $(2-x) = t \Rightarrow -dx = dt \Rightarrow dx = -dt \therefore I = -\int \dfrac{dt}{\sqrt{t^2+1}} = -\log\left|t+\sqrt{t^2+1}\right| + C = -\log\left|(2-x)+\sqrt{(2-\right.$

$C = \log\left|\dfrac{1}{(2-x)+\sqrt{x^2-4x+5}}\right| + C$

4. $\dfrac{1}{\sqrt{9-25x^2}}$

SOLUTION

: Let$I = \int \dfrac{dx}{\sqrt{9-25x^2}} = \dfrac{1}{5}\int \dfrac{dx}{\sqrt{\dfrac{9}{25}-x^2}} = \dfrac{1}{5}\int \dfrac{dx}{\sqrt{\left(\dfrac{3}{5}\right)^2-x^2}} = \dfrac{1}{5}\sin^{-1}\left(\dfrac{x}{3/5}\right) + C = \dfrac{1}{5}\sin^{-1}\left(\dfrac{5x}{3}\right) + C$ Alternative solution : Let

$I = \int \dfrac{dx}{\sqrt{9-25x^2}} = \int \dfrac{dx}{\sqrt{9-(5x)^2}}$ Put $5x = 3\sin\theta \Rightarrow 5dx = 3\cos\theta\, d\theta \therefore I = \dfrac{1}{5}\int \dfrac{3\cos\theta\, d\theta}{\sqrt{9-9\sin^2\theta}} = \dfrac{1}{5}\int \dfrac{\cos\theta\, d\theta}{\sqrt{1-\sin^2\theta}} = \dfrac{1}{5}\int d\theta =$

$\dfrac{1}{5}\theta + C = \dfrac{1}{5}\sin^{-1}\left(\dfrac{5x}{3}\right) + C$

5. $\dfrac{3x}{1+2x^4}$

SOLUTION

: Let $I=\int \frac{3x}{1+2x^4}dx$ Put $x^2=t \Rightarrow 2xdx=dt \Rightarrow xdx=\frac{dt}{2} \therefore I=\frac{3}{2}\int \frac{dt}{1+2t^2}=\frac{3}{4}\int \frac{dt}{\frac{1}{2}+t^2}=\frac{3}{4}\int \frac{dt}{\left(\frac{1}{\sqrt{2}}\right)^2+t^2}=\frac{3}{4}\cdot\frac{1}{1/\sqrt{2}}\tan^{-1}\left(\frac{t}{1/\sqrt{2}}\right)+$
$C=\frac{3}{2\sqrt{2}}\tan^{-1}\left(\sqrt{2}t\right)+C=\frac{3}{2\sqrt{2}}\tan^{-1}\left(\sqrt{2}x^2\right)+C$

6. $\frac{x^2}{1-x^6}$

SOLUTION

Let $I=\int \frac{x^2dx}{1-x^6}=\int \frac{x^2}{1-(x^3)^2}dx$ Put $x^3=t \Rightarrow 3x^2dx=dt \therefore I=\frac{1}{3}\int \frac{dt}{1-t^2}=\frac{1}{3}.\frac{1}{2}\log\left|\frac{1+t}{1-t}\right|+C=\frac{1}{6}\log\left|\frac{1+t}{1-t}\right|+C=\frac{1}{6}\log\left|\frac{1+x^3}{1-x^3}\right|+$
C

7. $\frac{x-1}{\sqrt{x^2-1}}$

SOLUTION

: Let $I=\int \frac{x-1}{\sqrt{x^2-1}}dx=\int \frac{x}{\sqrt{x^2-1}}dx-\int \frac{1}{\sqrt{x^2-1}}dx=I_1-I_2$ (say) Now, $I_1=\int \frac{x}{\sqrt{x^2-1}}dx$ Put $x^2-1=t \Rightarrow 2x\,dx=dt$
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$\therefore\ I=\frac{1}{2}\int \frac{dt}{\sqrt{t}}=\frac{1}{2}\int t^{-1/2}dt=\frac{1}{2}\times\frac{t^{1/2}}{1/2}+C_1=\sqrt{t}+C_1=\sqrt{x^2-1}+C_1$ And $I_2=\int \frac{1}{\sqrt{x^2-1}}dx=\log\left|x+\sqrt{x^2-1}\right|+C_2\ \therefore$
$I=\sqrt{x^2-1}-\log\left|x+\sqrt{x^2-1}\right|+C$ where,$C=C_1-C_2$

8. $\frac{x^2}{\sqrt{x^6+a^2}}$

SOLUTION

Let $I=\int \frac{x^2}{x^6+a^6}dx=\int \frac{x^2dx}{\sqrt{(x^3)^2+(a^3)^2}}$ Put $x^3=t \Rightarrow 3x^2dx=dt=\frac{1}{3}\int \frac{dt}{\sqrt{t^2+(a^3)^2}}=\frac{1}{3}\log\left|t+\sqrt{t^2+a^6}\right|+C=\frac{1}{3}\log\left|x^3+\sqrt{x^6+a^6}\right|+$
C

9. $\frac{\sec^2x}{\sqrt{\tan^2x+4}}$

SOLUTION

: Let $I=\frac{\sec^2x}{\sqrt{\tan^2x+4}}dx$ Put $x=t \Rightarrow \sec^2xdx=dt=\int \frac{dt}{\sqrt{t^2+(2)^2}}=\log\left|t+\sqrt{t^2+4}\right|+C=\log\left|\tan x+\sqrt{\tan^2x+4}\right|+C$

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10. $\dfrac{1}{\sqrt{x^2+2x+2}}$

SOLUTION

Let $I=\int \dfrac{1}{\sqrt{x^2+2x+2}}dx=\int \dfrac{dx}{\sqrt{(x+1)^2+1}} \Rightarrow I=\log\left|(x+1)+\sqrt{(x+1)^2+1}\right|+C=\log\left|(x+1)+\sqrt{x^2+2x+2}\right|+C$

11. $\dfrac{1}{9x^2+6x+5}$

SOLUTION

Let$I=\int \dfrac{1}{9x^2+6x+5}=\dfrac{1}{9}\int \dfrac{dx}{x^2+\frac{2}{3}x+\frac{5}{9}}=\dfrac{1}{9}\int \dfrac{dx}{\left(x^2+\frac{2}{3}x+\frac{1}{9}\right)+\left(\frac{5}{9}-\frac{1}{9}\right)}=\dfrac{1}{9}\int \dfrac{dx}{\left(x+\frac{1}{3}\right)^2+\left(\frac{2}{3}\right)^2}$

$=\dfrac{1}{9}\times\dfrac{1}{2/3}\tan^{-1}\left(\dfrac{x+\frac{1}{3}}{\frac{2}{3}}\right)+C=\dfrac{1}{6}\tan^{-1}\left(\dfrac{3x+1}{2}\right)+C$

12. $\dfrac{1}{\sqrt{7-6x-x^2}}$

SOLUTION

: Let $I=\int \dfrac{1}{\sqrt{7-6x-x^2}}dx=\int \dfrac{1}{\sqrt{7-(x^2+6x)}}dx=\int \dfrac{dx}{\sqrt{16-(x^2+6x+9)}}=\int \dfrac{1}{\sqrt{(4)^2-(x+3)^2}}dx=\sin^{-1}\left(\dfrac{x+3}{4}\right)+C$

13. $\dfrac{1}{\sqrt{(x-1)(x-2)}}$

SOLUTION

Let$=\int \dfrac{1}{\sqrt{(x-1)(x-2)}}dx=\int \dfrac{dx}{\sqrt{x^2-3x+2}}=\int \dfrac{dx}{\sqrt{\left(x^2-2\cdot\frac{3}{2}x+\frac{9}{4}\right)+2-\frac{9}{4}}}=\int \dfrac{dx}{\sqrt{\left(x-\frac{3}{2}\right)^2-\frac{1}{4}}}=\int \dfrac{dx}{\sqrt{\left(x-\frac{3}{2}\right)^2+\left(\frac{1}{2}\right)^2}}=$

$\log\left|\left(x-\dfrac{3}{2}\right)+\sqrt{\left(x-\dfrac{3}{2}\right)^2-\left(\dfrac{1}{2}\right)^2}\right|+C=\log\left|\left(x-\dfrac{3}{2}\right)+\sqrt{(x-1)(x-2)}\right|+C$

14. . $\dfrac{1}{\sqrt{8+3x-x^2}}$

SOLUTION

: Let $I=\int \dfrac{dx}{\sqrt{8+3x-x^2}}=\int \dfrac{dx}{\sqrt{8-(x^2-3x)}}=\int \dfrac{dx}{\sqrt{8-\left(x^2-2\cdot\frac{3}{2}\cdot x+\frac{9}{4}\right)+\frac{9}{4}}}=\int \dfrac{dx}{\sqrt{\frac{41}{4}-\left(x-\frac{3}{2}\right)^2}}=\int \dfrac{dx}{\sqrt{\left(\frac{\sqrt{41}}{2}\right)^2-\left(x-\frac{3}{2}\right)^2}}$

$=\sin^{-1}\left(\dfrac{x-\frac{3}{2}}{\frac{\sqrt{41}}{2}}\right)+C=\sin^{-1}\left(\dfrac{2x-3}{\sqrt{41}}\right)+C$

15. $\dfrac{1}{\sqrt{(x-a)(x-b)}}$

SOLUTION .: Let $I=\int \frac{1}{\sqrt{(x-a)(x-b)}}dx=\int \frac{dx}{\sqrt{x^2-(a+b)x+ab}}=\int \frac{dx}{\sqrt{x^2-2\left(\frac{a+b}{2}\right)x+\left(\frac{a+b}{2}\right)^2+ab-\left(\frac{a+b}{2}\right)^2}}$

$=\int \frac{dx}{\sqrt{\left(x-\left(\frac{a+b}{2}\right)\right)^2-\left(\frac{a-b}{2}\right)^2}}=\log\left|\left(x-\frac{a+b}{2}\right)+\sqrt{\left(x-\frac{a+b}{2}\right)^2-\left(\frac{a-b}{2}\right)^2}\right|+C=\log\left|\left(x-\frac{a+b}{2}\right)+\sqrt{(x-a)(x-b)}\right|+$
C

16\. . $\frac{4x+1}{\sqrt{2x^2+x-3}}$

SOLUTION

: Let $I=\int \frac{4x+1}{\sqrt{2x^2+x-3}}dx$ Put $2x^2+x-3=t \Rightarrow (4x+1)\,dx=dt \therefore I=\int \frac{dt}{\sqrt{t}}=\int t^{1/2}dt=2t^{1/2}+C=2\sqrt{2x^2+x-3}+C$

17\. $\frac{x+2}{\sqrt{x^2-1}}$

SOLUTION

: Let $I=\int \frac{x+2}{\sqrt{x^2-1}}dx=\int \frac{x}{\sqrt{x^2-1}}dx+\int \frac{2}{\sqrt{x^2-1}}dx=I_1+I_2$ (say) $\Rightarrow I=I_1+I_2$ …..(i) Now, $I_1=\int \frac{x}{\sqrt{x^2-1}}dx$ Put $x^2-1=t \Rightarrow 2x\,dx=dt \therefore I_1=\frac{1}{2}\int \frac{dt}{t}=\frac{1}{2}\int t^{-1/2}dt=\frac{1}{2}\times\frac{t^{1/2}}{1/2}=\sqrt{t}+C_1=\sqrt{x^2-1}+C_1$ ….(ii) And $I_2=\int \frac{2}{\sqrt{x^2-1}}dx=2\log\left|x+\sqrt{x^2-1}\right|+C_2$ ….(ii) From (i), (ii) and (iii), we get $I=\sqrt{x^2-1}+2\log\left|x+\sqrt{x^2-1}\right|+C$ Where, $C=C_1+C_2$

18\. $\frac{5x-2}{1+2x+3x^2}$

SOLUTION

: Let $I=\int \frac{5x-2}{3x^2+2x+1}dx$ We put , $5x-2=A\left(\frac{d}{dx}(3x^2+2x+1)\right)+B \Rightarrow 5x-2=A(6x+2)+B$ ….(i) Comparing coefficients of x in (i), we get $5=6A \Rightarrow A=\frac{5}{6}$

Comparing the constant terms in (i), we get $-2=2A+B \Rightarrow -2=2\times\frac{5}{6}+B \Rightarrow -2=\frac{5}{3}+B \Rightarrow B=-\frac{11}{3} \therefore I=\int \frac{\frac{5}{6}(6x+2)-\frac{11}{3}}{3x^2+2x+1}dx$ $\Rightarrow I=\frac{5}{6}\int \frac{6x+2}{3x^2+2x+1}dx-\frac{11}{3}\int \frac{dx}{3x^2+2x+1} \Rightarrow I=\frac{5}{6}I_1-\frac{11}{3}I_2$ …..(ii) Let $I_1=\int \frac{6x+2}{3x^2+2x+1}dx$ Put $3x^2+2x+1=t \Rightarrow (6x+2)\,dx=dt \therefore I_1=\int \frac{dt}{t}=\log|t|+C_1=\log\left|3x^2+2x+1\right|+C_1$ ….(iii) Let $I_2=\int \frac{dx}{3x^2+2x+1}=\frac{1}{3}\int \frac{dx}{x^2+\frac{2}{3}x+\frac{1}{3}}=\frac{1}{3}\int \frac{dx}{x^2+\frac{2}{3}x+\frac{1}{9}}$

$\frac{1}{3}\int \frac{dx}{\left(x+\frac{1}{3}\right)^2+\frac{2}{9}}=\frac{1}{3}\int \frac{dx}{\left(x+\frac{1}{3}\right)^2+\left(\frac{\sqrt{2}}{3}\right)^2}=\frac{1}{3}\times\frac{1}{\sqrt{2}/3}\tan^{-1}\left(\frac{x+\frac{1}{3}}{\frac{\sqrt{2}}{3}}\right)=\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)+C_2$ ….(iv) From (ii), (iii) and (iv) we get $I=\frac{5}{6}\log(3x^2+2x+1)-\frac{11}{3\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}+C\right)\left[C=\frac{5}{6}C_1-\frac{11}{3}C_2\right]$

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19\. . $\frac{6x+7}{\sqrt{(x-5)(x-4)}}$

SOLUTION

: Let $I=\int \frac{6x+7}{\sqrt{(x-5)(x-4)}}dx=\int \frac{(6x+7)\,dx}{\sqrt{x^2-9x+20}}$ Let $6x+7=A\times\left[\frac{d}{dx}(x^2-9x+20)\right]+B \Rightarrow 6x+7=A(2x-9)+B$ ....(i) Comparing coefficients of x in (i), we get $2A=6\Rightarrow A=3$ Comparing constant terms in (i), we get $7=-9A+B\Rightarrow 7=-9\times 3+B$ $\Rightarrow B=34 \therefore I=\int \frac{3(2x-9)+34}{\sqrt{x^2-9x+20}}dx$ $I=3\int \frac{2x-9}{\sqrt{x^2-9x+20}}dx+34\int \frac{dx}{\sqrt{x^2-9x+20}}$ $I=3I_1+34I_2$ (Say) ....(ii) Let $I_1=\int \frac{2x-9}{\sqrt{x^2-9x+20}}dx=\int \frac{dt}{t}=\int t^{1/2}dt=2t^{1/2}+C_1=2\sqrt{x^2-9x+20}+C_1$ .....(iii) $I_2=\int \frac{dx}{\sqrt{x^2-9x+20}}=\int \frac{dx}{\sqrt{x^2-9x+\left(\frac{9}{2}\right)^2-\left(\frac{9}{2}\right)}}$

$I_2=\int \frac{dx}{\sqrt{\left(x-\frac{9}{2}\right)^2-\frac{81}{4}+20}}=\int \frac{dx}{\sqrt{\left(x-\frac{9}{2}\right)^2-\left(\frac{1}{2}\right)^2}}=\log\left|x-\frac{9}{2}+\sqrt{\left(x-\frac{9}{2}\right)^2-\left(\frac{1}{2}\right)^2}\right|+C_2=\log\left|\left(x-\frac{9}{2}\right)+\sqrt{x^2-9x+20}\right|+$ C_2 .....(iv) From (ii),(iii) and (iv) we get $I=3\times 2\sqrt{x^2-9x+20}+34\log\left|\left(x-\frac{9}{2}\right)+\sqrt{x^2-9x+20}\right|+C$ Or $I=6\sqrt{x^2-9x+20}+$ $34\log\left|\left(x-\frac{9}{2}\right)+\sqrt{x^2-9x+20}\right|+C$ $[C_1=3C_1+34C_2]$

20. $\frac{x+2}{\sqrt{4x-x^2}}$

SOLUTION

: Let $I=\int \frac{x+2}{\sqrt{4x-x^2}}dx=\int \frac{x+2}{\sqrt{4-(x^2-4x+4)}}dx=\int \frac{(x-2)+4}{\sqrt{4-(x-2)^2}}dx=\int \frac{x-2}{\sqrt{4-(x-2)^2}}dx+4\int \frac{dx}{\sqrt{4-(x-2)^2}}=I_1+$ $4\sin^{-1}\left(\frac{x-2}{2}\right)+C_2$ ...(i) Where $I_1=\int \frac{(x-2)\,dx}{\sqrt{4-(x-2)^2}}$ Put $(x-2)^2=t\Rightarrow 2(x-2)\,dx=dt \therefore I_1=\frac{1}{2}\int \frac{dt}{\sqrt{4-t}}=\frac{1}{2}\left[\frac{(4-t)^{-1/2+1}}{-\left(\frac{-1}{2}+1\right)}\right]+$ $C_1=-\sqrt{(4-t)}+C_1=-\sqrt{4-(x-2)^2}+C_1=-\sqrt{4-x^2-4+4x}+C_1=-\sqrt{4x-x^2}+C_1$ ....(ii) From (i) and (ii) we get $I=-\sqrt{4x-x^2}+4\sin^{-1}\left(\frac{x-2}{2}\right)+C$ $[C=C_1+C_2]$

21. $\frac{x+2}{\sqrt{x^2+2x+3}}$

SOLUTION

: Let $I=\int \frac{x+2}{\sqrt{x^2+2x+3}}dx=\frac{1}{2}\int \frac{2x+4}{\sqrt{x^2+2x+3}}dx=\frac{1}{2}\int \frac{2x+2+2}{\sqrt{x^2+2x+3}}dx=\frac{1}{2}\int \frac{2x+2}{\sqrt{x^2+2x+3}}dx+\int \frac{dx}{\sqrt{x^2+2x+3}}$ Let $I=I_1+$ I_2 (say) ......(i) Where $I_1=\frac{1}{2}\int \frac{2x+2}{\sqrt{x^2+2x+3}}dx=\frac{1}{2}\int \frac{dt}{t}=\frac{1}{2}\int t^{-1/2}dt=\frac{1}{2}\times 2t^{1/2}=\frac{1}{2}\times 2\sqrt{x^2+2x+3}+C_1=\sqrt{x^2+2x+3}+$ C_1 ...(ii) Also, $I_2=\int \frac{dx}{\sqrt{x^2+2x+3}}=\int \frac{dx}{\sqrt{x^2+2x+1-1+3}}=\int \frac{dx}{\sqrt{(x+1)^2+\left(\sqrt{2}\right)^2}}=\log\left|(x+1)+\sqrt{(x+1)^2+2}\right|=\log\left|(x+1)+\right.$ C_2 ....(iii) Hence from (i), (ii) and (iii), we get $I=\sqrt{x^2+2x+3}+\log\left|(x+1)+\sqrt{x^2+2x+3}\right|+C$ $[C=C_1+C_2]$

22. . $\frac{x+3}{x^2-2x-5}$

SOLUTION

: Let$I=\int \frac{x+3}{x^2-2x-5}dx$ Let $x+3=A\left(\frac{d}{dx}(x^2-2x-5)\right)+B=A(2x-2)+B$ ....(i) Comparing the coefficients of x in (i), we get $1=2A\Rightarrow A=\frac{1}{2}$ Comparing the constant terms in (i), we get $3=B-2A\Rightarrow 3=B-1\Rightarrow B=4 \therefore I=\int \frac{\frac{1}{2}(2x-2)+4}{x^2-2x-5}dx$

$= \frac{1}{2}\int \frac{2x-2}{x^2-2x-5}dx + 4\int \frac{dx}{x^2-2x-5} \Rightarrow I = \frac{1}{2}I_1 + 4I_2$ ....(ii) Where $I_1 = \int \frac{2x-2}{x^2-2x-5}dx$ Put $x^2-2x-5 = t \Rightarrow (2x-2)\,dx = dt$ $\therefore I_1 = \int \frac{dt}{t} = \log|t| = \log|x^2-2x-5| + C_1$ ....(iii) And $I_2 = \int \frac{dx}{x^2-2x-5} = \int \frac{dx}{(x-1)^2-6} = \int \frac{dx}{(x-1)^2-\left(\sqrt{6}\right)^2} = \frac{1}{2\sqrt{6}}\log\left|\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right| + C_2$ ....(iv) Hence, from $(ii), (iii)$ and (iv),we get $I = \frac{1}{2}\log\left|(x^2-2x-5)\right| + \frac{2}{\sqrt{6}}\log\left|\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right| + C$ $\left[C = \frac{1}{2}C_1 + 4C_2\right]$

23. $\frac{5x+3}{\sqrt{x^2+4x+10}}$

SOLUTION

: Let $I = \int \frac{5x+3}{\sqrt{x^2+4x+10}}dx$ We write $5x+3 = A\frac{d}{dx}\left(x^2+4x+10\right) + B \Rightarrow 5x+3 = 2xA + 4A + B$ ....(i) Comparing the coefficients of x in (i), we get $5 = 2A \Rightarrow A = 5/2$ Comparing the constant terms in (i), we get $3 = 4A + B \Rightarrow B = -7$ $\therefore$ $I = \int \frac{\frac{5}{2}(2x+4)+(-7)}{\sqrt{x^2+4x+10}}dx = \frac{5}{2}\int \frac{2x+4}{\sqrt{x^2+4x+10}}dx - 7\int \frac{dx}{\sqrt{x^2+4x+10}} = \frac{5}{2}I_1 - 7I_2$ (say) $\Rightarrow I = \frac{5}{2}I_1 - 7I_2$ ....(ii) Now, $I_1 = \int \frac{2x+4}{\sqrt{x^2+4x+10}}dx$ Put $x^2+4x+10 = t \Rightarrow (2x+4)\,dx = dt \therefore I_1 = \int \frac{dt}{\sqrt{t}} = \int t^{-1/2}dt = 2\sqrt{t} = 2\sqrt{x^2+4x+10} + C_1$ ....(iii) And $I_2 = \int \frac{dx}{\sqrt{x^2+4x+10}} = \int \frac{dx}{\sqrt{(x+2)^2+\left(\sqrt{6}\right)^2}} = \log\left|x+2+\sqrt{(x+2)^2+\left(\sqrt{6}\right)^2}\right| = \log\left|x+2+\sqrt{x^2+4x+10}\right| + C_2$ ....(iv)

Hence, from (ii), (iii) and (iv) we get $I = 5\sqrt{x^2+4x+10} - 7\log\left|x+2+\sqrt{x^2+4x+10}\right| + C$ $\left[C = \frac{5}{2}C_1 - 7C_2\right]$

Choose the correct answer in each of the Exercises 24 and 25. :

24. $\int \frac{dx}{x^2+2x+2}$ equals

(a) $x\tan^{-1}(x+1) + C$
(b) $\tan^{-1}(x+1) + C$
(c) $(x+1)\tan^{-1}x + C$
(d) $\tan^{-1}x + C$

SOLUTION

:(B) Let $I = \int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1} = \tan^{-1}(x+1) + C$

25. $\int \frac{dx}{\sqrt{9x-4x^2}}$ equals

(a) $\frac{1}{9}\sin^{-1}\left(\frac{9x-8}{8}\right) + C$
(b) $\frac{1}{2}\sin^{-1}\left(\frac{8x-9}{9}\right) + C$
(c) $\frac{1}{3}\sin^{-1}\left(\frac{9x-8}{8}\right) + C$
(d) $\frac{1}{2}\sin^{-1}\left(\frac{9x-8}{9}\right) + C$

SOLUTION

: (B) Let $I = \int \frac{dx}{\sqrt{9x-4x^2}} = \frac{1}{2}\int \frac{dx}{\sqrt{-x^2+\frac{9}{4}x}} = \frac{1}{2}\int \frac{dx}{\sqrt{-\left(x^2-\frac{9}{4}x\right)}} = \frac{1}{2}\int \frac{dx}{\sqrt{\left(\frac{9}{8}\right)^2 - \left[x^2-\frac{9}{4}x+\left(\frac{9}{8}\right)^2\right]}} = \frac{1}{2}\int \frac{dx}{\sqrt{\left(\frac{9}{8}\right)^2 - \left(x-\frac{9}{8}\right)^2}}$

$C = \frac{1}{2}\sin^{-1}\left(\frac{8x-9}{9}\right) + C$