NCERT - Exercise 5.3

Find $\frac{dy}{dx}$ in the following :

1. 2x + 3y = sinx

SOLUTION

We are given that, 2x + 3y = sin x ...(i) Differentiating (i) on both sides w.r.t. x, we get $2+3\frac{dy}{dx}=\cos x \Rightarrow \frac{dy}{dx}=\frac{\cos x-2}{3}$

2. 2x + 3y = sin y

SOLUTION

We are given that, 2x + 3y = sin y ...(i) Differentiating (i) on both sides w.r.t. x, we get

$$2+3\frac{dy}{dx}=\cos y\frac{dy}{dx}$$

$$\Rightarrow \cos y\frac{dy}{dx}-3\frac{dy}{dx}=2 \Rightarrow \frac{dy}{dx}=\frac{2}{\cos y-3}$$

3. $ax+by^2=\cos y$

SOLUTION We are given that, $ax+by^2=\cos y$ ...(i) Differentiating (i) on both sides w.r.t. x, we get

$$a+b\left[2y\frac{dy}{dx}\right]=-\sin y\frac{dy}{dx} \Rightarrow a+2by\frac{dy}{dx}+\sin y\frac{dy}{dx}=0$$

$$\Rightarrow \frac{dy}{dx}[2by+\sin y]=-a \Rightarrow \frac{dy}{dx}=\frac{-a}{2by+\sin y}$$

4. $xy+y^2=\tan x+y$

SOLUTION

We are given that, $xy+y^2=\tan x+y$ ...(i) Differentiating (i) on both sides w.r.t. x, we get

$$x\frac{dy}{dx}+y+2y\frac{dy}{dx}=\sec^2 x+\frac{dy}{dx}$$

$$\Rightarrow x\frac{dy}{dx}+2y\frac{dy}{dx}-\frac{dy}{dx}=\sec^2 x-y$$

$$\Rightarrow \frac{dy}{dx}[x+2y-1]=\sec^2 x-y \Rightarrow \frac{dy}{dx}=\frac{\sec^2 x-y}{x+2y-1}$$

5. $x^2+xy+y^2=100$

SOLUTION

We are given that, $x^2+xy+y^2=100$

...(i) Differentiating (i) on both sides w.r.t. x, we get

$$2x+x\frac{dy}{dx}+y+2y\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}[x+2y]=-(2x+y)$$

$$\Rightarrow \frac{dy}{dx}=\frac{-(2x+y)}{(x+2y)}$$

6. $x^3+x^2y+xy^2+y^2=81$

SOLUTION

We are given that, $x^3+x^2y+xy^2+y^2=81 \ldots$ (i) Differentiating (i) on both sides w.r.t. x, we get

$$3x^2+x^2\frac{dy}{dx}+y(2x)+y^2+x\left(2y\frac{dy}{dx}\right)+3y^2\frac{dy}{dx}=0$$

$$\Rightarrow \frac{dy}{dx}[x^2+2xy+3y^2]=-(3x^2+2xy+y^2)$$

$$\Rightarrow \frac{dy}{dx}=\frac{-(3x^2+2xy+y^2)}{x^2+2xy+3y^2}$$

7. $\sin^2 y+\cos xy=k$

SOLUTION

We are given that, $\sin^2 y+\cos xy=k$ Differentiating (i) on both sides w.r.t. x, we get

$$2\sin y\frac{d}{dx}(\sin y)+(-\sin xy)\frac{d}{dx}(xy)=0\ 2$$

$$\Rightarrow 2\sin y\cos y\frac{dy}{dx}+(-\sin xy)\left[x\frac{dy}{dx}+y\right]=0$$

$$\Rightarrow 2\sin y\cos y\frac{dy}{dx}-x\sin xy\frac{dy}{dx}-y\sin xy=0$$

$$\Rightarrow \frac{dy}{dx}[2\sin y\cos y-x\sin xy]=y\sin xy$$

$$\Rightarrow \frac{dy}{dx}=\frac{y\sin xy}{\sin 2y-x\sin xy}$$

8. . $\sin^2 x+\cos^2 y=1$

SOLUTION We are given that, $\sin^2 x+\cos^2 y=1$ Differentiating (i) on both sides w.r.t. x, we get

$$2\sin x\frac{d}{dx}(\sin x)+2\cos y\frac{d}{dx}(\cos y)=0$$

$$\Rightarrow 2\sin x\cos x+2\cos y(-\sin y)\frac{dy}{dx}=0$$

$$\Rightarrow \sin 2x-\sin 2y\frac{dy}{dx}=0\Rightarrow \frac{dy}{dx}=\frac{\sin 2x}{\sin 2y}$$

9. $y=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

SOLUTION

$$y=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

Putting $x=\tan\theta$, we get

$$y=\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)=\sin^{-1}(\sin 2\theta)=2\theta=2\tan^{-1}x$$

$$\therefore \frac{dy}{dx}=\frac{2}{1+x^2}$$

10. $y=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right),-\frac{1}{\sqrt{3}}<x<\frac{1}{\sqrt{3}}$

SOLUTION

$y=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$ Putting $x=\tan\theta$, we get

$$y=\tan^{-1}\left(\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}\right)$$

$\Rightarrow y = \tan^{-1}(\tan 3\theta) \Rightarrow y = 3\theta \Rightarrow y = 3\tan^{-1}x$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{3}{1+x^2}$

11. $y = \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right), 0 < x < 1.$

SOLUTION

$y = \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$, where $0 < x < 1$

Putting $x = \tan\theta$, we get

$y = \cos^{-1}\left(\dfrac{1-\tan^2\theta}{1+\tan^2\theta}\right) \Rightarrow y = \cos^{-1}(\cos 2\theta) \Rightarrow y = 2\theta$

$\Rightarrow y = 2\tan^{-1}x \Rightarrow \dfrac{dy}{dx} = \dfrac{2}{1+x^2}$

12. $y = \sin^{-1}\left(\dfrac{1-x^2}{1+x^2}\right), 0 < x < 1.$

SOLUTION Putting $x = \tan\theta$,we get

$y = \sin^{-1}\left(\dfrac{1-\tan^2\theta}{1+\tan^2\theta}\right) \Rightarrow y = \sin^{-1}(\cos 2\theta)$

$\Rightarrow y = \sin^{-1}\left\{\sin\left(\dfrac{\pi}{2} - 2\theta\right)\right\} \Rightarrow y = \dfrac{\pi}{2} - 2\theta$

$\Rightarrow y = \dfrac{\pi}{2} - 2\tan^{-1}x \Rightarrow \dfrac{dy}{dx} = 0 - \dfrac{2}{1+x^2} \Rightarrow \dfrac{dy}{dx} = -\dfrac{2}{1+x^2}$

13. $y = \cos^{-1}\left(\dfrac{2x}{1+x^2}\right), -1 < x < 1.$

SOLUTION

Putting $x = \tan\theta$,we get $y = \cos^{-1}\left(\dfrac{2\tan\theta}{1+\tan^2\theta}\right) \Rightarrow y = \cos^{-1}(\sin 2\theta)$

$\Rightarrow y = \cos^{-1}\left\{\cos\left(\dfrac{\pi}{2} - 2\theta\right)\right\} \Rightarrow y = \dfrac{\pi}{2} - 2\theta$

$\Rightarrow y = \dfrac{\pi}{2} - 2\tan^{-1}x \Rightarrow \dfrac{dy}{dx} = -\dfrac{2}{1+x^2}$

14. $y = \sin^{-1}\left(2x\sqrt{1-x^2}\right), -\dfrac{1}{\sqrt{2}} < x < \dfrac{1}{\sqrt{2}}.$

SOLUTION

Putting $x = \sin\theta$, we get

$y = \sin^{-1}[2\sin\theta\sqrt{1-\sin^2\theta}] \Rightarrow y = \sin^{-1}(\sin 2\theta) \Rightarrow y = 2\theta$

$\Rightarrow y = 2\sin^{-1}x \Rightarrow \dfrac{dy}{dx} = \dfrac{2}{\sqrt{1-x^2}}$

15. . $y = \sec^{-1}\left(\dfrac{1}{2x^2-1}\right), 0 < x < \dfrac{1}{\sqrt{2}}.$

SOLUTION

Putting $x = \cos\theta$, we get

$$y = \sec^{-1}\left(\frac{1}{2\cos^2\theta - 1}\right)$$

$$\Rightarrow y = \cos^{-1}(2\cos^2\theta - 1) \Rightarrow y = \cos^{-1}(\cos 2\theta)$$

$$\Rightarrow y = 2\theta \Rightarrow y = 2\cos^{-1}x \Rightarrow \frac{dy}{dx} = -\frac{2}{\sqrt{1-x^2}}$$