NCERT - Exercise 5.4

Differentiate the following w.r.t. x

1. $\frac{e^x}{\sin x}$

SOLUTION Let $y = \frac{e^x}{\sin x}$

$\therefore \frac{dy}{dx} = \frac{d}{dx}\left(\frac{e^x}{\sin x}\right) = \frac{\sin x \frac{d}{dx}(e^x) - e^x \frac{d}{dx}(\sin x)}{\sin^2 x}$

$= \frac{\sin x \cdot e^x - e^x \cdot \cos x}{\sin^2 x} = \frac{e^x(\sin x - \cos x)}{\sin^2 x}, x \neq n\pi, n \in Z$

2. $e^{\sin^{-1}x}$

SOLUTION

Let $e^{\sin^{-1}x}$

$\therefore \frac{dy}{dx} = \frac{d}{dx}(e^{\sin^{-1}x}) = e^{\sin^{-1}x}\frac{d}{dx}(\sin^{-1}x)$

$= e^{\sin^{-1}x} \cdot \frac{1}{\sqrt{1-x^2}}, x \in (-1,1)$

3. e^{x^3}

SOLUTION

Let $y = e^{x^3}$

$\therefore \frac{dy}{dx} = \frac{d}{dx}\left(e^{x^3}\right) = e^{x^3}\frac{d}{dx}(x^3) = e^{x^3} \cdot 3x^2 = 3e^{x^3} \cdot x^2$

4. $\sin(\tan^{-1}e^{-x})$

SOLUTION

Let $y = \sin(\tan^{-1}e^{-x})$

$\therefore \frac{dy}{dx} = \frac{d}{dx}(\sin(\tan^{-1}(e^{-x}))) = \cos(\tan^{-1}e^{-x})\frac{d}{dx}(\tan^{-1}(e^{-x}))$

$= \cos(\tan^{-1}e^{-x}) \cdot \frac{1}{(1+e^{-2x})} \cdot \frac{d}{dx}e^{-x}$

$= \cos(\tan^{-1}e^{-x}) \cdot \frac{-e^{-x}}{(1+e^{-2x})} = \frac{-e^{-x}\cos(\tan^{-1}e^{-x})}{1+e^{-2x}}$

5. $\log(\cos e^x)$

SOLUTION Let $y = \log(\cos e^x)$

$\therefore \frac{dy}{dx} = \frac{d}{dx}\log(\cos e^x) = \frac{1}{\cos e^x}\frac{d}{dx}(\cos e^x)$

$= \frac{1}{\cos e^x} \cdot (-\sin e^x)\frac{d}{dx}(e^x) = -\tan e^x \cdot e^x = -e^x \tan e^x$, where $e^x \neq (2n+1)\frac{\pi}{2}, n \in N$

6. $e^x + e^{x^2} + ... + e^{x^5}$

SOLUTION

Let $y = e^x + e^{x^2} + ... + e^{x^5}$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(e^x + e^{x^2} + ... + e^{x^5})$$

$$= \frac{d}{dx}(e^x) + \frac{d}{dx}(e^{x^2}) + \frac{d}{dx}(e^{x^3}) + \frac{d}{dx}(e^{x^4}) + \frac{d}{dx}(e^{x^5})$$
$$= e^x + e^{x^2}(2x) + e^{x^3}(3x^2) + e^{x^4}(4x^3) + e^{x^5}(5x^4)$$
$$= e^x + 2xe^{x^2} + 3x^2e^{x^3} + 4x^3e^{x^4} + 5x^4e^{x^5}$$

7. $\sqrt{e^{\sqrt{x}}}, x > 0$

SOLUTION

Let $y = \sqrt{e^{\sqrt{x}}}$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{e^{\sqrt{x}}}\right) = \frac{d}{dx}(e^{\sqrt{x}})^{1/2} = \frac{1}{2}(e^{\sqrt{x}})^{-\frac{1}{2}} \cdot \frac{d}{dx}e^{\sqrt{x}}$$

$$= \frac{1}{2}(e^{\sqrt{x}})^{-\frac{1}{2}} \cdot e^{\sqrt{x}} \frac{d}{dx}\sqrt{x} = \frac{1}{2}(e^{\sqrt{x}})^{-1/2} \cdot e^{\sqrt{x}} \cdot \frac{1}{2}(x)^{-1/2}$$
$$= \frac{e^{\sqrt{x}}}{4\sqrt{e^x}\sqrt{x}}, x > 0$$

8. $\log(\log x), x > 1$

SOLUTION

Let y $= \log(\log x), x > 1$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}\log(\log x) = \frac{1}{(\log x)} \cdot \frac{d}{dx}(\log x)$$
$$= \frac{1}{\log x} \cdot \frac{1}{x} = \frac{1}{x \log x}, x > 1$$

9. $\frac{\cos x}{\log x}, x > 0$

SOLUTION

Let $y = \frac{\cos x}{\log x}$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}\left(\frac{\cos x}{\log x}\right) = \frac{\log x \frac{d}{dx}(\cos x) - \cos x \frac{d}{dx}(\log x)}{(\log x)^2}$$

$$= \frac{\log x(-\sin x) - \cos x\left(\frac{1}{x}\right)}{(\log x)^2} = -\left[\frac{\sin x \log x + \frac{1}{x}\cos x}{(\log x)^2}\right]$$
$$= \frac{-(x \sin x \cdot \log x + \cos x)}{x(\log x)^2}$$

10. $\cos(\log x + e^x), x > 0$

SOLUTION

Let $y = \cos(\log x + e^x)$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}\{\cos(\log x + e^x)\}$$
$$= -\sin(\log x + e^x)\frac{d}{dx}(\log x + e^x) = -\sin(\log x + e^x)\left[\frac{1}{x} + e^x\right]$$
$$= -\left(\frac{1}{x} + e^x\right)\sin(\log x + e^x), x > 0$$