NCERT - Exercise 5.2

Differentiate the functions with respect tox in Exercises 1 to 8.

1. $sin(x^2+5)$

   SOLUTION

   Let $y=\sin(x^2+5)$

   $\Rightarrow \dfrac{dy}{dx}=\dfrac{d}{dx}\sin(x^2+5)=\cos(x^2+5)\dfrac{d}{dx}(x^2+5)$

   $=\cos(x^2+5)(2x+0)=2x\cos(x^2+5)$

2. cos(sin x)

   SOLUTION Let $y=\cos(\sin x)$

   $\Rightarrow \dfrac{dy}{dx}=\dfrac{d}{dx}\cos(\sin x)=-\sin(\sin x)\dfrac{d}{dx}\sin x$

   $=-\sin(\sin x)\cos x$

3. sin(ax + b)

   SOLUTION Let $y=\sin(ax+b)$

   $\Rightarrow \dfrac{dy}{dx}=\dfrac{d}{dx}\sin(ax+b)=\cos(ax+b)\dfrac{d}{dx}(ax+b)=\cos(ax+b)(a+0)=a\cos(ax+b)$

4. $\sec(\tan(\sqrt{x}))$

   SOLUTION

   Let$y=\sec\{\tan(\sqrt{x})\}$

   $\Rightarrow \dfrac{dy}{dx}=\dfrac{d}{dx}\sec(\tan\sqrt{x})=\sec(\tan\sqrt{x})\tan(\tan\sqrt{x})\dfrac{d}{dx}\tan\sqrt{x}$

   $=\sec(\tan\sqrt{x})\cdot\tan(\tan\sqrt{x})\cdot\sec^2\sqrt{x}\dfrac{d}{dx}(\sqrt{x})$

   $=\sec(\tan\sqrt{x})\cdot\tan(\tan\sqrt{x})\cdot\sec^2\sqrt{x}\cdot\dfrac{1}{2}x^{\frac{1}{2}-1}$

   $=\sec(\tan\sqrt{x})\cdot\tan(\tan\sqrt{x})\cdot\sec^2\sqrt{x}\cdot\dfrac{1}{2\sqrt{x}}$

5. $\dfrac{\sin(ax+b)}{\cos(cx+d)}$

   SOLUTION

   Let $y=\dfrac{\sin(ax+b)}{\cos(cx+d)}$

   $\Rightarrow \dfrac{dy}{dx}=\dfrac{d}{dx}\left(\dfrac{\sin(ax+d)}{\cos(cx+d)}\right)$

   $=\dfrac{\cos(cx+d)\dfrac{d}{dx}\sin(ax+d)-\sin(ax+b)\dfrac{d}{dx}\cos(cx+d)}{\cos^2(cx+d)}$

   $=\dfrac{a\cos(cx+d)\cos(ax+b)+c\sin(ax+b)\sin(cx+d)}{\cos^2(cx+d)}$

   $=a\cos(ax+b)\sec(cx+d)+c\sin(ax+b)\tan(cx+d)\cdot\sec(cx+d)$

6. $\cos x^3 \cdot \sin^2(x^5)$

SOLUTION

Let $y = \cos x^3 \cdot \sin^2(x^5)$

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}[\cos x^3 \cdot \sin^2(x^5)]$

$= \cos x^3 \frac{d}{dx}\sin^2(x^5) + \sin^2(x^5)\frac{d}{dx}\cos x^3$

$= \cos x^3 \cdot 2\sin(x^5)\frac{d}{dx}\sin(x^5) + \sin^2(x^5)(-\sin x^3)\frac{d}{dx}(x^3)$

$= \cos x^3 \cdot 2\sin(x^5)\cos(x^5)\frac{d}{dx}(x^5) + \sin^2(x^5)(-\sin x^3)(3x^2)$

$= 10x^4 \cos x^3 \sin(x^5)\cos(x^5) - 3x^2 \sin^2(x^5)\sin x^3$.

7. $2\sqrt{\cot(x^2)}$

SOLUTION

Let $y = 2\sqrt{\cot(x^2)}$

$\Rightarrow \frac{dy}{dx} = 2\frac{d}{dx}\sqrt{\cot(x^2)} = 2 \cdot \frac{1}{2}\{\cot(x^2)\}^{\frac{-1}{2}} \cdot \frac{d}{dx}\cot(x^2)$

$= \frac{1}{\sqrt{\cot(x^2)}}\{-\cos ec^2(x^2)\}\frac{d}{dx}(x^2)$

$= \frac{1}{\sqrt{\cot(x^2)}}\{-\cos ec^2(x^2)\}(2x) = \frac{-2x\cos ec^2(x^2)}{\sqrt{\cot(x^2)}} = \frac{-2\sqrt{2}x}{\sin x^2\sqrt{\sin 2x^2}}$

8. $\cos(\sqrt{x})$

SOLUTION

Let $y = \cos\left(\sqrt{x}\right)$

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\cos\left(\sqrt{x}\right) = -\sin\sqrt{x} \cdot \frac{d}{dx}(\sqrt{x})$

$= -\sin\sqrt{x} \cdot \frac{1}{2}(x)^{\frac{-1}{2}} = \frac{-\sin\sqrt{x}}{2\sqrt{x}}$

9. Prove that the function f given by $f(x) = |x-1|, x \in R$ is not differentiable at x = 1.

SOLUTION

We have, f(x) =|x−1| $f(1) = |1-1| = 0$

$Rf'(1) = \lim_{h\to 0}\frac{f(1+h)-f(1)}{h} = \lim_{h\to 0}\frac{|1+h-1|-0}{h}$

$= \lim_{h\to 0}\frac{|h|}{h} = \lim_{h\to 0}\frac{h}{h} = 1$

and $Lf'(1) = \lim_{h\to 0}\frac{f(1-h)-f(1)}{-h} = \lim_{h\to 0}\frac{|1-h-1|-0}{-h}$

$= \lim_{h\to 0}\frac{|-h|}{h} = \lim_{h\to 0}\frac{h}{-h} = -1$

Thus,$Rf'(1) \neq Lf'(1)$

This shows that f(x) is not differentiable at $x = 1$.

10. Prove that the greatest integer function defined by $f(x) = [x], 0 < x < 3$ is not differentiable at x = 1 and x = 2.

SOLUTION

At x = 1: $Rf'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0} \frac{[1+h]-[1]}{h} = 0$ [$[1+h=1\, and\, [1]=1]$

and $Lf'(1) = \lim_{h\to 0} \frac{f(1-h)-f(1)}{-h} = \lim_{h\to 0} \frac{[1-h]-[1]}{-h} = \infty$

Thus $Rf'(1) \neq Lf'(1)$

Hence f(x)= $[x]$ is not differentiable at x = 1 At x = 2 :

$Rf'(2) = \lim_{h\to 0} \frac{f(2+h)-f(2)}{h}$

$= \lim_{h\to 0} \frac{[2+h]-[2]}{h} = 0$ $Lf'(2) = \lim_{h\to 0} \frac{f(2-h)-f(2)}{-h} = \lim_{h\to 0} \frac{[2-h]-[2]}{-h} = \frac{1-2}{0} = \infty$

$\therefore Rf'(2) \neq Lf'(2)$ Hence, $f(x) = [x]$ is not differentiable at x = 2.

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