NCERT - Exercise -7.3

Find the Integrate of the functions in Exercise 1 to 22.:

1. $\sin^2(2x+5)$

SOLUTION

$: \text{Let } I = \int \sin^2(2x+5)\,dx = \frac{1}{2}\int [1-\cos 2(2x+5)]dx$

$= \frac{1}{2}\int [1-\cos(4x+10)]\,dx = \frac{1}{2}\left[x - \frac{\sin(4x+10)}{4}\right] + C$

2. $\sin 3x \cos 4x$

SOLUTION

$: \text{Let} I = \int \sin 3x \cos 4x dx = \frac{1}{2}\int [\sin(3x+4x)+\sin(3x-4x)]dx = \frac{1}{2}\int [\sin 7x + \sin(-x)]dx$

$= \frac{1}{2}\int (\sin 7x - \sin x)\,dx = \frac{1}{2}\left(\frac{-\cos 7x}{7}\right) - \frac{1}{2}(-\cos x) + C = -\frac{1}{14}\cos 7x + \frac{1}{2}\cos x + C$

3. $\cos 2x \cos 4x \cos 6x$

SOLUTION

$: \text{Let} I = \int \cos 2x \cos 4x \cos 6x\,dx = \frac{1}{2}\int (2\cos 2x \cos 4x)\cos 6x\,dx = \frac{1}{2}\int (\cos 6x + \cos 2x)\cos 6x\,dx$

$= \frac{1}{4}\int 2\cos^2 6x dx + \frac{1}{4}\int (2\cos 2x cos6x)\,dx = \frac{1}{4}\int (1+\cos 12x)\,dx + \frac{1}{4}\int (\cos 8x + \cos 4x)\,dx = \frac{1}{4}x + \frac{1}{4}\left(\frac{\sin 12x}{12}\right) + \frac{1}{4}\left(\frac{\sin 8x}{8} + \frac{\sin 4x}{4}\right) +$

$C = \frac{1}{4}\left[x + \frac{1}{12}\sin 12x + \frac{1}{8}\sin 8x + \frac{1}{4}\sin 4x\right] + C$

4. $\sin^3(2x+1)$

SOLUTION

$: \text{Let } I = \int \sin^3(2x+1)dx = \frac{1}{4}\int [3\sin(2x+1) - \sin 3(2x+1)]\,dx$

$= \frac{3}{4}\left(-\frac{\cos(2x+1)}{2}\right) - \frac{1}{4}\left(\frac{-\cos 3(2x+1)}{6}\right) + C = -\frac{3}{8}\cos(2x+1) + \frac{1}{24}\cos 3(2x+1) + C = -\frac{3}{8}\cos(2x+1) + \frac{1}{24}[4\cos^3(2x+1) - 3\cos$

C

$= -\frac{3}{8}\cos(2x+1) + \frac{1}{6}\cos^3(2x+1) - \frac{1}{8}\cos(2x+1) + C = -\frac{1}{2}\cos(2x+1) + \frac{1}{6}\cos^3(2x+1) + C$

5. $\sin^3 x \cos^3 x$

SOLUTION

$: \text{Let } I = \int \sin^3 x \cos^3 x dx = \int \sin x \cdot \sin^2 x \cdot \cos^3 x dx = \int \sin x\left(1-\cos^2 x\right)\cos^3 x dx = \int \left(\cos^3 x - \cos^5 x\right)\sin x dx$ Put $\cos x = t \Rightarrow$

$-\sin x dx = dt \therefore I = -\int \left(t^3 - t^5\right)dt = \frac{-t^4}{4} + \frac{t^6}{6} + C = \frac{1}{6}\cos^6 x - \frac{1}{4}\cos^4 x + C$

6. $\sin x \sin 2x \sin 3x$

SOLUTION

$: \text{Let } I = \int \sin x \sin 2x \sin 3x dx = \frac{1}{2}\int (2\sin x \sin 2x)\sin 3x dx = \frac{1}{2}\int (\cos x - \cos 3x)\sin 3x\,dx$

$= \frac{1}{4}\int 2\sin 3x\, cosx\, dx - \frac{1}{4}\int 2sin3x\, cos3x\, dx$

$$= \frac{1}{4}\int (\sin 4x + \sin 2x)\,dx - \frac{1}{4}\int \sin 6x\,dx = -\frac{1}{16}\cos 4x - \frac{1}{8}\cos 2x + \frac{1}{24}\cos 6x + C = \frac{1}{4}\left[\frac{1}{6}\cos 6x - \frac{1}{4}\cos 4x - \frac{1}{2}\cos 2x\right] + C$$

7. $\sin 4x \sin 8x$

SOLUTION

.: Let $I = \int \sin 4x \sin 8x dx = \frac{1}{2}\int 2\sin 4x \sin 8x dx = \frac{1}{2}\int (\cos 4x - \cos 12x)dx$

$$= \frac{1}{2}\left[\frac{1}{4}\sin 4x - \frac{1}{12}\sin 12x\right] + C$$

8. $\dfrac{1-\cos x}{1+\cos x}$

SOLUTION

: Let $I = \int \frac{1-\cos x}{1+\cos x}dx = \int \frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}dx = \int \tan^2\frac{x}{2}dx = \int \left(\sec^2\frac{x}{2} - 1\right)dx = \frac{\tan\frac{x}{2}}{(1/2)} - x + C = 2\tan\frac{x}{2} - x + C$

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9. $\dfrac{\cos x}{1+\cos x}$

SOLUTION

: Let $I=\int \frac{\cos x}{1+\cos x}dx=\int \frac{(1+\cos x)-1}{1+\cos x}dx=\int (1)dx-\int \frac{1}{1+\cos x}dx$

$=x-\int \frac{1}{2\cos^2\frac{x}{2}}dx+C=x-\frac{1}{2}\int \sec^2\frac{x}{2}dx+C=x-\frac{1}{2}\cdot\frac{\tan\frac{x}{2}}{(1/2)}+C=x-\tan\frac{x}{2}+C$

10. $\sin^4 x$

SOLUTION

: Let $I=\int \sin^4 x\,dx=\int\left(\frac{1-\cos 2x}{2}\right)^2 dx=\frac{1}{4}\int\left(1+\cos^2 2x-2\cos x2x\right)dx=\frac{1}{4}\int\left[1+\frac{1+\cos 4x}{2}-2\cos 2x\right]dx=\frac{1}{4}\int(1)\,dx+$
$\frac{1}{8}\int(1+\cos 4x)\,dx-\frac{2}{4}\int\cos 2xdx$

$=\frac{3}{8}\int(1)\,dx+\frac{1}{8}\int\cos 4x\,dx-\frac{1}{2}\int\cos 2x\,dx=\frac{3}{8}x+\frac{1}{32}\sin 4x-\frac{1}{4}\sin 2x+C$

11. $\cos^4 2x$

SOLUTION

Let $I=\int\cos^4 2x\,dx=\int\left(\frac{1+\cos 4x}{2}\right)^2 dx=\frac{1}{4}\int\left(1+\cos^2 4x+2\cos 4x\right)dx=\frac{1}{4}\int\left[1+\frac{1+\cos 8x}{2}+2\cos 4x\right]dx=\frac{3}{8}\int dx+$
$\frac{1}{8}\int\cos 8x\,dx+\frac{1}{2}\int\cos 4x\,dx\cdot=\frac{3}{8}x+\frac{1}{64}\sin 8x+\frac{1}{8}\sin 4x+C$

12. $\dfrac{\sin^2 x}{1+\cos x}$

SOLUTION

: Let $I=\int\frac{\sin^2 x}{1+\cos x}dx=\int\frac{1-\cos^2 x}{1+\cos x}dx=\int(1-\cos x)\,dx=x-\sin x+C$

13. . $\dfrac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}$

SOLUTION

: Let $I=\int\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}dx=\int\frac{(2\cos^2 x-1)-(2\cos^2\alpha-1)}{\cos x-\cos\alpha}dx=\int\frac{2\left(\cos^2 x-\cos^2\alpha\right)}{\cos x-\cos\alpha}dx=\int\frac{2(\cos x-\cos\alpha)(\cos x+\cos\alpha)}{\cos x-\cos\alpha}dx$
$=2\int(\cos x+\cos\alpha)\,dx=2\int\cos xdx+2\cos\alpha\int dx=2\sin x+2x\cos\alpha+C=2(\sin x+x\cos\alpha)+C$

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14. $\dfrac{\cos x-\sin x}{1+\sin 2x}$

SOLUTION

: Let $I=\int\frac{\cos x-\sin x}{1+\sin 2x}dx=\int\frac{\cos x-\sin x}{1+2\sin x\cos x}dx$

$=\int\frac{\cos x-\sin x}{(\cos x+\sin x)^2}dx$ Put $\cos x+\sin x=t\Rightarrow(-\sin x+\cos x)\,dx=dt\;\therefore I=\int\frac{dt}{t^2}=\frac{t^{-2+1}}{-2+1}+C=\frac{-1}{t}+C=-\frac{1}{\cos x+\sin x}+C$

15. $\tan^3 2x\sec 2x$

SOLUTION

.: Let $I=\int\tan^3 2x\sec 2xdx\;I=\int\tan^2 2x\cdot\tan 2x\cdot\sec 2xdx=\int\left(\sec^2 2x-1\right)\cdot\sec 2x\tan 2x\,dx$ Put $\sec 2x=t\Rightarrow 2\sec 2x\tan 2x\,dx=dt$

$\therefore I=\frac{1}{2}\int\left(t^2-1\right)dt=\frac{1}{2}\left(\frac{t^3}{3}-t\right)+C=\frac{1}{6}\sec^3 2x-\frac{1}{2}\sec 2x+C$

16. $\tan^4 x$

SOLUTION

: Let$I = \int \tan^4 x dx = \int (\sec^2 x - 1)^2 dx = \int (\sec^4 x - 2\sec^2 x + 1) dx = \int \sec^4 x dx - 2\int \sec^2 x dx + \int (1) dx = \int \sec^4 x dx - 2\tan x + x + C_1$
$\Rightarrow I = I_1 - 2\tan x + x + C_1 \ldots$(i) Where $I_1 = \int \sec^4 x dx$ Now, $I_1 = \int \sec^4 x dx = \int \sec^2 x \cdot \sec^2 x dx = \int (1 + \tan^2 x)\sec^2 x dx$
Put $\tan x = t \Rightarrow \sec^2 x dx = dt \therefore I_1 = \int (1 + t^2) dt = t + \frac{t^3}{3} + C_2 = \tan x + \frac{1}{3}\tan^3 x + C_2 \ldots$(ii) From (i) and (ii), we have
$I = \tan x + \frac{1}{3}\tan^3 x + C_2 - 2\tan x + x + C_1 = \frac{1}{3}\tan^3 x - \tan x + x + C$,where $C = C_1 + C_2$

17. $\frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}$

SOLUTION

: Let $I = \int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx = \int \left(\frac{\sin^3 x}{\sin^2 x \cos^2 x} + \frac{\cos^3 x}{\sin^2 x \cos^2 x} \right) dx = \int (\sec x \tan x + \cos ec x \cot x) dx = \sec x - \cos ec x + C$

18. $\frac{\cos 2x + 2\sin^2 x}{\cos^2 x}$

SOLUTION

: Let $I = \int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx = \int \frac{(\cos^2 x + \sin^2 x) + 2\sin^2 x}{\cos^2 x} dx = \int \frac{\cos^2 x + \sin^2 x}{\cos^2 x} dx = \int \frac{1}{\cos^2 x} dx = \int \sec^2 x dx = \tan x + C$

19. $\frac{1}{\sin x \cos^3 x}$

SOLUTION

.: Let $I = \int \frac{1}{\sin x \cos^3 x} dx = \int \frac{\sin^2 x + \cos^2 x}{\sin x \cos^3 x} dx = \int \left(\frac{\sin^2 x}{\sin x \cos^3 x} + \frac{\cos^2 x}{\sin x \cos^3 x} \right) dx = \int \left(\frac{\sin x}{\cos^3 x} + \frac{\cos x}{\sin x \cos^2 x} \right) dx = \int \left(\tan x \sec^2 x + \frac{\sec^2 x}{\tan x} \right)$
$\int \left(\tan x + \frac{1}{\tan x} \right) \sec^2 x dx$ Put $\tan x = t \Rightarrow \sec^2 x dx = dt \therefore I = \int \left(t + \frac{1}{t} \right) dt = \frac{t^2}{2} + \log|t| + C = \log|\tan x| + \frac{1}{2}\tan^2 x + C$

20. . $\frac{\cos 2x}{(\cos x + \sin x)^2}$

SOLUTION

: Let$I = \int \frac{\cos 2x}{(\cos x + \sin x)^2} dx = \int \frac{\cos^2 x - \sin^2 x}{(\cos x + \sin x)^2} dx \int \frac{(\cos x - \sin x)(\cos x + \sin x)}{(\cos x + \sin x)^2} dx = \int \frac{\cos x - \sin x}{\cos x + \sin x} dx$ Put $\cos x + \sin x = t \Rightarrow$
$(-\sin x + \cos x) dx = dt \therefore I = \int \frac{dt}{t} = \log|t| + C = \log|\cos x + \sin x| + C$

21. $\sin^{-1}(\cos x)$

SOLUTION

: Let $I = \int \sin^{-1}(\cos x) dx = \int \sin^{-1}\left[\sin\left(\frac{\pi}{2} - x\right)\right] dx = \int \left(\frac{\pi}{2} - x\right) dx = \frac{\pi}{2}\int dx - \int x dx = \frac{\pi x}{2} - \frac{x^2}{2} + C$

22. $\frac{1}{\cos(x-a)\cos(x-b)}$

SOLUTION

Let $I = \int \frac{1}{\cos(x-a)\cos(x-b)} dx = \frac{1}{\sin(a-b)} \int \frac{\sin[(x-b)-(x-a)]}{\cos(x-a)\cos(x-b)} dx = \frac{1}{\sin(a-b)} \int \frac{\sin(x-b)\cos(x-a) - \cos(x-b)\sin(x-a)}{\cos(x-a)\cos(x-b)} dx$
$= \frac{1}{\sin(a-b)} \left[\int \tan(x-b) dx - \int \tan(x-a) dx \right] = \frac{1}{\sin(a-b)} [-\log|\cos(x-b)| + \log|\cos(x-a)|] + C = \frac{1}{\sin(a-b)} \log\left| \frac{\cos(x-a)}{\cos(x-b)} \right| +$
C

Choose the correct answer in Exercises 23 and 24.:

23. $\int \frac{\sin^2 x - \cos^2 x}{sin^2 x \cos^2 x} dx$ is equals to

(a) $\tan x + \cot x + C$

(b) $\tan x + \text{cosec} x + C$

(c) $-\tan x + \cot x + C$

(d) $\tan x + \sec x + C$

SOLUTION

: Let $I = \int \frac{\sin^2 x - \cos^2 x}{sin^2 x \cos^2 x} dx = \int \left(\sec^2 x - \cos ec^2 x\right) dx = \tan x + \cot x + C$

24. $\int \frac{e^x (1+x)}{\cos^2 (e^x x)} dx$ equals

(a) $-\cot (ex^x) + C$

(b) $\tan (xe^x) + C$

(c) $\tan (e^x) + C$

(d) $\cot (e^x) + C$

SOLUTION

: Let $I = \int \frac{e^x (1+x)}{\cos^2 (e^x x)} dx$ Put $xe^x = t \Rightarrow (e^x \cdot 1 + e^x x)\, dx = dt \therefore I = \int \frac{dt}{\cos^2 t} = \int \sec^2 t\, dt = \tan t + C = tan (xe^x) + C$

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