NCERT - Exercise – 7.1

Find an anti derivative (or integral) of the following functions by the method of inspection

1. $\sin 2x$

SOLUTION

Let $I = \int \sin 2x dx = -\frac{\cos 2x}{2} + C$

2. . $\cos 3x$

SOLUTION

: Let $I = \int \cos 3x dx = \frac{\sin 3x}{3} + C$

3. e^{2x}

SOLUTION

: Let $I = \int e^{2x} dx = \frac{e^{2x}}{2} + C$

4. $(ax+b)^2$

SOLUTION

: Let $I = \int (ax+b)^2 dx = \frac{(ax+b)^3}{3a} + C$

5. $\sin 2x - 4e^{3x}$

SOLUTION

: Let $I = \int \left(\sin 2x - 4e^{3x}\right) dx = -\frac{\cos 2x}{2} - \frac{4e^{3x}}{3} + C$

Find the following integrals in Exercises 6 to 20

6. $\int \left(4e^{3x} + 1\right) dx$

SOLUTION

: Let $I = \int \left(4e^{3x} + 1\right) dx = \frac{4e^{3x}}{3} + x + C$

7. $\int x^2 \left(1 - \frac{1}{x^2}\right) dx$

SOLUTION

: Let$I = \int x^2 \left(1 - \frac{1}{x^2}\right) dx = \int \left(x^2 - 1\right) dx = \frac{x^3}{3} - x + C$

8. $\int \left(ax^2 + bx + c\right) dx$

SOLUTION

: Let$I = \int \left(ax^2 + bx + c\right) dx = \frac{ax^3}{3} + \frac{bx^2}{2} + cx + C$

9. $\int \left(2x^2 + e^x\right) dx$

SOLUTION

: Let $I = \int \left(2x^2 + e^x\right) dx = \frac{2x^3}{3} + e^x + C$

10. $\int \left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 dx$

SOLUTION

Let$I=\int \left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 dx=\int \left(x+\frac{1}{x}-2\right) dx=\frac{x^2}{2}+\log x-2x+C$

11. $\int \frac{x^3+5x^2-4}{x^2}dx$

SOLUTION

: Let$I=\int \left(\frac{x^3+5x^2-4}{x^2}\right) dx=\int \left(x+5-4x^{-2}\right) dx=\frac{x^2}{2}+5x-\frac{4x^{-1}}{-1}+C=\frac{x^2}{2}+5x+\frac{4}{x}+C$

12. $\int \frac{x^3+3x+4}{\sqrt{x}}dx$

SOLUTION

: Let$I=\int \left(\frac{x^3+3x+4}{\sqrt{x}}\right)dx=\int \left(x^{5/2}+3x^{1/2}+4x^{-1/2}\right) dx=\frac{x^{7/2}}{7/2}+\frac{3x^{3/2}}{3/2}+\frac{4x^{1/2}}{1/2}+C=\frac{2}{7}x^{7/2}+2x^{3/2}+8\sqrt{x}+C$

13. $\int \frac{x^3-x^2+x-1}{x-1}dx$

SOLUTION

: Let $I=\int \left(\frac{x^3-x^2+x-1}{x-1}\right) dx=\int \frac{x^2(x-1)+(x-1)}{x-1}dx=\int \left(x^2+1\right) dx=\frac{x^3}{3}+x+C$

14. . $\int (1-x)\sqrt{x}dx$

SOLUTION

: Let $I=\int (1-x)\sqrt{x}dx=\int \left(x^{1/2}-x^{3/2}\right)dx=\frac{x^{3/2}}{3/2}-\frac{x^{5/2}}{5/2}+C=\frac{2}{3}x^{3/2}-\frac{2}{5}x^{5/2}+C$

15. $\int \sqrt{x}\left(3x^2+2x+3\right) dx$

SOLUTION

Let $I=\int \sqrt{x}\left(3x^2+2x+3\right) dx=\int \left(3x^{5/2}+2x^{3/2}+3x^{1/2}\right) dx=\frac{3x^{7/2}}{7/2}+\frac{2x^{5/2}}{5/2}+\frac{3x^{3/2}}{3/2}+C=\frac{6}{7}x^{7/2}+\frac{4}{5}x^{5/2}+2x^{3/2}+C$

16. $\int (2x-3\cos x+e^x) dx$

SOLUTION

Let $I=\int (2x-3\cos x+e^x) dx=2\frac{x^2}{2}-3\sin x+e^x+C=x^2-3\sin x+e^x+C$

17. $\int \left(2x^2-3\sin x+5\sqrt{x}\right) dx$

SOLUTION

: Let $I=\int \left(2x^2-3\sin x+5\sqrt{x}\right) dx=\frac{2x^3}{3}-3(-\cos x)+\frac{5x^{3/2}}{3/2}+C=\frac{2}{3}x^3+3\cos x+\frac{10}{3}x^{3/2}+C$

18. $\int \sec x(\sec x+\tan x) dx$

SOLUTION

: Let $I=\int \sec x(\sec x+\tan x) dx=\int \left(\sec^2 x+\sec x\tan x\right) dx=\tan x+\sec x+C$

19. $\int \frac{\sec^2 x}{\cos ec^2 x} dx$

SOLUTION

: Let $I = \int \frac{\sec^2 x}{\cos ec^2 x} dx = \int \frac{1}{\cos^2 x} \cdot \sin^2 x dx = \int \tan^2 x dx = \int \left(\sec^2 x - 1\right) dx = \tan x - x + C$

20. $\int \frac{2 - 3\sin x}{\cos^2 x} dx$

SOLUTION

: Let $I = \int \frac{2 - 3\sin x}{\cos^2 x} dx = \int \left(\frac{2}{\cos^2 x} - 3\frac{\sin x}{\cos^2 x}\right) dx = \int \left(2\sec^2 x - 3\sec x \tan x\right) dx = 2\tan x - 3\sec x + C$

Choose the correct answer in Exercises 21 and 22

21. The anti derivative of $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$ equals

(a) $\frac{1}{3}x^{1/3} + 2x^{1/2} + C$

(b) $\frac{2}{3}x^{2/3} + \frac{1}{2}x^{1/2} + C$

(c) $\frac{2}{3}x^{3/2} + 2x^{1/2} + C$

(d) $\frac{3}{2}x^{3/2} + \frac{1}{2}x^{1/2} + C$

SOLUTION

: (C) Let $I = \int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) dx = \int \left(x^{1/2} + x^{-1/2}\right) dx = \frac{2}{3}x^{3/2} + 2x^{1/2} + C$

22. If $\frac{d}{dx} f(x) = 4x^2 - \frac{3}{x^4}$,such that $f(2) = 0$ then, $f(x)$ is

(a) $x^4 + \frac{1}{x^3} - \frac{129}{8}$

(b) $x^3 + \frac{1}{x^4} + \frac{129}{8}$

(c) $x^4 + \frac{1}{x^3} + \frac{129}{8}$

(d) $x^3 + \frac{1}{x^4} - \frac{129}{8}$

SOLUTION

: (A) $f(x) = \int \left(4x^3 - \frac{3}{x^4}\right) dx = 4\int x^3 dx - 3\int x^{-4} dx = \frac{4x^4}{4} - \frac{3x^{-3}}{-3} + C = x^4 + \frac{1}{x^3} + C \therefore f(2) = (2)^4 + \frac{1}{(2)^3} + C = 0 \Rightarrow$
$C = -16 - \frac{1}{8} = -\frac{129}{8} \therefore f(x) = x^4 + \frac{1}{x^3} - \frac{129}{8}$