

NCERT - Exercise 5.7

Find the second order derivatives of the functions given in questions 1 to 10.

1. $x^2 + 3x + 2$

SOLUTION

$$\text{Let } y = x^2 + 3x + 2 \Rightarrow \frac{dy}{dx} = 2x + 3 \therefore \frac{d^2y}{dx^2} = 2$$

2. x^{20}

SOLUTION

$$\text{Let } y = x^{20} \Rightarrow \frac{dy}{dx} = 20x^{19} \therefore$$

$$\frac{d^2y}{dx^2} = 20 \cdot 19x^{18} = 380x^{18}$$

3. $x \cdot \cos x$

SOLUTION

$$\text{Let } y = x \cos x \Rightarrow \frac{dy}{dx} = x \cdot (-\sin x) + \cos x = -x \sin x + \cos x$$

$$\Rightarrow \frac{d^2y}{dx^2} = -[x \cos x + \sin x] - \sin x = -(x \cos x + 2 \sin x)$$

4. $\log x$

SOLUTION

$$\text{Let } y = \log x \Rightarrow \frac{dy}{dx} = \frac{1}{x} \therefore$$

$$\frac{d^2y}{dx^2} = -\frac{1}{x^2}$$

5. $x^3 \log x$

SOLUTION

$$\text{Let } y = x^3 \log x$$

$$\Rightarrow \frac{dy}{dx} = x^3 \cdot \frac{1}{x} + \log x \cdot 3x^2 = x^2 + 3x^2 \log x$$

$$\Rightarrow \frac{d^2y}{dx^2} = 2x + 3 \left[x^2 \cdot \frac{1}{x} + \log x \cdot 2x \right]$$

$$= 2x + 3[x + 2x \log x] = 2x + 3x + 6x \log x = 5x + 6x \log x = x(5 + 6 \log x)$$

6. $e^x \sin 5x$

SOLUTION

$$\text{Let } y = e^x \sin 5x \Rightarrow \frac{dy}{dx} = e^x \cdot \cos 5x \cdot 5 + \sin 5x \cdot e^x$$

$$= e^x [5 \cos 5x + \sin 5x] \therefore \frac{d^2y}{dx^2} = e^x [5(-\sin 5x) \cdot 5 + \cos 5x \cdot 5] + [5 \cos 5x + \sin 5x] e^x$$

$$= e^x [-25 \sin 5x + 5 \cos 5x + 5 \cos 5x + \sin 5x] = e^x [10 \cos 5x - 24 \sin 5x] = 2e^x [5 \cos 5x - 12 \sin 5x]$$

7. $e^{6x} \cos 3x$

SOLUTION

$$\text{Let } y = e^{6x} \cos 3x \Rightarrow \frac{dy}{dx} = e^{6x}(-\sin 3x) \cdot 3 + \cos 3x \cdot e^{6x} \cdot 6 = 6e^{6x} \cos 3x - 3e^{6x} \sin 3x$$

$$\Rightarrow \frac{d^2y}{dx^2} = 6[e^{6x}(-\sin 3x) \cdot 3 + \cos 3x \cdot e^{6x} \cdot 6] - 3[e^{6x} \cos 3x \cdot 3 + \sin 3x e^{6x} \cdot 6]$$

$$= -18e^{6x} \sin 3x + 36e^{6x} \cos 3x - 9e^{6x} \cos 3x - 18e^{6x} \sin 3x = 27e^{6x} \cos 3x - 36e^{6x} \sin 3x = 9e^{6x}(3 \cos 3x - 4 \sin 3x)$$

8. $\tan^{-1}x$

SOLUTION

$$\text{Let } y = \tan^{-1}x \Rightarrow \frac{dy}{dx} = \frac{1}{(1+x^2)}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{(1+x^2) \cdot 0 - 1 \cdot (2x)}{(1+x^2)^2} = \frac{-2x}{(1+x^2)^2}$$

9. $\log(\log x)$

SOLUTION

$$\text{Let } y = \log(\log x) \Rightarrow \frac{dy}{dx} = \frac{1}{\log x} \cdot \frac{1}{x} \therefore$$

$$\frac{d^2y}{dx^2} = \frac{1}{\log x} \left(-\frac{1}{x^2} \right) + \frac{1}{x} \frac{d}{dx} \left(\frac{1}{\log x} \right)$$

$$= \frac{-1}{x^2 \log x} + \frac{1}{x} \left[\frac{\log x \cdot 0 - 1 \cdot \frac{1}{x}}{(\log x)^2} \right] = \frac{-1}{x^2 \log x} + \frac{1}{x} \left[\frac{-\frac{1}{x}}{(\log x)^2} \right]$$

$$= \frac{-1}{x^2 \log x} - \frac{1}{x^2 (\log x)^2} = \frac{-1}{x^2 \log x} \left[1 + \frac{1}{\log x} \right] = \frac{-(1 + \log x)}{(x \log x)^2}$$

10. $\sin(\log x)$

SOLUTION

$$\text{Let } y = \sin(\log x) \Rightarrow \frac{dy}{dx} = \cos(\log x) \cdot \frac{1}{x}$$

$$\therefore \frac{d^2y}{dx^2} = \cos(\log x) \cdot \left(-\frac{1}{x^2} \right) + \frac{1}{x} \cdot \{-\sin(\log x)\} \cdot \frac{1}{x}$$

$$= \frac{-\cos(\log x)}{x^2} - \frac{\sin(\log x)}{x^2}$$

$$= -\frac{1}{x^2} [\cos(\log x) + \sin(\log x)]$$

11. If $y = 5 \cos x - 3 \sin x$, then prove that $\frac{d^2y}{dx^2} + y = 0$.

SOLUTION

We have, $y = 5 \cos x - 3 \sin x$

$$\Rightarrow \frac{dy}{dx} = 5(-\sin x) - 3(\cos x) = -5 \sin x - 3 \cos x$$

$$\Rightarrow \frac{d^2y}{dx^2} = -5 \cos x - 3(-\sin x) = -5 \cos x + 3 \sin x$$

$$\Rightarrow \frac{d^2y}{dx^2} + y = -5 \cos x + 3 \sin x + 5 \cos x - 3 \sin x = 0 \text{ Hence proved.}$$

12. If $y = \cos^{-1}x$, find $\frac{d^2y}{dx^2}$ in terms of y alone.

SOLUTION

$$y = \cos^{-1}x \Rightarrow x = \cos y \dots \text{(i)}$$

Differentiating (i) w.r.t. x , we get $1 = -\sin y \frac{dy}{dx}$

$$\Rightarrow \frac{dy}{dx} = -\operatorname{cosec} y \dots \text{(ii)}$$

Differentiating (ii) w.r.t. x , we get $\frac{d^2y}{dx^2} = \operatorname{cosec} y \cot y \frac{dy}{dx} = -\operatorname{cosec}^2 y \cot y$

13. If $y = 3 \cos(\log x) + 4 \sin(\log x)$, then show that $x^2 y_2 + x y_1 + y = 0$

SOLUTION

We have, $y = 3 \cos(\log x) + 4 \sin(\log x)$, ... (i)

$$\frac{dy}{dx} = 3[-\sin(\log x)] \frac{1}{x} + 4[\cos(\log x)] \frac{1}{x} \dots \text{(ii)}$$

Differentiating (ii) w.r.t. x , we get

$$\frac{d^2y}{dx^2} = 3 \left[\left(-\sin(\log x) \left(-\frac{1}{x^2} \right) \right) + \frac{1}{x} \left(-\cos(\log x) \right) \frac{1}{x} \right] + 4 \left[\cos(\log x) \cdot \left(\frac{-1}{x^2} \right) + \frac{1}{x} \left\{ -\sin(\log x) \right\} \frac{1}{x} \right]$$

$$= \frac{1}{x^2} [3 \sin(\log x) - 3 \cos(\log x) - 4 \cos(\log x) - 4 \sin(\log x)]$$

$$\Rightarrow x^2 \frac{d^2y}{dx^2} = [3 \sin(\log x) - 4 \cos(\log x) - \{3 \cos(\log x) + 4 \sin(\log x)\}]$$

$$\Rightarrow x^2 \frac{d^2y}{dx^2} = -x \frac{dy}{dx} - y$$

$$\Rightarrow x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0 \Rightarrow x^2 y_2 + x y_1 + y = 0 \text{ Hence proved.}$$

14. If $y = Ae^{mx} + Be^{nx}$, then show that $\frac{d^2y}{dx^2} - (m+n) \frac{dy}{dx} + mny = 0$.

SOLUTION

Let $y = Ae^{mx} + Be^{nx}$... (i)

$$\frac{dy}{dx} = Ae^{mx} \cdot m + Be^{nx} \cdot n = Ame^{mx} + Bne^{nx} \dots \text{(ii)}$$

Differentiating (ii) w.r.t. x , we get

$$\frac{d^2y}{dx^2} = Ame^{mx} \cdot m + Bne^{nx} \cdot n = Am^2 e^{mx} + Bn^2 e^{nx} \dots \text{(iii)}$$

$$\text{Now, } \frac{d^2y}{dx^2} - (m+n) \frac{dy}{dx} + mny = Am^2 e^{mx} + Bn^2 e^{nx}$$

$$- [(m+n)(Ame^{mx} + Bne^{nx})] + mn(Ae^{mx} + Be^{nx}) \text{ [from (i), (ii) and (iii)]}$$

$$= Am^2 e^{mx} + Bn^2 e^{nx} - Am^2 e^{mx} - Bmne^{nx} - Amne^{mx} - Bn^2 e^{nx} + Amne^{mx} + Bmne^{nx} = 0 \text{ Hence proved.}$$

15. If $y = 500e^{7x} + 600e^{-7x}$, then show that $\frac{d^2y}{dx^2} = 49y$.

SOLUTION

Let $y = 500e^{7x} + 600e^{-7x}$... (i)

$$\frac{dy}{dx} = 500 \cdot e^{7x} \cdot 7 + 600 \cdot e^{-7x} \cdot (-7)$$

$$= 3500e^{7x} - 4200e^{-7x} \dots \text{(ii)}$$

Differentiating (ii) w.r.t. x , we get

$$\frac{d^2y}{dx^2} = 3500 \cdot 7 \cdot e^{7x} - 4200 \cdot (-7)e^{-7x} = 24500e^{7x} + 29400e^{-7x}$$

$$= 49(500e^{7x} + 600e^{-7x}) = 49y \therefore \frac{d^2y}{dx^2} = 49y$$

16. If $e^y(x+1) = 1$, then show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$.

SOLUTION

$xe^y + e^y = 1$... (i) Differentiating (i) w.r.t. x , we get

$$xe^y \frac{dy}{dx} + e^y + e^y \frac{dy}{dx} = 0 \Rightarrow$$

$$\frac{dy}{dx} = \frac{-1}{x+1} \dots (ii)$$

From (ii), $\left(\frac{dy}{dx}\right)^2 = \left(\frac{-1}{x+1}\right)^2 = \frac{1}{(x+1)^2} \dots (iii)$

Differentiating (ii) w.r.t. x , we get $\frac{d^2y}{dx^2} = \frac{1}{(x+1)^2} \dots (iv)$

From (iii) and (iv), we get $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$

Hence proved. .

17. If $y = (\tan^{-1}x)^2$, show that $(x^2 + 1)^2 y_2 + 2x(x^2 + 1)y_1 = 2$.

SOLUTION

Let $y = (\tan^{-1}x)^2$... (i) Differentiating (i) w.r.t. x , we get

$$\frac{dy}{dx} = 2 \tan^{-1}x \cdot \frac{1}{(1+x^2)} \dots (ii)$$

Differentiating (ii) w.r.t. x , we get

$$\frac{d^2y}{dx^2} = 2 \left[\tan^{-1}x \cdot \frac{\{1+x^2\} \cdot 0 - 2x}{(1+x^2)^2} + \frac{1}{(1+x^2)^2} \cdot \frac{1}{(1+x^2)} \right]$$

$$= 2 \left[\frac{-2x \tan^{-1}x}{(1+x^2)^2} + \frac{1}{(1+x^2)^2} \right] = 2 \left[\frac{-2x \tan^{-1}x + 1}{(1+x^2)^2} \right]$$

Now, $(x^2 + 1)^2 \frac{d^2y}{dx^2} + 2x(x^2 + 1) \frac{dy}{dx}$

$$= (x^2 + 1)^2 \cdot 2 \left[\frac{-2x \tan^{-1}x + 1}{(1+x^2)^2} \right] + 2x(x^2 + 1) \cdot 2 \tan^{-1}x \cdot \frac{1}{(1+x^2)} = -4x \tan^{-1}x + 2 + 4x \tan^{-1}x = 2 \text{ Hence proved.}$$



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