NCERT - Exercise 6.4

1. Using differentials, find the approximate value of each of the following up to 3 places of decimal. (i) $\sqrt{25.3}$

(ii) $\sqrt{49.5}$

(iii) $\sqrt{0.6}$

(iv) $(0.009)^{1/3}$

(v) $(0.999)^{1/10}$

(vi) $(15)^{114}$

(vii) $(26)^{1/3}$

(viii) $(255)^{1/4}$

(ix) $(82)^{1/4}$

(x) $(401)^{1/2}$

(xi) $(0.0037)^{1/2}$

(xii) $(26.57)^{1/3}$

(xiii) $(81.5)^{1/4}$

(xiv) $(3.968)^{3/2}$

(xv) $(32.15)^{1/5}$

SOLUTION

(i) Let $y=\sqrt{x}, x=25, \Delta x=0.3$

$\Rightarrow \frac{dy}{dx}=\frac{1}{2\sqrt{x}}$, and $\Delta y=\left(\frac{dy}{dx}\right)\Delta x=\left(\frac{1}{2\sqrt{x}}\right)\Delta x=\frac{1}{2\times 5}\times 0.3=0.03$

Also $\Delta y=\sqrt{x+\Delta x}-\sqrt{x} \Rightarrow 0.03=\sqrt{253}-\sqrt{25} \Rightarrow \sqrt{253}=0.03+5=5.030$

(ii) Let $y=\sqrt{x}, \Rightarrow \frac{dy}{dx}=\frac{1}{2\sqrt{x}}$ and $\Delta y=\left(\frac{dy}{dx}\right)\Delta x=\left(\frac{1}{2\sqrt{49}}\right)0.5=\frac{1}{14}\times 0.5$

Also, $\Delta y=\sqrt{x+\Delta x}-\sqrt{x}$

$\Rightarrow \frac{0.5}{14}=\sqrt{495}-7$, or $\sqrt{495}=\frac{5}{140}+7=7+0.036=7.036$

(iii) Let $y=\sqrt{x}, \Rightarrow \frac{dy}{dx}=\frac{1}{2\sqrt{x}}$ and $\Delta y=\left(\frac{dy}{dx}\right)\Delta x$

$\Rightarrow \Delta y=\frac{1}{2\sqrt{064}}\times \Delta r=\frac{1}{2(0.8)}\times(-0.04)=\frac{-0.\cdot 04}{16}=-0.025$

Also, $\Delta y=\sqrt{x+\Delta x}-\sqrt{x}$ $-0.025=\sqrt{0.6}-\sqrt{0.64}$ or $\sqrt{0.6}=0.8-0.025=0.775$

(iv) Let $y=x^{1/3}, x=0.008, \Delta x=0.001$

$\Rightarrow \frac{dy}{dx}=\frac{1}{3}x^{-2/3}$ and $\Delta y=\left(\frac{dy}{dx}\right)\Delta x$

$=\frac{1}{3}(0.008)^{-2/3}(0.001)=\frac{1}{3(0.2)^{3\times(2/3)}}\times 0.001=\frac{1}{3(0.04)}\times 0.001=\frac{0.001}{0.12}=\frac{1}{120}=0.008$

Also, $\Delta y=\sqrt[3]{x+\Delta x}-\sqrt[3]{x} \Rightarrow 0.008=\sqrt[3]{0009}-\sqrt[3]{0008} \Rightarrow \sqrt[3]{0009}=0.008+0.2=0.208$

(v) Let $y=x^{1/10}, x=1, \Delta x=-0.001 \Rightarrow \frac{dy}{dx}=\frac{1}{10}x^{-9/10}$ and $\Delta y=\left(\frac{dy}{dx}\right)\Delta x$

$\Rightarrow \Delta y = \dfrac{1}{10x^{9/10}} \times (-0.001) = \dfrac{-0.001}{10(1)^{9/10}} = \dfrac{-0.001}{10} = -0.0001$

Also, $\Delta y = (x+\Delta x)^{1/10} - (x)^{1/10} \Rightarrow -0.0001 = (0.999)^{1/10} - (1)^{1/10}$

$\Rightarrow (0.999)^{1/10} = 1 - 0.0001 = 0.999$

(vi) Let $y = x^{1/4}, x = 16, \Delta x = -1 \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{4}x^{-3/4}$ and $\Delta y = \left(\dfrac{dy}{dx}\right)\Delta x$

$\Rightarrow \Delta y = \left(\dfrac{1}{4}x^{-3/4}\right)(-1) = \left(\dfrac{-1}{4(16)^{3/4}}\right) = \dfrac{-1}{4 \times 2^3} = \dfrac{-1}{32} = -0.03125$

Also $\Delta y = (x+\Delta x)^{1/4} - (x)^{1/4} \Rightarrow -0.03125 = (15)^{1/4} - (16)^{1/4} \Rightarrow (15)^{1/4} = -0.03125 + 2 = 1.96875 \cong 1.969$

(vii) Let $y = x^{1/3}, x = 27, \Delta x = -1 \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{3}x^{-2/3}$ and $\Delta y = \left(\dfrac{dy}{dx}\right)\Delta x$

$\Rightarrow \Delta y = \left(\dfrac{1}{3}x^{-2/3}\right)(-1) = \left(\dfrac{-1}{3(27)^{2/3}}\right) = \dfrac{-1}{3 \times 9} = \dfrac{-1}{27} = -0.037$

AIso, $\Delta y = (x+\Delta x)^{1/3} - (x)^{1/3} \Rightarrow -0.0370 = (26)^{1/3} - (27)^{1/3} \Rightarrow (26)^{1/3} = 3 - 0.037 = 2.963$

(viii) Let $y = x^{1/4}, x = 256, \Delta x = -1$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{4}x^{-3/4}$ and $\Delta y = \left(\dfrac{dy}{dx}\right)\Delta x$

$\Rightarrow \Delta y = \left(\dfrac{1}{4}x^{-3/4}\right)(-1) = \left(\dfrac{-1}{4(256)^{3/4}}\right) = \dfrac{-1}{4 \times 64} = \dfrac{-1}{256}$ Also, $\Delta y = (x+\Delta x)^{1/4} - (x)^{1/4}$

$\Rightarrow \dfrac{-1}{256} = (255)^{1/4} - (256)^{1/4} \Rightarrow (255)^{1/4} = 4 - \dfrac{1}{256} = \dfrac{1023}{256} = 3.996$

(ix) Let $y = x^{1/4}, \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{4x^{3/4}}$ and $\Delta y = \left(\dfrac{dy}{dx}\right)\Delta x$

$\Rightarrow \Delta y = \left(\dfrac{1}{4(81)^{3/4}}\right)(1) = \dfrac{1}{4(27)} = \dfrac{1}{108}$

Also, $\Delta y = (x+\Delta x)^{1/4} - (x)^{1/4}$

$\Rightarrow \dfrac{1}{108} = (82)^{1/4} - (81)^{1/4} \Rightarrow (82)^{1/4} = 3 + \dfrac{1}{108} = 3.009$

(x) Let $y = x^{1/2}, x = 400, \Delta x = 1 \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{2x^{1/2}}$ and $\Delta y = \left(\dfrac{dy}{dx}\right)\Delta x$

$\Rightarrow \Delta y = \left(\dfrac{1}{2(400)^{1/2}}\right)(1) = \dfrac{1}{2(20)} = \dfrac{1}{40}$

Also, $\Delta y = (x+\Delta x)^{1/2} - (x)^{1/2} \Rightarrow \dfrac{1}{40} = (401)^{1/2} - (400)^{1/2}$

$\Rightarrow (401)^{1/2} = 20 + \dfrac{1}{40} = 20.025.$ (xi) Let $y = x^{1/2}, x = 0.0036, \Delta x = 0.0001 \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{2\sqrt{x}}$ and $\Delta y = \left(\dfrac{dy}{dx}\right)\Delta x$

$\Rightarrow \Delta y = \dfrac{1}{2(0.0036)^{1/2}} \times (0.0001) = \dfrac{0.0001}{2(0.06)} = \dfrac{1}{1200}$

Also,$\Delta y = (x+\Delta x)^{1/2} - (x)^{1/2} \Rightarrow \dfrac{1}{1200} = (0.0037)^{1/2} - (0.0036)^{1/2} \Rightarrow (0.0037)^{1/2} = (0.0036)^{1/2} + \dfrac{1}{1200} = 0.06 + \dfrac{1}{1200} =$ $0.06083 \cong 0.061$

(xii) Let $y = x^{1/3}, x = 27, \Delta x = -0.43$

$\Rightarrow \frac{dy}{dx} = \frac{1}{3x^{2/3}}$ and $\Delta y = \left(\frac{dy}{dx}\right)\Delta x \Rightarrow \Delta y = \left(\frac{1}{3(27)^{2/3}}\right)(-0.43) = \frac{-0.43}{3(9)} = \frac{-0.43}{27} = -0.015926$

Also, $\Delta y = (x+\Delta x)^{1/3} - (x)^{1/3} \Rightarrow -0.015926 = (26.57)^{1/3} - (27)^{1/3} \Rightarrow (26.57)^{1/3} = -0.015926 + 3 = 2.984$

(xiii) Let $y = x^{1/4}, x = 81, \Delta x = 0.5 \Rightarrow \frac{dy}{dx} = \frac{1}{4x^{3/4}}$ and $\Delta y = \left(\frac{dy}{dx}\right)\Delta x$

$\Rightarrow \Delta y = \left(\frac{1}{4(81)^{3/4}}\right)(0.5) = \frac{1}{4\times 3^3} \times 0.5 = \frac{0.5}{108} = \frac{5}{1080}$

Also, $\Delta y = (x+\Delta x)^{1/4} - (x)^{1/4} \Rightarrow \frac{5}{1080} = (81.5)^{1/4} - (81)^{1/4} \Rightarrow (81.5)^{1/4} = 3 + \frac{5}{1080} = 3.0046 \cong 3.005$

(xiv) Let $y = x^{3/2}, x = 4, \Delta x = -0.032 \Rightarrow \frac{dy}{dx} = \frac{3x^{1/2}}{2}$ and $\Delta y = \left(\frac{dy}{dx}\right)\Delta x$

$\Rightarrow \Delta y = \frac{3}{2}(2) \times (-0.032) = -0.096$ Also, $\Delta y = (x+\Delta x)^{3/2} - (x)^{3/2} \Rightarrow -0.096 = (3.968)^{3/2} - (4)^{3/2} \Rightarrow (3.968)^{3/2} = 8 - 0.096 = 7.904$

(xv) Let $y = x^{1/5}, x = 32, \Delta x = 0.15 \Rightarrow \frac{dy}{dx} = \frac{1}{5x^{4/5}}$ and $\Delta y = \left(\frac{dy}{dx}\right)\Delta x$

$\Rightarrow \Delta y = \frac{1}{5(32)^{4/5}} \times (0.15) = \frac{1}{5(16)} \times 0.15 = \frac{15}{8000}$

Also, $\Delta y = (x+\Delta x)^{1/5} - (x)^{1/5} \Rightarrow \frac{15}{8000} = (32.15)^{1/5} - (32)^{1/5} \Rightarrow \&(32.15)^{1/5} = \frac{15}{8000} + 2 = 2.00187$

2. Find the approximate value of $f(2.01)$, where $f(x) = 4x^2 + 5x + 2$.

SOLUTION

We have, $f(x) = 4x^2 + 5x + 2 \Rightarrow f'(x) = 8x + 5$

Also, $f(x+\Delta x) \approx f(x) + \Delta x f'(x)$

Taking $x = 2$ and $\Delta x = 0.01$, we get $f(2.01) \approx f(2) + (0.01) f'(2) = \left(4\times 2^2 + 5\times 2 + 2\right) + \frac{1}{100}(8\times 2 + 5) = 28.21$

$\Rightarrow f(2.01) \approx 28.21$

3. Find the approximate value of $f(5.001)$, where $f(x) = x^3 - 7x^2 + 15$.

SOLUTION

Given, $f(x) = x^3 - 7x^2 + 15\& \Rightarrow f'(x) = 3x^2 - 14x$

Also, $f(x+\Delta x) \approx f(x) + \Delta x f'(x)$

Taking $x = 5$ and $\Delta x = 0.001$, we get $f(5.001) \approx 5^3 - 7\cdot 5^2 + 15 + (0.001)\left(3\cdot 5^2 - 14\cdot 5\right)$

$= 125 - 175 + 15 + \frac{1}{100}(5) = -35 + 0.005 = -34.995$

4. Find the approximate change in the volume V of a cube of side x metres caused by increasing the side by 1%.

SOLUTION

We have $V = x^3 \Rightarrow \frac{dV}{dx} = 3x^2$ and $\Delta V = \left(\frac{dV}{dx}\right)\Delta x = 3x^2\left(\frac{x}{100}\right)$

$= \frac{3x^3}{100} \therefore$ Change in volume $= 0.03x^3 m^3$

5. Find the approximate change in the surface area of a cube of side x metres caused by decreasing the side by 1%.

SOLUTION

Surface area S of given cube , $S = 6x^2 \Rightarrow \frac{dS}{dx} = 12x$

Hence, $\Delta S \approx 12x\Delta x = 12x\left(-\frac{x}{100}\right)$

$= -\frac{12x^2}{100}\, m^2 \therefore$ Change in surface area$= 0.12x^2 m^2$

6. If the radius of a sphere is measured as 7m with an error of 0.02m, then find the approximate error in calculating its volume.

SOLUTION

The volume V of a sphere of radius r is $V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dr} = \left(\frac{4}{3}\pi\right)\left(3r^2\right) = 4\pi r^2$

Let Δr be error in measuring radius $\Rightarrow r = 7m$ and $\Delta r = 0.02m$.

Hence, $\Delta V \approx \left(4\pi r^2\right)\Delta r = \left(4\pi\left(7^2\right)\right)(\pm 0.02)\, m^3 = \pm 3.92\pi\, m^3$

$\therefore$ Error in calculating the volume $= \pm 3.92\pi\, m^3$

7. If the radius of a sphere is measured as 9m with an error of 0.03m, then find the approximate error in calculating its surface area.

SOLUTION

The surface area S of a sphere of radius r is given by $S = 4\pi r^2 \Rightarrow \frac{dS}{dr} = 8\pi r$

Let Δr be the error in measuring radius $\Rightarrow r = 9, \Delta r = 0.03$

Hence, $\Delta S \approx (8\pi r)\Delta r = \{8\pi(9\text{m})\}(\pm 0.03\text{m}) = \pm(2.16\pi)\,\text{m}^2$ Error in calculating the surface area$= \pm 2.16\pi\text{m}^2$.

8. If $f(x) = 3x^2 + 15x + 5$, then the approximate value of $f(3.02)$ is

(A) 47.66

(B) 57.66

(C) 67.66

(D) 77.66

SOLUTION

(D) Given, $f(x) = 3x^2 + 15x + 5 \Rightarrow f\prime(x) = 6x + 15$

Also, $f(x + \Delta x) \approx f(x) + \Delta x f\prime(x)$,

Taking $x = 3$ and $\Delta x = 0.02$, we get $f(3.02) \approx 3 \times 3^2 + 15 \times 3 + 5 + 0.02(6 \times 3 + 15) = 77 + 0.66$

$\Rightarrow f(3.02) \approx 77.66$

9. The approximate change in the volume of a cube of side x metres caused by increasing the side by 3% is

(A) $0.06x^3\text{m}^3$

(B) $0.6x^3\text{m}^3$

(C) $0.09x^3\text{m}^3$

(D) $0.9x^3\text{m}^3$

SOLUTION

(C) We know that the volume V of a cube with edge x is given by $V = x^3 \Rightarrow \frac{dV}{dx} = 3x^2$

Hence, $\Delta V \approx 3x^2\Delta x = 3x^2\left(\frac{3}{100}x\right) = \frac{9x^3}{100}$

$\therefore$ Approximate change in volume $= \frac{9x^3}{100}\text{m}^3 = 0.09x^3\text{m}^3$