

NCERT - Exercise 5.2

Differentiate the functions with respect to x in Exercises 1 to 8.

1. $\sin(x^2 + 5)$

SOLUTION

$$\begin{aligned} \text{Let } y &= \sin(x^2 + 5) \\ \Rightarrow \frac{dy}{dx} &= \frac{d}{dx} \sin(x^2 + 5) = \cos(x^2 + 5) \frac{d}{dx}(x^2 + 5) \\ &= \cos(x^2 + 5)(2x + 0) = 2x \cos(x^2 + 5) \end{aligned}$$

2. $\cos(\sin x)$

SOLUTION Let $y = \cos(\sin x)$

$$\begin{aligned} \Rightarrow \frac{dy}{dx} &= \frac{d}{dx} \cos(\sin x) = -\sin(\sin x) \frac{d}{dx} \sin x \\ &= -\sin(\sin x) \cos x \end{aligned}$$

3. $\sin(ax + b)$

SOLUTION Let $y = \sin(ax + b)$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx} \sin(ax + b) = \cos(ax + b) \frac{d}{dx}(ax + b) = \cos(ax + b)(a + 0) = a \cos(ax + b)$$

4. $\sec(\tan(\sqrt{x}))$

SOLUTION

$$\begin{aligned} \text{Let } y &= \sec\{\tan(\sqrt{x})\} \\ \Rightarrow \frac{dy}{dx} &= \frac{d}{dx} \sec(\tan \sqrt{x}) = \sec(\tan \sqrt{x}) \tan(\tan \sqrt{x}) \frac{d}{dx} \tan \sqrt{x} \\ &= \sec(\tan \sqrt{x}) \cdot \tan(\tan \sqrt{x}) \cdot \sec^2 \sqrt{x} \frac{d}{dx}(\sqrt{x}) \\ &= \sec(\tan \sqrt{x}) \cdot \tan(\tan \sqrt{x}) \cdot \sec^2 \sqrt{x} \cdot \frac{1}{2} x^{-\frac{1}{2}} \\ &= \sec(\tan \sqrt{x}) \cdot \tan(\tan \sqrt{x}) \cdot \sec^2 \sqrt{x} \cdot \frac{1}{2\sqrt{x}} \end{aligned}$$

5. $\frac{\sin(ax + b)}{\cos(cx + d)}$

SOLUTION

$$\begin{aligned} \text{Let } y &= \frac{\sin(ax + b)}{\cos(cx + d)} \\ \Rightarrow \frac{dy}{dx} &= \frac{d}{dx} \left(\frac{\sin(ax + b)}{\cos(cx + d)} \right) \\ &= \frac{\cos(cx + d) \frac{d}{dx} \sin(ax + b) - \sin(ax + b) \frac{d}{dx} \cos(cx + d)}{\cos^2(cx + d)} \\ &= \frac{a \cos(cx + d) \cos(ax + b) + c \sin(ax + b) \sin(cx + d)}{\cos^2(cx + d)} \\ &= a \cos(ax + b) \sec(cx + d) + c \sin(ax + b) \tan(cx + d) \cdot \sec(cx + d) \end{aligned}$$

6. $\cos x^3 \cdot \sin^2(x^5)$

SOLUTION

Let $y = \cos x^3 \cdot \sin^2(x^5)$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}[\cos x^3 \cdot \sin^2(x^5)]$$

$$= \cos x^3 \frac{d}{dx} \sin^2(x^5) + \sin^2(x^5) \frac{d}{dx} \cos x^3$$

$$= \cos x^3 \cdot 2 \sin(x^5) \frac{d}{dx} \sin(x^5) + \sin^2(x^5) (-\sin x^3) \frac{d}{dx} (x^3)$$

$$= \cos x^3 \cdot 2 \sin(x^5) \cos(x^5) \frac{d}{dx} (x^5) + \sin^2(x^5) (-\sin x^3) (3x^2)$$

$$= 10x^4 \cos x^3 \sin(x^5) \cos(x^5) - 3x^2 \sin^2(x^5) \sin x^3.$$

7. $2\sqrt{\cot(x^2)}$

SOLUTION

Let $y = 2\sqrt{\cot(x^2)}$

$$\Rightarrow \frac{dy}{dx} = 2 \frac{d}{dx} \sqrt{\cot(x^2)} = 2 \cdot \frac{1}{2} \{\cot(x^2)\}^{-\frac{1}{2}} \cdot \frac{d}{dx} \cot(x^2)$$

$$= \frac{1}{\sqrt{\cot(x^2)}} \{-\operatorname{cosec}^2(x^2)\} \frac{d}{dx} (x^2)$$

$$= \frac{1}{\sqrt{\cot(x^2)}} \{-\operatorname{cosec}^2(x^2)\} (2x) = \frac{-2x \operatorname{cosec}^2(x^2)}{\sqrt{\cot(x^2)}} = \frac{-2\sqrt{2}x}{\sin x^2 \sqrt{\sin 2x^2}}$$

8. $\cos(\sqrt{x})$

SOLUTION

Let $y = \cos(\sqrt{x})$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx} \cos(\sqrt{x}) = -\sin \sqrt{x} \cdot \frac{d}{dx} (\sqrt{x})$$

$$= -\sin \sqrt{x} \cdot \frac{1}{2} (x)^{-\frac{1}{2}} = \frac{-\sin \sqrt{x}}{2\sqrt{x}}$$

9. Prove that the function f given by $f(x) = |x-1|, x \in R$ is not differentiable at $x = 1$.

SOLUTION

We have, $f(x) = |x-1|, f(1) = |1-1| = 0$

$$Rf'(1) = \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \rightarrow 0} \frac{|1+h-1| - 0}{h}$$

$$= \lim_{h \rightarrow 0} \frac{|h|}{h} = \lim_{h \rightarrow 0} \frac{h}{h} = 1$$

$$\text{and } Lf'(1) = \lim_{h \rightarrow 0} \frac{f(1-h) - f(1)}{-h} = \lim_{h \rightarrow 0} \frac{|1-h-1| - 0}{-h}$$

$$= \lim_{h \rightarrow 0} \frac{|-h|}{h} = \lim_{h \rightarrow 0} \frac{h}{-h} = -1$$

Thus, $Rf'(1) \neq Lf'(1)$

This shows that $f(x)$ is not differentiable at $x = 1$.

10. Prove that the greatest integer function defined by $f(x) = [x]$, $0 < x < 3$ is not differentiable at $x = 1$ and $x = 2$.

SOLUTION

$$\text{At } x = 1: Rf'(1) = \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \rightarrow 0} \frac{[1+h] - [1]}{h} = 0 \quad [1+h = 1 \text{ and } [1] = 1]$$

$$\text{and } Lf'(1) = \lim_{h \rightarrow 0} \frac{f(1-h) - f(1)}{-h} = \lim_{h \rightarrow 0} \frac{[1-h] - [1]}{-h} = \infty$$

Thus $Rf'(1) \neq Lf'(1)$

Hence $f(x) = [x]$ is not differentiable at $x = 1$ At $x = 2$:

$$Rf'(2) = \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{[2+h] - [2]}{h} = 0 \quad Lf'(2) = \lim_{h \rightarrow 0} \frac{f(2-h) - f(2)}{-h} = \lim_{h \rightarrow 0} \frac{[2-h] - [2]}{-h} = \frac{1-2}{0} = \infty$$

$\therefore Rf'(2) \neq Lf'(2)$ Hence, $f(x) = [x]$ is not differentiable at $x = 2$.



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