NCERT - Exercise 5.6

If x andy are connected parametrically by the equations given in questions 1 to 10, without eliminating the parameter, find $\frac{dy}{dx}$.

1. $x = 2at^2, y = at^4$

   SOLUTION

   Here, $x = 2at^2$ ....(i) and $y = at^4$ ...(ii) Differentiating (i) & (ii) w.r.t. t, we get

   $\frac{dx}{dt} = 2a(2t) = 4at$ and $\frac{dy}{dt} = 4at^3$

   $\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4at^3}{4at} = t^2$

2. $x = a\cos\theta, y = b\cos\theta$

   SOLUTION

   Here, $x = a\cos\theta$ ...(i) and $y = b\cos\theta$ ...(ii) Differentiating (i) & (ii) w.r.t. 0, we get

   $\frac{dx}{d\theta} = a(-\sin\theta) = -a\sin\theta$ and $\frac{dy}{d\theta} = b(-\sin\theta) = -b\sin\theta$

   $\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{-b\sin\theta}{-a\sin\theta} = \frac{b}{a}$

3. $x = \sin t, y = \cos 2t$

   SOLUTION

   Here,$x = \sin t \ldots$(i) and $y = \cos 2t \ldots$(ii) Differentiating (i) & (ii) w.r.t. t, we get

   $\frac{dx}{dt} = \cos t$ and

   $\frac{dy}{dt} = -\sin 2t \cdot 2 = -2\sin 2t$

   $\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-2\sin 2t}{\cos t} = \frac{-2\cdot 2\sin t\cos t}{\cos t} = -4\sin t$

4. . $x = 4t, y = \frac{4}{t}$

   SOLUTION

   Here,$x = 4t \ldots$(i) $y = \frac{4}{t}$ Differentiating (i) & (ii) w.r.t. t, we get

   $\frac{dx}{dt} = 4$ and $\frac{dy}{dt} = \frac{-4}{t^2}$

   $\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-4}{t^2} \times \frac{1}{4} = \frac{-1}{t^2}$

5. $x = \cos\theta - \cos 2\theta, y = \sin\theta - \sin 2\theta$

   SOLUTION

   Here, $x = \cos\theta - \cos 2\theta$ ..(i) and $y = \sin\theta - \sin 2\theta \ldots$(ii) Differentiating (i) & (ii) w.r.t. θ , we get

   $\frac{dx}{d\theta} = -\sin\theta - (-\sin 2\theta)\cdot 2$

   $\frac{dy}{d\theta} = \cos\theta - \cos 2\theta \cdot 2 = \cos\theta - 2\cos 2\theta$

   $\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\cos\theta - 2\cos 2\theta}{2\sin 2\theta - \sin\theta}$

6. $x = a(\theta - \sin\theta), y = a(1 + \cos\theta)$

SOLUTION

Here,$x = a(\theta - \sin\theta)$ and $\ldots$(i) $y = a(1 + \cos\theta)$ $\ldots$(ii) Differentiating (i) & (ii) w.r.t. θ, we get

$\frac{dx}{d\theta} = a[1 - \cos\theta]$ and

$\frac{dy}{d\theta} = a[-\sin\theta] = -a\sin\theta$

$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{-a\sin\theta}{a(1-\cos\theta)} = \frac{-\sin\theta}{1-\cos\theta}$

$= \frac{-2\sin\theta/2\cos\theta/2}{2\sin^2\theta/2} = -\cot\frac{\theta}{2}$

7. $x = \frac{\sin^3 t}{\sqrt{\cos 2t}}, y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$

SOLUTION

Here $x = \frac{\sin^3 t}{\sqrt{\cos 2t}} \ldots$(i) and $y = \frac{\cos^3 t}{\sqrt{\cos 2t}} \ldots$(ii)

Differentiating (i) & (ii) w.r.t. t, we get

$$\frac{dx}{dt} = \frac{\sqrt{\cos 2t}\frac{d}{dt}(\sin^3 t) - \sin^3 t\frac{d}{dt}(\sqrt{\cos 2t})}{\cos 2t}$$

$$= \frac{\sqrt{\cos 2t}3\sin^2 t\cos t - \sin^3 t\cdot\frac{1}{2\sqrt{\cos 2t}}\cdot(-\sin 2t)\cdot 2}{\cos 2t}$$

$$= \frac{\sqrt{\cos 2t}3\sin^2 t\cos t + \frac{\sin^3 t\sin 2t}{\sqrt{\cos 2t}}}{\cos 2t}$$

$$= \frac{3\cos 2t\sin^2 t\cos t + \sin^3 t\sin 2t}{(\cos 2t)^{3/2}}$$

$$\frac{dy}{dt} = \frac{\sqrt{\cos 2t}\frac{d}{dt}(\cos^3 t) - \cos^3 t\frac{d}{dt}(\sqrt{\cos 2t})}{\cos 2t}$$

$$= \frac{\sqrt{\cos 2t}\cdot 3\cos^2 t(-\sin t) - \cos^3 t\cdot\frac{1}{2\sqrt{\cos 2t}}\cdot(-\sin 2t)\cdot 2}{\cos 2t}$$

$$= \frac{-3\cos^2 t\cdot\sin t\cdot\sqrt{\cos 2t} + \frac{\cos^3 t\sin 2t}{\sqrt{\cos 2t}}}{\cos 2t}$$

$$= \frac{\cos^3 t\sin 2t - 3\cos^2 t\cdot\sin t\cos 2t}{(\cos 2t)^{3/2}}$$

$$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cos^3 t\sin 2t - 3\cos^2 t\cdot\sin t\cos 2t}{3\cos 2t\sin^2 t\cos t + \sin^3 t\sin 2t} = -\cot 3t$$

8. $x = a\left(\cos t + \log\tan\frac{t}{2}\right), y = a\sin t$

SOLUTION

Here, $x = a\left(\cos t + \log\tan\frac{t}{2}\right) \ldots$(1) and $y = a\sin t \ldots$(2) Differentiating (1) & (2) w.r.t. t, we get

$$\frac{dx}{dt}=a\left[-\sin t+\frac{1}{\tan\frac{t}{2}}\frac{d}{dt}\left(\tan\frac{t}{2}\right)\right]$$

$$=a\left[-\sin t+\frac{1}{\tan\frac{t}{2}}\sec^2\frac{t}{2}\cdot\frac{1}{2}\right]=\frac{a\cos^2 t}{\sin t}\ \frac{dy}{dt}=a\cos t$$

$$\therefore\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{a\cos t\sin t}{a\cos^2 t}=\tan t$$

9. $x=a\sec\theta, y=b\tan\theta$

SOLUTION

Here, $x=a\sec\theta$, ...(1) and $y=b\tan\theta$ ...(2) Differentiating (1) & (2) w.r.t. θ, we get

$$\frac{dx}{d\theta}=a\sec\theta\tan\theta \text{ and } \frac{dy}{d\theta}=b\sec^2\theta$$

$$\therefore\frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}=\frac{b\sec^2\theta}{a\sec\theta\tan\theta}=\frac{b\sec\theta}{a\tan\theta}=\frac{b}{a}\cos ec\theta$$

10. $x=a(\cos\theta+\theta\sin\theta), y=a(\sin\theta-\theta\sec\theta)$

SOLUTION

Here, $x=a(\cos\theta+\theta\sin\theta)$ .....(1) and $y=a(\sin\theta-\theta\sec\theta)$ ....(2) Differentiating (1) & (2) w.r.t. θ, we get

$$\frac{dx}{d\theta}=a[-\sin\theta+\theta\cdot\cos\theta+\sin\theta]=a\theta\cos\theta\ \frac{dy}{d\theta}=a[\cos\theta-(\theta(-\sin\theta)+\cos\theta)]$$

$$=a[\cos\theta+\theta\sin\theta-\cos\theta]=a\theta\sin\theta$$

$$\therefore\frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}=\frac{a\theta\sin\theta}{a\theta\cos\theta}=\tan\theta$$

11. If $x=\sqrt{a^{\sin^{-1}t}}, y=\sqrt{a^{\cos^{-1}}}$, show that $\frac{dy}{dx}=-\frac{y}{x}$.

SOLUTION

Given : $x=\sqrt{a^{\sin^{-1}t}}$ and $y=\sqrt{a^{\cos^{-1}}}$, Differentiating x and y w.r.t. t,we get

$$\frac{dx}{dt}=\frac{1}{2}\cdot\frac{1}{\sqrt{a^{\sin^{-1}}}}\cdot\frac{d}{dt}a^{\sin^{-1}t}=\frac{1}{2}\cdot\frac{1}{\sqrt{a^{\sin^{-1}t}}}a^{\sin^{-1}t}\cdot\log a\cdot\frac{d}{dt}\sin^{-1}t$$

$$=\frac{\sqrt{a^{\sin^{-1}t}}}{2}\cdot\log a\frac{1}{\sqrt{1-t^2}}$$

$$\frac{dy}{dt}=\frac{1}{2}\frac{1}{\sqrt{a^{\cos^{-1}t}}}\cdot\frac{d}{dt}a^{\cos^{-1}t}=\frac{1}{2}\cdot\frac{1}{\sqrt{a^{\cos^{-1}}t}}\cdot a^{\cos^{-1}t}\cdot\log a\frac{-1}{\sqrt{1-t^2}}$$

$$=\frac{\sqrt{a^{\cos^{-1}}}}{2}\log a\frac{-1}{\sqrt{1-t^2}}$$

$$\therefore\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$=\frac{\frac{\sqrt{a^{\cos^{-1}t}}}{2}\cdot\log a\cdot\frac{-1}{\sqrt{1-t^2}}}{\frac{\sqrt{a^{\sin^{-1}t}}}{2}\cdot\log a\cdot\frac{1}{\sqrt{1-t^2}}}$$

$$=-\frac{\sqrt{a^{\cos^{-1}t}}}{\sqrt{a^{\sin^{-1}t}}}=\frac{-y}{x}$$

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