NCERT - Exercise 5.8

1. Verify Rolle's theorem for the function $f(x) = x^2 + 2x - 8, x[-4, 2]$.

   SOLUTION

   Consider, $f(x) = x^2 + 2x - 8\, in\, [-4, 2]$ which isa polynomial function and so

   (i) Functionf(x) is continuous in [−4, 2]

   (ii) f(x) is derivable in (−4, 2) and

   (iii) f(−4) = 0 and f(2) = 0 ⇒ f(−4) =f(2).

   Hence,conditions of Rolle's theorem are satisfied. Hence there exists, at least one $c \in (-4, 2)$such that $f(-4) = f(2)$.
   Now, $f'(c) = 0 \Rightarrow 2c + 2 = 0 \Rightarrow c = -1 \in (-4, 2)$. Thus, $-1 \in (-4, 2)$such that $f'(-1) = 0$. Hence, Rolle's theorem is verified.

2. Examine if Rolle's theorem is applicable to any of the following functions. Can you say some thing about the converse of Rolle's theorem from these examples?

   (i) f(x) = [x] for x ∈ [5, 9]

   (ii) f(x) = [x] for x∈[-2, 2]

   (iii) f(x) = x^2−1 for x∈ [1, 2].

   SOLUTION

   (i) Being greatest integer function, the given function is not differentiable and continuous. Hence, Rolle's theorem is not applicable.

   (ii) Being greatest integer function, the given function is not differentiable and continuous. Hence, Rolle's theorem is not applicable.

   (iii) $f(x) = x^2 - 1, x \in$[1, 2] is a polynomial function. So,
   (a) It is continuous in [1, 2]

   (b) It is derivable in (1, 2) and

   (c) $f(1) = (1)^2 - 1 = 1 - 1 = 0$

   $f(2) = (2)^2 - 1 = 4 - 1 = 3$ As $f(1) \neq f(2)$, hence Rolle's theorem is not applicable.

3. If $f : [-5, 5] \rightarrow R$ is a differentiable function and if $f'(x)$does not vanish anywhere, then prove that $f(-5) \neq f(5)$.

   SOLUTION

   For Rolle's theorem, if

   (i) f is continuous in [a, b]

   (ii) f is derivable in (a, b)

   (iii) $f(a) = f(b)$ then $f'(c) = 0, c \in (a, b)$
   We are given/is continuous and derivable but $f'(c) \neq 0 \Rightarrow f(a) \neq f(b)$ i.e. $f(-5) \neq f(5)$ Hence proved.

4. Verify Mean Value Theorem, if $f(x) = x^2 - 4x - 3$ in the interval [a,b], where a = 1 and b = 4.

   SOLUTION

   We have, $f(x) = x^2 - 4x - 3$, $x \in$[1, 4] which is a polynomial function. So

   (i) f(x) is continuous in [1, 4]

(ii) f(x) is derivable in (1, 4) and, hence conditions of mean value theorem are satisfied, so there exists, at least one $c \in (1,4)$ such that

$f'(c) = \dfrac{f(4)-f(1)}{4-1} \Rightarrow 2c-4 = \dfrac{(-3)-(-6)}{3} \Rightarrow 2c-4=1$

$\Rightarrow 2c=5 \Rightarrow c=\dfrac{5}{2} \in (1,4)$ Hence, mean value theorem is verified.

5. Verify Mean Value Theorem, if $f(x)=x^3-5x^2-3x$ in the interval [a, b], where a = 1 and b = 3.

SOLUTION

We have, $f(x)=x^3-5x^2-3x, x \in (1,3)$, which is a polynomial function, so (i) It is continuous in [1, 3].

(ii) It is derivable in (1, 3). So, all conditions of mean value theorem are verified.

Hence thereexists, at least onec, such that $f'(c)=\dfrac{f(3)-f(1)}{3-1} \Rightarrow 3c^2-10c-3=\dfrac{-27-(-7)}{2}$

$\Rightarrow 3c^2-10c-3=-10 \Rightarrow 3c^2-10c+7=0$

$\Rightarrow c=\dfrac{10\pm\sqrt{100-84}}{6}=\dfrac{10\pm4}{6}$

$\Rightarrow c=\dfrac{7}{3},1$ But, $c=\dfrac{7}{3}\in(1,3)$ So, mean value theorem is verified.

6. Examine the applicability of Mean Value Theorem for all three functions given in the above question 2.

SOLUTION

(i)f(x) = [x] for x ∈ [5, 9] f(x) = [x] in the interval [5, 9] is neither continuous, nor differentiable. ∴ Mean value theorem is not applicable.

(ii) f(x) = $[x]$ for x ∈ [−2, 2]

Again,f(x)= [x] in the interval [−2, 2] is neither continuous, nor differentiable. Hence, mean value theorem is not applicable.

(iii) $f(x)=x^2-1$ for $x\in$ [1, 2] It is a polynomial. Therefore, it is continuous in the interval [1, 2] and differentiable in the interval (1, 2).

So all conditions of mean value theorem are satisfied. Therefore, there exists at least one $c\in(1,2)$ such that $f'(c)=\dfrac{f(2)-f(1)}{2-1}\Rightarrow$ $2c=\dfrac{3-0}{2-1}=\dfrac{3}{1}$

As $c=\dfrac{3}{2}\in(1,2)$,so mean value theorem is verified.

Our Mathematics E-Books

1. J.E.E. (Join Entrance Exam)
   ★ Chapter Tests (Full Syllabus- Fully Solved)
   ★ Twenty Mock Tests (Full Length - Fully Solved)
2. B.I.T.S.A.T. Twenty Mock Tests (Fully Solved)
3. C.BS.E.
   ★ Work-Book Class XII (Fully Solved)
   ★ Objective Type Questions Bank C.B.S.E. Class XII (Fully Solved)
   ★Chapter Test Papers Class XII (Fully Solved)
   ★ Past Fifteen Years Topicwise Questions (Fully Solved)
   ★ Sample Papers Class XII (Twenty Papers Fully Solved- includes 2020 solved paper)
   ★ Sample Papers Class X (Twenty Papers Fully Solved -includes 2020 solved paper)
4. I.C.S.E. & I.S.C.
   ★Work-Book Class XII (Fully Solved)
   ★ Chapter Test Papers Class XII (Fully Solved)
   ★ Sample Papers Class XII (Twenty Papers Fully Solved -includes 2020 solved paper)
   ★ Sample Papers Class X (Twenty Papers Fully Solved -includes 2020 solved paper)
5. Practice Papers for SAT -I Mathematics (15 Papers - Fully Solved)
6. SAT - II Subject Mathematics (15 Papers - Fully Solved)