NCERT - Exercise – 7.2

Integrate the functions in Exercises 1 to 37 :

1. $\frac{2x}{1+x^2}$

SOLUTION

.: Let $I=\int\frac{2x}{1+x^2}dx$ Put $1+x^2=t\Rightarrow 2xdx=dt\;\therefore I=\int\frac{dt}{t}=\log t+C=\log\left(1+x^2\right)+C$

2. $\frac{(\log x)^2}{x}$

SOLUTION

: Let $I=\int\frac{(\log x)^2}{x}dx$ Put $\log x=t\Rightarrow\frac{1}{x}dx=dt\;\therefore I=\int t^2dt=\frac{t^3}{3}+C=\frac{1}{3}(\log x)^3+C$

3. $\frac{1}{x+x\log x}$

SOLUTION

: Let $I=\int\left(\frac{1}{x+x\log x}\right)dx$ Put $1+\log x=t\Rightarrow\frac{1}{x}dx=dt\;\therefore I=\int\frac{1}{x(1+\log x)}dx=\int\frac{1}{t}dt=\log t+C=\log(1+\log x)+C$

4. $\sin x\sin(\cos x)$

SOLUTION

.: Let $I=\int\sin x\sin(\cos x)\,dx$ Put $\cos x=t\Rightarrow-\sin xdx=dt\;\therefore I=-\int\sin t\,dt=\cos t+C=\cos(\cos x)+C$

5. $\sin(ax+b)\cos(ax+b)$

SOLUTION

.: Let $I=\int\sin(ax+b)\cos(ax+b)\,dx$ Put $\sin(ax+b)=t\Rightarrow a\cos(ax+b)\,dx=dt\;\therefore I=\frac{1}{a}\int tdt=\frac{1}{a}\cdot\frac{t^2}{2}+C=\frac{1}{2a}t^2+C=\frac{1}{2a}\sin^2(ax+b)+C$ Aliter : Put $\cos(ax+b)=t\Rightarrow-a\sin(ax+b)\,dx=dt\;\therefore I=\frac{-1}{a}\int tdt=\frac{-1}{a}\cdot\frac{t^2}{2}+C=\frac{-\cos^2(ax+b)}{2a}+C$

6. $\sqrt{ax+b}$

SOLUTION

.: Let $I=\int\sqrt{ax+b}dx=\int(ax+b)^{1/2}dx=\frac{(ax+b)^{3/2}}{\frac{3}{2}a}+C=\frac{2}{3a}(ax+b)^{3/2}+C$

7. $x\sqrt{x+2}$

Soln.: Let $I=\int x\sqrt{x+2}dx$ Put $x+2=t\Rightarrow dx=dt$ Also, $x=t-2\;\therefore I=\int(t-2)\sqrt{t}dt=\int\left(t^{3/2}-2t^{1/2}\right)dt=\frac{2}{5}t^{5/2}-2\times\frac{2}{3}t^{3/2}+C=\frac{2}{5}(x+2)^{5/2}-\frac{4}{3}(x+2)^{3/2}+C$

8. $x\sqrt{1+2x^2}$

SOLUTION

: Let $I=\int x\sqrt{1+2x^2}dx$ Put $1+2x^2=t\Rightarrow 4xdx=dt\;\therefore I=\frac{1}{4}\int\sqrt{1+2x^2}4xdx=\frac{1}{4}\int\sqrt{t}dt=\frac{1}{4}\frac{t^{3/2}}{3/2}+C=\frac{1}{6}t^{3/2}+C=\frac{1}{6}\left(1+2x^2\right)^{3/2}+C$

9. $(4x+2)\sqrt{x^2+x+1}$

SOLUTION

: Let$I=\int(4x+2)\sqrt{x^2+x+1}dx$ Put $x^2+x+1=t \Rightarrow (2x+1)\,dx=dt$ $I=2\int(2x+1)\sqrt{x^2+x+1}dx=2\int\sqrt{t}dt=\frac{2t^{3/2}}{3/2}+C=\frac{4}{3}t^{3/2}+C=\frac{4}{3}\left(x^2+x+1\right)^{3/2}+C$

10. $\frac{1}{x-\sqrt{x}}$

SOLUTION

: Let $I=\int\frac{1}{x-\sqrt{x}}dx=\int\frac{1}{\sqrt{x}(\sqrt{x}-1)}dx$ Put $\sqrt{x}-1=t \Rightarrow \frac{1}{2\sqrt{x}}dx=dt$ $\therefore I=2\int\frac{dx}{(\sqrt{x}-1)2\sqrt{x}}=2\int\frac{dt}{t}=2\log t+C=2\log\left(\sqrt{x}-1\right)+$

11. $\frac{x}{\sqrt{x+4}}, x>0$

SOLUTION

Let $I=\int\frac{x}{\sqrt{x+4}}dx$ Put $x+4=t \Rightarrow dx=dt$ $\therefore I=\int\frac{x}{\sqrt{x+4}}dx=\int\frac{t-4}{\sqrt{t}}dt=\int\left(t^{1/2}-4t^{-1/2}\right)dt=\frac{2}{3}t^{3/2}-4\times 2t^{1/2}+C \Rightarrow$
$I=\frac{2}{3}(x+4)^{3/2}-8(x+4)^{1/2}+C=\frac{2}{3}(x+4)^{1/2}[x+4-12]+C=\frac{2}{3}(x+4)^{1/2}(x-8)+C$

12. $\left(x^3-1\right)^{1/3}x^5$

SOLUTION

: Let $I=\int\left(x^3-1\right)^{1/3}x^5dx$ Put $x^3-1=t \Rightarrow 3x^2dx=dt$ Also $x^3=t+1$ $\therefore I=\frac{1}{3}\int\left(x^3-1\right)^{1/3}x^3\cdot 3x^2dx=\frac{1}{3}\int t^{1/3}(t+1)\,dt$
$=\frac{1}{3}\left[\frac{3}{7}t^{7/3}+\frac{3}{4}t^{4/3}\right]+C=\frac{1}{7}t^{7/3}+\frac{1}{4}t^{4/3}+C=\frac{1}{7}\left(x^3-1\right)^{7/3}+\frac{1}{4}\left(x^3-1\right)^{4/3}+C$

13. $\frac{x^2}{\left(2+3x^3\right)^3}$

SOLUTION

: Let$I=\int\frac{x^2}{\left(2+3x^3\right)^3}dx$ Put $2+3x^2=t \Rightarrow 9x^2dx=dt$ $\therefore I=\frac{1}{9}\int\frac{9x^2\,dx}{\left(2+3x^3\right)^3}=\frac{1}{9}\int\frac{dt}{t^3}=\frac{1}{9}\int t^{-3}dt=\frac{1}{9}\frac{t^{-2}}{(-2)}+C=\frac{1}{18}t^{-2}+C=-\frac{1}{18\left(2+3x^3\right)^2}+C$

14. $\frac{1}{x(\log x)^m}, x>0$

SOLUTION

.: Let $I=\int\frac{1}{x(\log x)^m}dx$ Put $\log x=t \Rightarrow \frac{1}{x}dx=dt$ $\therefore I=\int\frac{dt}{t^m}=\int t^{-m}dt=\frac{t^{-m+1}}{-m+1}+C=\frac{(\log x)^{1-m}}{1-m}+C$

15. $\frac{x}{9-4x^2}$

SOLUTION

: Let$I=\int\frac{x}{9-4x^2}dx$ Put $9-4x^2=t \Rightarrow -8x\,dx=dt$ $\therefore I=-\frac{1}{8}\int\frac{dt}{t}=-\frac{1}{8}\log|t|+C=\frac{1}{8}\log\frac{1}{|t|}+C=\frac{1}{8}\log\frac{1}{|9-4x^2|}+C$

16. e^{2x+3}

SOLUTION

: Let$I=\int e^{2x+3}dx$ Put $2x+3=t \Rightarrow 2\,dx=dt$ $\therefore I=\frac{1}{2}\int e^t\,dt=\frac{1}{2}e^t+C=\frac{1}{2}e^{2x+3}+C$

17. $\dfrac{x}{e^{x^2}}$

SOLUTION

: Let $I=\int \frac{x}{e^{x^2}}dx$ Put $x^2=t \Rightarrow 2x\,dx=dt \therefore I=\frac{1}{2}\int \frac{dt}{e^t}=\frac{1}{2}\int e^{-t}dt=\frac{1}{2}\left(\frac{e^{-t}}{-1}\right)+C=-\frac{1}{2e^t}+C=-\frac{1}{2e^{x^2}}+C$

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18. $\dfrac{e^{\tan^{-1}x}}{1+x^2}$

SOLUTION

: Let $I=\int \dfrac{e^{\tan^{-1}x}}{1+x^2}dx$ Put $\tan^{-1}x=t \Rightarrow \dfrac{1}{1+x^2}dx=dt \therefore I=\int e^t dt=e^t+C=e^{\tan^{-1}x}+C$

19. $\dfrac{e^{2x}-1}{e^{2x}+1}$

SOLUTION

: Let $I=\int \dfrac{e^{2x}-1}{e^{2x}+1}dx=\int \dfrac{e^x\left(e^x-e^{-x}\right)}{e^x\left(e^x+e^{-x}\right)}dx=\int \dfrac{e^x-e^{-x}}{e^x+e^{-x}}dx$ Put $e^x+e^{-x}=t \Rightarrow \left(e^x-e^{-x}\right)dx=dt \therefore I=\int \dfrac{dt}{t}=\log|t|+C=$ $\log\left|e^x+e^{-x}\right|+C$

20. $\dfrac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}$

SOLUTION

: Let $I=\int \dfrac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}dx$ Put $e^{2x}+e^{-2x}=t \Rightarrow \left(2e^{2x}-2e^{-2x}\right)dx=dt \Rightarrow \left(e^{2x}-e^{-2x}\right)dx=\dfrac{dt}{2} \therefore I=\dfrac{1}{2}\int \dfrac{1}{t}dt=\dfrac{1}{2}\log|t|+C=$ $\dfrac{1}{2}\log\left|e^{2x}+e^{-2x}\right|+C$

21. $\tan^2(2x-3)$

SOLUTION

: Let $I=\int \tan^2(2x-3)\,dx=\int\left[\sec^2(2x-2)-1\right]dx=\int \sec^2(2x-3)\,dx-\int dx=\int \sec^2(2x-3)dx-x+C_1=I_1-x+C_1$ ....(i) Where $I_1=\int \sec^2(2x-3)\,dx$ Put $2x-3=t \Rightarrow 2dx=dt \Rightarrow I_1=\dfrac{1}{2}\int \sec^2 t\,dt=\dfrac{1}{2}\tan t+C_2=\dfrac{1}{2}\tan(2x-3)+C_2$ .....(ii)
From (i) and (ii) , we get $I=I_1-x+C_1=\dfrac{1}{2}\tan(2x-3)-x+C$ where $C=C_1+C_2$

22. $\sec^2(7-4x)$

SOLUTION

: Let $I=\int \sec^2(7-4x)\,dx$ Put $7-4x=t \Rightarrow -4dx=dt \therefore I=-\dfrac{1}{4}\int \sec^2 t\,dt=-\dfrac{1}{4}\tan t+C=-\dfrac{1}{4}\tan(7-4x)+C$

23. $\dfrac{\sin^{-1}x}{\sqrt{1-x^2}}$

SOLUTION

Let$I=\int \dfrac{\sin^{-1}x}{\sqrt{1-x^2}}dx$ Put $\sin^{-1}x=t \Rightarrow \dfrac{1}{\sqrt{1-x^2}}dx=dt \Rightarrow I=\int t\,dt=\dfrac{1}{2}t^2+C=\dfrac{1}{2}\left(\sin^{-1}x\right)^2+C$

24. $\dfrac{2\cos x-3\sin x}{6\cos x+4\sin x}$

SOLUTION

: Let $I=\int \dfrac{2\cos x-3\sin x}{6\cos x+4\sin x}dx=\dfrac{1}{2}\int \dfrac{2\cos x-3\sin x}{2\sin x+3\cos x}dx$ Put $2\sin x+3\cos x=t \Rightarrow (2\cos x-3\sin x)\,dx=dt \therefore I=\dfrac{1}{2}\int \dfrac{dt}{t}=\dfrac{1}{2}\log|t|+C=\dfrac{1}{2}\text{lo}$

25. $\dfrac{1}{\cos^2 x(1-\tan x)^2}$

SOLUTION

: Let $I=\int \dfrac{1}{\cos^2 x(1-\tan x)^2}dx=I=\int \dfrac{\sec^2 x}{(1-\tan x)^2}dx$ Put $1-\tan x=t \Rightarrow -\sec^2 x\,dx=dt \therefore I=-\int \dfrac{dt}{t^2}=-\dfrac{t^{-2+1}}{-2+1}+C=$ $\dfrac{1}{t}+C=\dfrac{1}{1-\tan x}+C$

26. $\frac{\cos\sqrt{x}}{\sqrt{x}}$

SOLUTION

: Let $I=\int \frac{\cos\sqrt{x}}{\sqrt{x}}dx$ Put $\sqrt{x}=t \Rightarrow \frac{1}{2\sqrt{x}}dx=dt \therefore I=2\int \cos t\,dt=2\sin t+C=2\sin\sqrt{x}+C$

27. $\sqrt{\sin 2x}\cos 2x$

SOLUTION

: Let $I=\int \sqrt{\sin 2x}\cos 2x dx$ Put $\sin 2x=t \Rightarrow 2\cos 2x dx=dt \therefore I=\frac{1}{2}\int t^{1/2}dt=\frac{1}{2}\cdot\frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1}+C=\frac{1}{2}\times\frac{2}{3}t^{3/2}+C=\frac{1}{3}t^{3/2}+C=$
$\frac{1}{3}(\sin 2x)^{3/2}+C$

28. . $\frac{\cos x}{\sqrt{1+\sin x}}$

SOLUTION

: Let $I=\int \frac{\cos x}{\sqrt{1+\sin x}}dx$ Put $1+\sin x=t \Rightarrow \cos x dx=dt \therefore I=\int \frac{dt}{t^{1/2}}=\frac{t^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+C=2t^{1/2}+C=2\sqrt{1+\sin x}+C$

29. $\cot x\log\sin x$

SOLUTION

: Let$I=\int \cot x\log\sin x dx$ Put $\log\sin x=t \Rightarrow \frac{1}{\sin x}\cos x dx=dt \Rightarrow \cot x dx=dt \therefore I=\int t\,dt=\frac{t^2}{2}+C=\frac{1}{2}(\log\sin x)^2+C$

30. $\frac{\sin x}{1+\cos x}$

SOLUTION

: Let $I=\int \frac{\sin x}{1+\cos x}dx$ Put $1+\cos x=t \Rightarrow -\sin x dx=dt \therefore I=-\int \frac{dt}{t}=-\log|t|+C=-\log|1+\cos x|+C=\log\left(\frac{1}{|1+\cos x|}\right)+$
C

31. $\frac{\sin x}{(1+\cos x)^2}$

SOLUTION

: Let$I=\int \frac{\sin x}{(1+\cos x)^2}dx$ Put $1+\cos x=t \Rightarrow -\sin x dx=dt \therefore I=-\int \frac{dt}{t^2}=-\frac{t^{-2+1}}{-2+1}+C=\frac{1}{t}+C=\frac{1}{1+\cos x}+C$

32. $\frac{1}{1+\cot x}$

SOLUTION

: Let$I=\int \frac{1}{1+\cot x}dx=\int \frac{1}{1+\frac{\cos x}{\sin x}}dx=\int \frac{\sin x}{\sin x+\cos x}dx=\frac{1}{2}\int \frac{2\sin x}{\sin x+\cos x}dx=\frac{1}{2}\int \frac{(\sin x+\cos x)-(\cos x-\sin x)}{(\sin x+\cos x)}dx=\frac{1}{2}\int(1)\,dx-$
$\frac{1}{2}\int \frac{\cos x-\sin x}{\sin x+\cos x}dx=\frac{1}{2}x-\frac{1}{2}\int \frac{\cos x-\sin x}{\sin x+\cos x}dx+C_1 \therefore I=\frac{x}{2}-\frac{1}{2}I_1+C_1$ .....(i) Let $I_1=\int \frac{\cos x-\sin x}{\sin x+\cos x}dx$ Put $\sin x+\cos x=t$
$\Rightarrow (\cos x-\sin x)\,dx=dt \Rightarrow I_1=\int \frac{dt}{t}=\log|t|+C_2=\log|\cos x+\sin x|+C_2$ .....(ii) From (i) and (ii), we get $\Rightarrow I=\frac{1}{2}x-$
$\frac{1}{2}\log|\cos x+\sin x|+C$ Where, $C=C_1+C_2$

33. $\dfrac{1}{1-\tan x}$

SOLUTION

: Let $I=\int \frac{1}{1-\tan x}dx=\int \frac{1}{1-\frac{\sin x}{\cos x}}dx=\int \frac{\cos x}{\cos x-\sin x}dx=\frac{1}{2}\int \frac{2\cos x}{\cos x-\sin x}dx=\frac{1}{2}\int \frac{(\cos x-\sin x)-(-\sin x-\cos x)}{\cos x-\sin x}dx=$ $\frac{1}{2}\int (1)\,dx-\frac{1}{2}\int \frac{-\sin x-\cos x}{\cos x-\sin x}dx=\frac{x}{2}-\frac{1}{2}\int \frac{-\sin x-\cos x}{\cos x-\sin x}dx+C_1 \therefore I=\frac{x}{2}-\frac{1}{2}I_1+C_1$ ....(i) Let $I_1=\int \frac{-\sin x-\cos x}{\cos x-\sin x}dx$ Put $\cos x-\sin x=t \Rightarrow (-\sin x-\cos x)\,dx=dt \therefore I_1=\int \frac{dt}{t}=\log|t|+C_2=\log|\cos x-\sin x|+C_2$ ....(ii) From (i) and (ii) we get $\Rightarrow$ $I=\frac{x}{2}-\frac{1}{2}\log|\cos x-\sin x|+C$ where $C=C_1+C_2$

34. $\dfrac{\sqrt{\tan x}}{\sin x\cos x}$

SOLUTION

: Let $I=\int \frac{\sqrt{\tan x}}{\sin x\cos x}dx=\int \frac{\sqrt{\tan x}}{\frac{\sin x}{\cos x}\cdot\cos^2 x}dx=\int \frac{\sqrt{\tan x}}{\tan x}\cdot\sec^2 x dx=\int (\tan x)^{-1/2}\cdot\sec^2 x dx$ Put $\tan x=t \Rightarrow \sec^2 x dx=dt \therefore$ $I=\int t^{-\frac{1}{2}}dt=\frac{t^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+C=2t^{1/2}+C=2\sqrt{\tan x}+C$

35. $\dfrac{(1+\log x)^2}{x}$

SOLUTION

: Let $I=\int \frac{(1+\log x)^2}{x}dx$ Put $1+\log x=t \Rightarrow \frac{1}{x}dx=dt \therefore I=\int t^2dt=\frac{t^3}{3}+C=\frac{1}{3}(1+\log x)^3+C$

36. $\dfrac{(x+1)(x+\log x)^2}{x}$

SOLUTION

: Let $I=\int \frac{(x+1)(x+\log x)^2}{x}dx=\int (x+\log x)^2\left(1+\frac{1}{x}\right)dx$ Put $x+\log x=t \Rightarrow \left(1+\frac{1}{x}\right)dx=dt \therefore I=\int t^2dt=\frac{t^3}{3}+C=$ $\frac{1}{3}(x+\log x)^3+C$

37. $\dfrac{x^3\sin\left(\tan^{-1}x^4\right)}{1+x^8}$

SOLUTION

: Let $I=\int \frac{x^3\sin\left(\tan^{-1}x^4\right)}{1+x^8}dx$ Put $\tan^{-1}x^4=t \Rightarrow \frac{1}{1+x^8}\cdot 4x^3dx=dt \therefore I=\frac{1}{4}\int \sin t dt=\frac{1}{4}(-\cos t)+C=-\frac{1}{4}\cos\left(\tan^{-1}x^4\right)+C$

Choose the correct answer in Exercises 38 and 39.:

38. $\int\left(\dfrac{10x^9+10^x\log_e 10}{x^{10}+10^x}\right)dx$equals

(a) $10^x-x^{10}+C$

(b) $10^x+x^{10}+C$

(c) $\left(10^x-x^{10}\right)^{-1}+C$

(d) $\log\left(10^x-x^{10}\right)+C$

SOLUTION

(D) Let $I=\int\left(\frac{10x^{9}+10^{x}\log_{e}10}{x^{10}+10^{x}}\right)dx$ Put $x^{10}+10^{x}=t \Rightarrow \left(10x^{9}+\log_{e}10\cdot 10^{x}\right)dx=dt \Rightarrow I=\int\frac{dt}{t}=\log|t|+C=\log\left(x^{10}+10^{x}\right)+C$

39. $\int\frac{dx}{\sin^{2}x\cos^{2}x}$ equals

(a) $\tan x+\cot x+C$

(b) $\tan x-\cot x+C$

(c) $\tan x\cot x+C$

(d) $\tan x-\cot 2x+C$

SOLUTION

(D) Let $I=\int\frac{dx}{\sin^{2}x\cos^{2}x}=\int\frac{\sin^{2}x+\cos^{2}x}{\sin^{2}x\cos^{2}x}dx=\int\left(\sec^{2}x+\cos ec^{2}x\right)dx=\tan x-\cot x+C$