NCERT - Exercise 5.5

Differentiate the functions given in Exercises 1 to 11 w.r.t. x

1. $cosx \cdot cos2x \cdot cos3x$

SOLUTION

Let $y = cosx \cdot cos2x \cdot cos3x$

Taking log on both sides, we get

$\log y = \log(\cos x \cdot cos2x \cdot cos3x)$

Then, $log\, y\; log\,(cosx) + log\,(cos2x) + log\,(cos3x)$ ...(i)

On differentiating (i) both sides w.r.t. x, we get $\frac{1}{y}.\frac{dy}{dx}$

$= \frac{1}{\cos x}(-\sin x) + \frac{1}{\cos 2x}(-\sin 2x)(2) + \frac{1}{\cos 3x}(-\sin 3x)(3)$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = -\tan x - 2\tan 2x - 3\tan 3x$

$\therefore \frac{dy}{dx} = -\cos x \cdot \cos 2x \cdot \cos 3x(\tan x + 2\tan 2x + 3\tan 3x)$

2. $\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$

SOLUTION

Let $y = \sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$ ....(i)

Taking log on both sides of (i), we get

$\log y = \frac{1}{2}[\log(x-1) + \log(x-2) - \log(x-3) - \log(x-4) - \log(x-5)]$ ... (ii)

Now, differentiating (ii) on both sides w.r.t, x, we get

$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{(x-1)} + \frac{1}{(x-2)} - \frac{1}{(x-3)} - \frac{1}{(x-4)} - \frac{1}{(x-5)}\right]$

$\therefore \frac{dy}{dx} = \frac{1}{2}\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}} \times \left[\frac{1}{(x-1)} + \frac{1}{(x-2)} - \frac{1}{(x-3)} - \frac{1}{(x-4)} - \frac{1}{(x-5)}\right]$

3. $(\log x)^{\cos x}$

SOLUTION Let $y = (\log x)^{\cos x}$ ...(i) Taking log on both sides of (i), we get $\log y = \cos x \log(\log x)$ ....(ii)

On differentiating (ii) both sides w.r.t. x, we get

$\frac{1}{y}\frac{dy}{dx} = \cos x \frac{d}{dx}\log(\log x) + \log(\log x)\frac{d}{dx}\cos x = \cos x \cdot \frac{1}{\log x}\frac{1}{x} + \log(\log x)(-\sin x)$

$= \frac{\cos x}{x \log x} - \sin x \log(\log x)$

$\Rightarrow \frac{dy}{dx} = (\log x)^{\cos x}\left[\frac{\cos x}{x\log x} - \sin x \log(\log x)\right]$

4. $x^x - 2^{\sin x}$

SOLUTION

$y = x^x - 2^{\sin x} \Rightarrow y = u - v$, where $u = x^x$

and $v = 2^{\sin x}$

$\therefore \frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx} \ldots$(i) Now, $u = x^x$

Taking log on both sides, we get log u = x log x Differentiating w.r.t. x, we get

$\Rightarrow \frac{1}{u}\frac{du}{dx} = x\frac{1}{x} + \log x$

$\Rightarrow \frac{du}{dx} = x^x[1 + \log x]$ ....(ii) and $v = 2^{\sin x}$ Taking log on both sides, we get $\Rightarrow \log v = \sin x \log 2$

Differentiating w.r.t.x, we get

$\Rightarrow \frac{1}{v}\frac{dv}{dx} = \log 2(\cos x) \Rightarrow \frac{dv}{dx} = 2^{\sin x}(\cos x \cdot \log 2)$ ....(iii)

From (i), (ii) and (iii), we get

$\frac{dy}{dx} = x^x[1 + \log x] - 2^{\sin x}(\cos x \cdot \log 2)$

5. $(x+3)^2 \cdot (x+4)^3 \cdot (x+5)^4$

SOLUTION

Let $y = (x+3)^2 \cdot (x+4)^3 \cdot (x+5)^4$

Taking log on both sides, we get

$\log y = \log[(x+3)^2(x+4)^3(x+5)^4] \Rightarrow \log y = 2\log(x+3) + 3\log(x+4) + 4\log(x+5) \ldots$(i)

Differentiating (i) on both sides w.r.t. x, we get

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = 2 \cdot \frac{1}{(x+3)} + 3 \cdot \frac{1}{(x+4)} + 4 \cdot \frac{1}{(x+5)}$

$\Rightarrow \frac{dy}{dx} = (x+3)^2 \cdot (x+4)^3 \cdot (x+4)^4 \left[\frac{2}{x+3} + \frac{3}{x+4} + \frac{4}{x+5}\right]$

$= (x+3)(x+4)^2(x+5)^3(9x^2 + 70x + 133)$

6. $\left(x + \frac{1}{x}\right)^x + x^{\left(1 + \frac{1}{x}\right)}$

SOLUTION

Let $y = \left(x + \frac{1}{x}\right)^x + x^{\left(1 + \frac{1}{x}\right)} = u + v$ where $u = \left[x + \frac{1}{x}\right]^x$ and $v = x^{\left(1 + \frac{1}{x}\right)}$

$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ ....(i)

Now, $u = \left(x + \frac{1}{x}\right)^x \Rightarrow \log u = x \log\left(x + \frac{1}{x}\right)$ .....(ii)

Differentiating (ii) w.r.t. x, we get

$\Rightarrow \frac{1}{u}\frac{du}{dx} = x\frac{d}{dx}\log\left(x + \frac{1}{x}\right) + \log\left(x + \frac{1}{x}\right)(1)$

$= \frac{x}{x + \frac{1}{x}}\left(1 - \frac{1}{x^2}\right) + \log\left(x + \frac{1}{x}\right)$

$$\Rightarrow \frac{du}{dx} = \left(x+\frac{1}{x}\right)^x \left[\frac{x}{x+\frac{1}{x}}\left(x-\frac{1}{x^2}\right) + \log\left(x+\frac{1}{x}\right)\right] \ldots .(iii)$$

Also, $v = x^{\left(x+\frac{1}{x}\right)}$

Taking log on both the sides, we get

$$\Rightarrow \log v = \left(x+\frac{1}{x}\right)\log x \ldots (iv)$$

Differentiating (iv) on both sides w.r.t. x, we get

$$\Rightarrow \frac{1}{v}\frac{dv}{dx} = \left(x+\frac{1}{x}\right)\frac{d}{dx}\log x + \log x \frac{d}{dx}\left(x+\frac{1}{x}\right)$$

$$= \left(x+\frac{1}{x}\right)\frac{1}{x} + \log x\left(-\frac{1}{x^2}\right)$$

$$\Rightarrow \frac{dv}{dx} = x^{\left(x+\frac{1}{x}\right)}\left[\left(x+\frac{1}{x}\right)\frac{1}{x} + \log x\left(\frac{-1}{x^2}\right)\right] \ldots (v)$$

Substituting the values of (iii) and (v) in (i), we get

$$\frac{dy}{dx} = \left(x+\frac{1}{x}\right)^x \left[\frac{x}{x+\frac{1}{x}}\left(x-\frac{1}{x^2}\right) + \log\left(x+\frac{1}{x}\right)\right] + x^{\left(x+\frac{1}{x}\right)}\left[\left(x+\frac{1}{x}\right)\frac{1}{x} + \log x\left(\frac{-1}{x^2}\right)\right]$$

$$= \left(x+\frac{1}{x}\right)^x \left[\frac{x^2-1}{x^2+1} + \log\left(x+\frac{1}{x}\right)\right] + x^{\left(x+\frac{1}{x}\right)}\left[\frac{x+1-\log x}{x^2}\right]$$

7. $(\log x)^x + x^{\log x}$

SOLUTION

Let $y = (\log x)^x + x^{\log x}$

$\Rightarrow y = u + v$, where$u = (\log x)^x, v = x^{\log x}$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \ldots .(i)$$

Now, $u = (\log x)^x$

Taking log on both sides, we get $\therefore \log u = x\log(\log x)$

Differentiating w.r.t. x, we get

$$\frac{1}{u}\cdot\frac{du}{dx} = x\frac{d}{dx}\log(\log x) + \log(\log x)$$

$$= x\cdot\frac{1}{\log x}\cdot\frac{1}{x} + \log(\log x) = \frac{1}{\log x} + \log(\log x)$$

$$\therefore \frac{du}{dx} = (\log x)^x\left[\frac{1}{\log x} + \log(\log x)\right] \ldots (ii)$$

Also, $v = x^{\log x}$

Taking log on both sides, we get

$\log v = \log x \log x = (\log x)^2$

Differentiating w.r.t. x, we get

$$\frac{1}{v}\frac{dv}{dx} = 2\log x\cdot\frac{1}{x}.$$

$\therefore \frac{dv}{dx} = x^{\log x}\left[\frac{2\log x}{x}\right] \ldots$(iii)

From (i), (ii) & (iii), we get

$\frac{dy}{dx} = (\log x)^x\left[\frac{1}{\log x} + \log(\log x)\right] + x^{\log x}\left[\frac{2\log x}{x}\right]$

$= (\log x)^{x-1}[1 + \log x \cdot \log(\log x)] + 2x^{\log x - 1} \cdot \log x$

8. $(\sin x)^x + \sin^{-1}\sqrt{x}$

SOLUTION

Let $y = (\sin x)^x + \sin^{-1}\sqrt{x} = u + v$, where

$u = (\sin x)^x, v = \sin^{-1}\sqrt{x}$

$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \ldots.(i)$

Now, $u = (\sin x)^x$

Taking log on both sides, we get

$\log u = x \log \sin x$

Differentating w.r.t. x, we get

$\frac{1}{u} \cdot \frac{du}{dx} = x\frac{d}{dx}(\log \sin x) + \log \sin x$

$= x \cdot \frac{1}{\sin x} \cdot \cos x + \log \sin x = x \cot x + \log \sin x$

$\therefore \frac{du}{dx} = (\sin x)^x[x \cot x + \log \sin x]$ …..(ii)

Also, $v = \sin^{-1}\sqrt{x} \Rightarrow \frac{dv}{dx} = \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}} \ldots$(iii)

From (i), (ii) & (iii), we get

$\therefore \frac{dy}{dx} = (\sin x)^x[x \cdot \cot x + \log \sin x] + \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}}$

$= (\sin x)^x[x \cot x + \log \sin x] + \frac{1}{2\sqrt{x - x^2}}$

9. $x^{\sin x} + (\sin x)^{\cos x}$

SOLUTION

Let$y = x^{\sin x} + (\sin x)^{\cos x} = u + v$, where

$u = x^{\sin x} and\ v = (\sin x)^{\cos x} \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

$\ldots$(i)

Now, $u = x^{\sin x}$ Taking log on both sides, we get

$\log u = \sin x \log x \ldots$(ii)

Differentiating (ii) w.r.t. x, we get

$\Rightarrow \frac{1}{u}\frac{du}{dx} = \sin x \frac{d}{dx}(\log x) + \log x \frac{d}{dx}(\sin x) = \sin x \cdot \frac{1}{x} + \log x \cos x$

$\therefore \frac{du}{dx} = x^{\sin x}\left[\frac{\sin x}{x} + \log x \cos x\right] \ldots$(iii)

Also, $v = (\sin x)^{\cos x}$ Taking log on both sides, we get

$\log v = \cos x \log \sin x \ldots$(iv)

Differentiating (iv) w.r.t. x, we get

$\frac{1}{v}\frac{dv}{dx} = \cos x \frac{d}{dx} \log \sin x + \log \sin x \frac{d}{dx} \cos x$

$= \cos x \frac{1}{\sin x} \cos x + \log \sin x(-\sin x)$

$= \cos x \cot x - \sin x \log \sin x$

$\therefore \frac{dv}{dx} = (\sin x)^{\cos x}[\cos x \cot x - \sin x \log \sin x] \ldots .(\mathrm{v})$

Substituting the values of (iii) & (v) in (i), we get

$\frac{dy}{dx} = x^{\sin x}\left[\frac{\sin x}{x} + \log x \cos x\right] + (\sin x)^{\cos x}[\cos x \cot x - \sin x \log \sin x]$

10. $x^{x \cos x} + \frac{x^2+1}{x^2-1}$

SOLUTION

Let$y = x^{x \cos x} + \frac{x^2+1}{x^2-1} = u + v,$

where $u = x^{x \cos x}$ and $v = \frac{x^2+1}{x^2-1} \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \ldots (\mathrm{i})$

Now, $u = x^{x \cos x}$

Taking log on both sides, we get

$\log u = x \cos x \cdot \log x \ldots (\mathrm{ii})$

Differentiating (ii) w.r.t.x, we,get

$\frac{1}{u}\frac{du}{dx} = x \cos x \frac{d}{dx}(\log x) + x \log x \frac{d}{dx} \cos x + \cos x \log x \frac{d}{dx}(x)$

$\Rightarrow \frac{1}{u}\frac{du}{dx} = x \cos x \cdot \frac{1}{x} + x \log x(-\sin x) + \cos x \log x$

$= \cos - x \sin x \log x + \cos x \log x \therefore \frac{du}{dx} = x^{x \cos x}[\cos x - x \sin x \log x + \cos x \log x] \ldots ..(\mathrm{iii})$

Also, $v = \frac{x^2+1}{x^2-1} \ldots .(\mathrm{iv})$

Differentiating (iv) w.r.t. x, we get

$\Rightarrow \frac{dv}{dx} = \frac{(x^2-1)\frac{d}{dx}(x^2+1) - (x^2+1)\frac{d}{dx}(x^2-1)}{(x^2-1)^2}$

$= \frac{(x^2-1)(2x) - (x^2+1)(2x)}{(x^2-1)^2} = \frac{2x[x^2-1-x^2-1]}{(x^2-1)^2} = \frac{-4x}{(x^2-1)^2} \ldots (\mathrm{v})$

Substituting the values of (iii) & (v) in (i), we get

$\frac{dy}{dx} = x^{x \cos x}[\cos x - x \sin x \log x + \cos x \log x] - \frac{4x}{(x^2-1)^2}$

$= x^{x \cos x}[\cos x(1 + \log x) - x \sin x \log x] - \frac{4x}{(x^2-1)^2}$

11. $(x \cos x)^x + (x \sin x)^{1/x}$

SOLUTION

Let $y = (x \cos x)^x + (x \sin x)^{1/x} = u + v,$

where$u = (x \cos x)^x, v = (x \sin x)^{1/x}$

$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

$\ldots (\mathrm{i})$ Now, $u = (x \cos x)^x$. Taking log on both sides, we get log u = x log (x cos x) ….(ii)

Differentiating (ii) on both sides w.r.t. x, we get

$\frac{1}{u}\frac{du}{dx} = x\frac{d}{dx}\log(x\cos x) + \log(x\cos x)$

$= x\left[\frac{1}{x\cos x}\frac{d}{dx}(x\cos x)\right] + \log(x\cos x)$

$= x\left[\frac{1}{x\cos x}\left(x\frac{d}{dx}\cos x + \cos x\right)\right] + \log(x\cos x)$

$= x\left[\frac{1}{x\cos x}\{x(-\sin x) + \cos x\}\right] + \log(x\cos x)$

$= \sec x(\cos x - \sin x) + \log(x\cos x) = 1 - x\tan x + \log(x\cos x) \therefore \frac{du}{dx} = (x\cos x)^x[1 - x\tan x + \log(x\cos x)] \ldots$.(iii) Now, $v = (x\sin x)^{1/x}$

Taking log on both sides, we get $\log v = \frac{1}{x}(x\sin x)$

$\ldots$(iv) Differentiating (iv) w.r.t.x, we get

$\frac{1}{v}\frac{dv}{dx} = \frac{1}{x}\frac{d}{dx}\log(x\sin x) + \log(x\sin x)\frac{d}{dx}\left(\frac{1}{x}\right)$

$= \frac{1}{x}\cdot\frac{1}{x\sin x}\frac{d}{dx}(x\sin x) + \log(x\sin x)\cdot\left(\frac{-1}{x^2}\right)$

$= \frac{1}{x^2\sin x}\left[x\frac{d}{dx}(\sin x) + \sin x\right] - \frac{\log(x\sin x)}{x^2} =$

$= \frac{\cot x}{x} + \frac{1}{x^2} - \frac{\log(x\sin x)}{x^2} = \frac{x\cot x + 1 - \log(x\sin x)}{x^2}$

$\therefore \frac{dv}{dx} = (x\sin x)^{1/x}\left[\frac{x\cot x + 1 - \log(x\sin x)}{x^2}\right] \ldots$(v)

Substituting the values of (iii) and (v) in (i), we get $\frac{dy}{dx} = (x\cos x)^x[1 - x\tan x + \log(x\cos x)] + (x\sin x)^{1/x}\left[\frac{x\cot x + 1 - \log(x\sin x)}{x^2}\right]$

Find $\frac{dy}{dx}$ of the functions given in Exercises 12 to 15.

12. $x^y + y^x = 1$

SOLUTION

$x^y + y^x = 1 \ldots$(i)

Differentiating (i) w.r.t. x, we get

$\frac{d}{dx}(x^y) + \frac{d}{dx}(y^x) = 0 \ldots$(ii)

Let $u = x^y \therefore \log u = y\log x$

Differentiating w.r.t. x, we get

$\Rightarrow \frac{1}{u}\frac{du}{dx} = y\cdot\frac{1}{x} + \log x\frac{dy}{dx}$

$\Rightarrow \frac{du}{dx} = x^y\left[\frac{y}{x} + \log x\frac{dy}{dx}\right] \ldots$.(iii)

Let $v = y^x \Rightarrow \log v = x\log y$

Differentiating w.r.t. x,we get

$\frac{1}{v}\cdot\frac{dv}{dx} = x\cdot\frac{1}{y}\cdot\frac{dy}{dx} + \log y \therefore \frac{dv}{dx} = y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right) \ldots$.(iv)

Substituting the values of (iii) and (iv) in (ii), we get

$x^y\left(\frac{y}{x}+\log x\frac{dy}{dx}\right)+y^x\left(\frac{x}{y}\frac{dy}{dx}+\log y\right)=0$

$\Rightarrow (x^y\log x+xy^{x-1})\frac{dy}{dx}=-(y^x\log y+yx^{y-1})$

$\Rightarrow \frac{dy}{dx}=-\left(\frac{y^x\log y+yx^{y-1}}{x^y\log x+xy^{x-1}}\right)$

13. . $y^x=x^y$

SOLUTION

We are given that, $y^x=x^y$ Taking log on both sides, we get x log y = y log x . . . (i)

Differentiating (i) on both sides w.r.t. x, we get

$x\frac{d}{dx}\log y+\log y\cdot 1=y\frac{d}{dx}\log x+\log x\frac{dy}{dx}$

$\Rightarrow x\cdot\frac{1}{y}\cdot\frac{dy}{dx}+\log y=\frac{y}{x}+\log x\cdot\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx}\left[\log x-\frac{x}{y}\right]=\log y-\frac{y}{x}\Rightarrow\frac{dy}{dx}=\frac{y(x\log y-y)}{x(y\log x-x)}$

14. $(\cos x)^y=(\cos y)^x$

SOLUTION

We have, $(\cos x)^y=(\cos y)^x$ Taking log on both sides, we get

y log (cos x) = x log (cos y) . . . (i) Differentiating (i) on both sides w.r.t. x, we get $\Rightarrow y\frac{d}{dx}\log(\cos x)+\log(\cos x)\frac{dy}{dx}$

$=x\frac{d}{dx}(\log(\cos y))+\log(\cos y)$

$\Rightarrow y\frac{1}{\cos x}(-\sin x)+\log(\cos x)\frac{dy}{dx}$

$=x\cdot\frac{1}{\cos y}(-\sin xy)\frac{dy}{dx}+\log(\cos y)\Rightarrow\frac{dy}{dx}[\log(\cos x)+x\tan y]=\log(\cos y)+y\tan x$

$\Rightarrow\frac{dy}{dx}=\frac{\log(\cos y)+y\tan x}{\log(\cos x)+x\tan y}$

15. $xy=e^{(x-y)}$

SOLUTION

We have, $xy=e^{(x-y)}$ Taking log on both sides, we get $\log(xy)=\log e^{(x-y)}$

or log x + logy $=x-y$ ..(i)

Differentiating (i) on both sides w.r.t. x, we get

$\frac{1}{x}+\frac{1}{y}\frac{dy}{dx}=\left(1-\frac{dy}{dx}\right)\Rightarrow\frac{dy}{dx}\left(\frac{1}{y}+1\right)=1-\frac{1}{x}$

$\Rightarrow\frac{dy}{dx}=\frac{y(x-1)}{x(y+1)}$

16. Find the derivative of the function given by $f(x)=(1+x)(1+x^2)(1+x^4)(1+x^8)$ and hence find $f'(1)$.

SOLUTION

Let $f(x)=y$

$\Rightarrow y=(1+x)(1+x^2)(1+x^4)(1+x^8)$

Taking log on both sides, we get

$\log y=\log[(1+x)(1+x^2)(1+x^4)(1+x^8)]$

$\Rightarrow\log y=\log((1+x)+\log(1+x^2)+\log(1+x^4)+\log(1+x^8)$

Differentiating w.r.t. x, we get

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{(1+x)} + \frac{1}{(1+x^2)}(2x) + \frac{1}{(1+x^4)}(4x^3) + \frac{1}{(1+x^8)}(8x^7)$

$\Rightarrow \frac{dy}{dx} = y\left[\frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{(1+x^4)} + \frac{8x^7}{(1+x^8)}\right]$

$\therefore f'(x) = (1+x)(1+x^2)(1+x^4)(1+x^8) \times \left[\frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{2x^3}{1+x^4} + \frac{8x^7}{1+x^8}\right]$

$\Rightarrow f'(1) = (1+1)(1+1)(1+1)(1+1)$

$\left[\frac{1}{1+1} + \frac{2(1)}{1+1} + \frac{4(1)}{1+1} + \frac{8(1)}{1+1}\right]$

$\Rightarrow f'(1) = (2)(2)(2)(2)\left[\frac{1}{2} + \frac{2}{2} + \frac{4}{2} + \frac{8}{2}\right]$

$\Rightarrow f'(1) = 16\left\{\frac{1+2+4+8}{2}\right\} = \frac{16}{2}(15) = 8(15) = 120$

17. Differentiate $(x^2-5x+8)(x^3+7x+9)$ in three ways mentioned below:

(i) by using product rule

(ii) by expanding the product to obtain a single polynomial,

(iii) by logarithmic differentiation. Do they all give the same answer?

SOLUTION

(i) Let $f(x) = (x^2-5x+8)(x^3+7x+9)$ By using product rule,

$f'(x) = x^2-5x+8)\frac{d}{dx}(x^3+7x+9) + (x^3+7x+9)\frac{d}{dx}(x^2-5x+8)$

$\Rightarrow f'(x) = (x^2-5x+8)(3x^2+7) + (x^3+7x+9)(2x-5)$

$\Rightarrow f'(x) = 3x^4 - 15x^3 + 24x^2 + 7x^2 - 35x + 56 + 2x^4 + 14x^2 + 18x - 5x^3 - 35x - 45 \Rightarrow f'(x) = 5x^4 - 20x^3 + 45x^2 - 52x + 11$

(ii) By expanding the product to obtain a single polynomial,

$f(x) = (x^2-5x+8)(x^3+7x+9) \Rightarrow f(x) = x^5 + 7x^3 + 9x^2 - 5x^4 - 35x^2 - 45x + 8x^3 + 56x + 72$

$\Rightarrow f(x) = x^5 - 5x^4 + 15x^3 - 26x^2 + 11x + 72 \therefore f'(x) = 5x^4 - 20x^3 + 45x^2 - 52x + 11$

(iii) By logarithmic differentiation, Let $f(x) = y \Rightarrow y = (x^2-5x+8)(x^3+7x+9)$

Taking log on both the sides, we get $\log y = \log\{(x^2-5x+8)(x^3+7x+9)\}$ $\log y = \log\{(x^2-5x+8) + \log(x^3+7x+9)\}$ Differentiating w.r.t. x, we get

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{(x^2-5x+8)}(2x-5) + \frac{1}{(x^3+7x+9)}(3x^2+7)$

$\Rightarrow \frac{dy}{dx} = y\left\{\frac{2x-5}{x^2-5x+8} + \frac{3x^2+7}{x^3+7x+9}\right\}$

$\Rightarrow \frac{dy}{dx} = (x^2-5x+8)(x^3+7x+9)\left[\frac{2x-5}{x^2-5x+8} + \frac{3x^2+7}{x^3+7x+9}\right]$

$= (2x-5)(x^3+7x+9) + (3x^2+7)(x^2-5x+8) \therefore \frac{dy}{dx} = 5x^4 - 20x^3 + 45x^2 - 52x + 11$ Yes, the answer is same in all the three cases.

18. If u, v and w are functions of x, then show that

$\frac{d}{dx}(u \cdot v \cdot w) = \frac{du}{dx}v \cdot w + u \cdot \frac{dv}{dx} \cdot w + u \cdot v\frac{dw}{dx}$ in two ways-first by repeated application of product rule, second by logarithmic differentiation.

SOLUTION

(i) Let $y = u \cdot v \cdot w = u \cdot (vw)$ ...(i) Differentiating (i) on both sides w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}u \cdot (vw) = u\frac{d}{dx}(vw) = u' \cdot (vw) + u[v'w + vw']$$

$$= u' \cdot v \cdot w + uv'w = \frac{du}{dx} \cdot v \cdot w + u \cdot \frac{dv}{dx} \cdot w + u \cdot v \cdot \frac{dw}{dx}$$

(ii) $y = u \cdot v \cdot w$ Taking log on both sides, we get $\log y = \log u + \log v + \log w$ ....(ii) Differentiating (ii) on both sides w.r.t. x, we get

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{u}\frac{du}{dx} + \frac{1}{v}\frac{dv}{dx} + \frac{1}{w}\frac{dw}{dx}$$

$$\Rightarrow \frac{dy}{dx} = y\left(\frac{1}{u}\frac{du}{dx} + \frac{1}{v}\frac{dv}{dx} + \frac{1}{w}\frac{dw}{dx}\right)$$

$$= uvw\left(\frac{1}{u}\frac{du}{dx} + \frac{1}{v}\frac{dv}{dx} + \frac{1}{w}\frac{dw}{dx}\right)$$

$$= vw\frac{du}{dx} + uw\frac{dv}{dx} + uv\frac{dw}{dx} = \frac{du}{dx} \cdot v \cdot w + u \cdot \frac{dv}{dx} \cdot w + u \cdot v \cdot \frac{dw}{dx}.$$

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