

## NCERT - Exercise 5.3

Find  $\frac{dy}{dx}$  in the following :

1.  $2x + 3y = \sin x$

**SOLUTION**

We are given that,  $2x + 3y = \sin x$  ... (i) Differentiating (i) on both sides w.r.t.  $x$ , we get  $2 + 3\frac{dy}{dx} = \cos x \Rightarrow \frac{dy}{dx} = \frac{\cos x - 2}{3}$

2.  $2x + 3y = \sin y$

**SOLUTION**

We are given that,  $2x + 3y = \sin y$  ... (i) Differentiating (i) on both sides w.r.t.  $x$ , we get

$$2 + 3\frac{dy}{dx} = \cos y \frac{dy}{dx}$$

$$\Rightarrow \cos y \frac{dy}{dx} - 3\frac{dy}{dx} = 2 \Rightarrow \frac{dy}{dx} = \frac{2}{\cos y - 3}$$

3.  $ax + by^2 = \cos y$

**SOLUTION**

We are given that,  $ax + by^2 = \cos y$  ... (i) Differentiating (i) on both sides w.r.t.  $x$ , we get

$$a + b \left[ 2y \frac{dy}{dx} \right] = -\sin y \frac{dy}{dx} \Rightarrow a + 2by \frac{dy}{dx} + \sin y \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} [2by + \sin y] = -a \Rightarrow \frac{dy}{dx} = \frac{-a}{2by + \sin y}$$

4.  $xy + y^2 = \tan x + y$

**SOLUTION**

We are given that,  $xy + y^2 = \tan x + y$  ... (i) Differentiating (i) on both sides w.r.t.  $x$ , we get

$$x \frac{dy}{dx} + y + 2y \frac{dy}{dx} = \sec^2 x + \frac{dy}{dx}$$

$$\Rightarrow x \frac{dy}{dx} + 2y \frac{dy}{dx} - \frac{dy}{dx} = \sec^2 x - y$$

$$\Rightarrow \frac{dy}{dx} [x + 2y - 1] = \sec^2 x - y \Rightarrow \frac{dy}{dx} = \frac{\sec^2 x - y}{x + 2y - 1}$$

5.  $x^2 + xy + y^2 = 100$

**SOLUTION**

We are given that,  $x^2 + xy + y^2 = 100$

... (i) Differentiating (i) on both sides w.r.t.  $x$ , we get

$$2x + x \frac{dy}{dx} + y + 2y \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} [x + 2y] = -(2x + y)$$

$$\Rightarrow \frac{dy}{dx} = \frac{-(2x + y)}{(x + 2y)}$$

6.  $x^3 + x^2y + xy^2 + y^2 = 81$

**SOLUTION**

We are given that,  $x^3 + x^2y + xy^2 + y^2 = 81$  ... (i) Differentiating (i) on both sides w.r.t.  $x$ , we get

$$3x^2 + x^2 \frac{dy}{dx} + y(2x) + y^2 + x \left( 2y \frac{dy}{dx} \right) + 3y^2 \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} [x^2 + 2xy + 3y^2] = -(3x^2 + 2xy + y^2)$$

$$\Rightarrow \frac{dy}{dx} = \frac{-(3x^2 + 2xy + y^2)}{x^2 + 2xy + 3y^2}$$

7.  $\sin^2 y + \cos xy = k$

**SOLUTION**

We are given that,  $\sin^2 y + \cos xy = k$  Differentiating (i) on both sides w.r.t.  $x$ , we get

$$2 \sin y \frac{d}{dx}(\sin y) + (-\sin xy) \frac{d}{dx}(xy) = 0$$

$$\Rightarrow 2 \sin y \cos y \frac{dy}{dx} + (-\sin xy) \left[ x \frac{dy}{dx} + y \right] = 0$$

$$\Rightarrow 2 \sin y \cos y \frac{dy}{dx} - x \sin xy \frac{dy}{dx} - y \sin xy = 0$$

$$\Rightarrow \frac{dy}{dx} [2 \sin y \cos y - x \sin xy] = y \sin xy$$

$$\Rightarrow \frac{dy}{dx} = \frac{y \sin xy}{\sin 2y - x \sin xy}$$

8.  $\sin^2 x + \cos^2 y = 1$

**SOLUTION** We are given that,  $\sin^2 x + \cos^2 y = 1$  Differentiating (i) on both sides w.r.t.  $x$ , we get

$$2 \sin x \frac{d}{dx}(\sin x) + 2 \cos y \frac{d}{dx}(\cos y) = 0$$

$$\Rightarrow 2 \sin x \cos x + 2 \cos y (-\sin y) \frac{dy}{dx} = 0$$

$$\Rightarrow \sin 2x - \sin 2y \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{\sin 2x}{\sin 2y}$$

9.  $y = \sin^{-1} \left( \frac{2x}{1+x^2} \right)$

**SOLUTION**

$$y = \sin^{-1} \left( \frac{2x}{1+x^2} \right)$$

Putting  $x = \tan \theta$ , we get

$$y = \sin^{-1} \left( \frac{2 \tan \theta}{1 + \tan^2 \theta} \right) = \sin^{-1}(\sin 2\theta) = 2\theta = 2 \tan^{-1} x$$

$$\therefore \frac{dy}{dx} = \frac{2}{1+x^2}$$

10.  $y = \tan^{-1} \left( \frac{3x-x^3}{1-3x^2} \right), -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

**SOLUTION**

$$y = \tan^{-1} \left( \frac{3x-x^3}{1-3x^2} \right) \text{ Putting } x = \tan \theta, \text{ we get}$$

$$y = \tan^{-1} \left( \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta} \right)$$

$$\Rightarrow y = \tan^{-1}(\tan 3\theta) \Rightarrow y = 3\theta \Rightarrow y = 3\tan^{-1}x$$

$$\Rightarrow \frac{dy}{dx} = \frac{3}{1+x^2}$$

11.  $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1.$

**SOLUTION**

$$y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), \text{ where } 0 < x < 1$$

Putting  $x = \tan \theta$ , we get

$$y = \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) \Rightarrow y = \cos^{-1}(\cos 2\theta) \Rightarrow y = 2\theta$$

$$\Rightarrow y = 2\tan^{-1}x \Rightarrow \frac{dy}{dx} = \frac{2}{1+x^2}$$

12.  $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1.$

**SOLUTION** Putting  $x = \tan \theta$ , we get

$$y = \sin^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) \Rightarrow y = \sin^{-1}(\cos 2\theta)$$

$$\Rightarrow y = \sin^{-1}\left\{\sin\left(\frac{\pi}{2}-2\theta\right)\right\} \Rightarrow y = \frac{\pi}{2}-2\theta$$

$$\Rightarrow y = \frac{\pi}{2}-2\tan^{-1}x \Rightarrow \frac{dy}{dx} = 0 - \frac{2}{1+x^2} \Rightarrow \frac{dy}{dx} = -\frac{2}{1+x^2}$$

13.  $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right), -1 < x < 1.$

**SOLUTION**

$$\text{Putting } x = \tan \theta, \text{ we get } y = \cos^{-1}\left(\frac{2\tan \theta}{1+\tan^2\theta}\right) \Rightarrow y = \cos^{-1}(\sin 2\theta)$$

$$\Rightarrow y = \cos^{-1}\left\{\cos\left(\frac{\pi}{2}-2\theta\right)\right\} \Rightarrow y = \frac{\pi}{2}-2\theta$$

$$\Rightarrow y = \frac{\pi}{2}-2\tan^{-1}x \Rightarrow \frac{dy}{dx} = -\frac{2}{1+x^2}$$

14.  $y = \sin^{-1}(2x\sqrt{1-x^2}), -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}.$

**SOLUTION**

Putting  $x = \sin \theta$ , we get

$$y = \sin^{-1}[2\sin \theta \sqrt{1-\sin^2\theta}] \Rightarrow y = \sin^{-1}(\sin 2\theta) \Rightarrow y = 2\theta$$

$$\Rightarrow y = 2\sin^{-1}x \Rightarrow \frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}$$

15.  $y = \sec^{-1}\left(\frac{1}{2x^2-1}\right), 0 < x < \frac{1}{\sqrt{2}}.$

**SOLUTION**

Putting  $x = \cos \theta$ , we get

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$$y = \sec^{-1} \left( \frac{1}{2\cos^2\theta - 1} \right)$$

$$\Rightarrow y = \cos^{-1}(2\cos^2\theta - 1) \Rightarrow y = \cos^{-1}(\cos 2\theta)$$

$$\Rightarrow y = 2\theta \Rightarrow y = 2\cos^{-1}x \Rightarrow \frac{dy}{dx} = -\frac{2}{\sqrt{1-x^2}}$$



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