## NCERT - Exercise 6.2

1. Show that the function given by $f(x) = 3x + 17$ is strictly increasing on $R$.

SOLUTION

We have, $f(x) = 3x + 17$ (i) $f(x)$ being a polynomial function, is continuous and derivable on $R$.
Differentiating (i), w.r.t. $x$, we get $f(x) = 3 > 0 \forall x \in R \Rightarrow f$ is strictly increasing on $R$.

2. Show that the function given by $f(x) = e^{2x}$ is strictly increasing on $R$.

SOLUTION

We have, $f(x) = e^{2x}$ (i) $f(x)$ being an exponential function, is continuous and derivable on $R$.
Differentiating (i) w.r.t. $x$, we get $f(x) = e^{2x} \cdot 2 = 2P > 0$ for all $x \in R \Rightarrow f$ is strictly increasing on $R$.

3. Show that the function given by $f(x) = \sin x$ is

(a) strictly increasing in $\left(0, \frac{\pi}{2}\right)$

(b) strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$

(c) neither increasing nor decreasing in $(0, \pi)$

SOLUTION

We have, $f(x) = \sin x$ (i) which is continuous and derivable on $R$
Differentiating (i) w.r.t. $x$, we get $f'(x) = \cos x$

(a) For all $x \in \left(0, \frac{\pi}{2}\right)$, $\cos x > 0 \Rightarrow f'(x) > 0$ Therefore, $f(x) = \sin x$ is strictly increasing on $\left(0, \frac{\pi}{2}\right)$.

(b) For all $x \in \left(\frac{\pi}{2}, \pi\right)$, $\cos x < 0 \Rightarrow f\prime(x) < 0$ therefore $f(x) = \sin x$ is strictly decreasing on $\left(\frac{\pi}{2}, \pi\right)$.

(c) From parts (a) and (b), we conclude that $f(x) = \sin x$ is neither increasing nor decreasing on $(0, \pi)$ .

4. Find the intervals in which the function $f$ given by $f(x) = 2x^2 - 3x$ is (a) strictly increasing (b) strictly decreasing

SOLUTION

We have, $f(x) = d - 3x$ (i) $f(x)$ is a polynomial function. Hence, $f(x)$ is continuous and derivable on $R$.
Differentiating (i) w.r.t. $x$, we get $f(x) = 4x - 3$.

(a) For strictly increasing, $f'(x) > 0 \Rightarrow 4x - 3 > 0 \Rightarrow x > \frac{3}{4} \therefore f$ is strictly increasing on $\left(\frac{3}{4}, \infty\right)$

(b) For strictly decreasing $f'(x) < 0 \Rightarrow 4x - 3 < 0 \Rightarrow x < \frac{3}{4} \therefore f$ is strictly decreasing on $\left(-\infty, \frac{3}{4}\right)$

5. Find the intervals in which the function $f$ given by $f(x) = 2x^3 - 3x^2 - 36x + 7$ is

(a) strictly increasing

(b) strictly decreasing

SOLUTION

We have, $f(x) = 2x^3 - 3x^2 - 36x + 7$ (i) $f(x)$ is a polynomial function. Hence, $f(x)$ is continuous and derivable on $R$. Differentiating (i) w.r.t. $x$, we get $f'(x) = 6x^2 - 6x - 36 = 6(x^2 - x - 6) = 6(x-3)(x+2)$

(a) For increasing, $f'(x\rangle > 0 \Rightarrow 6\,(x-3)(x+2) > 0$

$\Rightarrow (x-3)(x-(-2)) > 0 \Rightarrow x < -2$ or $x > 3$. $\Rightarrow x \in (-\infty, -2) \cup (3, \infty) \therefore f$ is strictly increasing on $(-\infty, -2) \cup (3, \infty)$

(b) For decreasing, $f'(x) < 0 \Rightarrow 6\,(x-3)(x+2) < 0$

$\Rightarrow (x-3)(x-(-2)) < 0 \Rightarrow x < 3, x > -2$

$\Rightarrow -2 < x < 3 \Rightarrow x \in (-2,3)$

$\therefore f$ is strictly decreasing on $(-2,3)$

6. Find the intervals in which the following functions are strictly increasing or decreasing:

(a) $x^2+2x-5$

(b) $10-6x-2x^2$

(c) $-2x^3-9x^2-12x+1$

(d) $6-9x-x^2$

(e) $(x+1)^3(x-3)^3$

SOLUTION

(a) We have, $f(x) = x^2+2x-5$ (i) $f(x)$ being a polynomial, is continuous and derivable on s$R$.

Differentiating (i), w.r.t. $x$, we get, $f'(x) = 2x+2$ For increasing, $f'(x) > 0 \Rightarrow 2x+2 > 0 \Rightarrow x > -1$

For decreasing, $f'(x) < 0 \Rightarrow 2x+2 < 0 \Rightarrow x < -1 \therefore f(x)$ is strictly increasing for $x > -1$.

$f(x)$ is strictly decreasing for $x < -1$.

(b) We have, $f(x) = 10-6x-2x^2$ (i)

$f(x)$ being a polynomial, is continuous and derivable on $R$

Differentiating (i) w.r.t. $x$, we get $f(x) = 0-6-2(2x) = -6-4x$

For increasing, $f(x) > 0 \Rightarrow -6-4x > 0 \Rightarrow -4x > 6 \Rightarrow x < -\frac{3}{2} \Rightarrow x \in \left(-\infty, -\frac{3}{2}\right)$

$\therefore f(x)$ is strictly increasing if $x < -\frac{3}{2}$.

For decreasing, $f(x) < 0 \Rightarrow -6-4x < 0 \Rightarrow -4x < 6 \Rightarrow x > -\frac{3}{2} \Rightarrow x \in \left(-\frac{3}{2}, \infty\right)$

$\Rightarrow f(x)$ is strictly decreasing if $x > -\frac{3}{2}$.

(c) We have, $f(x) = -2x^3-9x^2-12x+1$ (i) $f(x)$ being a polynomial, is continuous and derivable on $R$

Differentiating (i) w.r.t. $x$, we get $f(x) = -2(3x^2)-9(2x)-12 = -6x^2-18x-12 = -6(x^2+3x+2) = -6(x+1)(x+2)$

For critical points, putting $f'(x) = 0 \Rightarrow -6\,(x+1)\,(x+2) = 0 \Rightarrow x = -1, x = -2$

The points $x = -1, -2$ divide the real line into following disjoint intervals $(-\infty,\ -2), (-2,\ -1)$ and $(-1,\ \infty)$ Interval Sign of $f\prime(x) = -6\,(x+1)\,(x+2)$

Nature of function $(-\infty,-2)$ $(-ve)\,(-ve)\,(-ve) = -ve$

$f$ is strictly decreasing $(-2,-1)$ $(-ve)\,(-ve)\,(+ve) = +ve$

$f$ is strictly increasing $(-1,\infty)$ $(-ve)\,(+ve)\,(+ve) = -ve$

$f$ is strictly decreasing

Hence, $f$ is strictly increasing on $(-2,-1)$ and strictly decreasing on $(-\infty,-2) \cup (-1,\infty)$

(d) We have, $f\,(x) = 6-9x-x^2$ (i) $f\,(x)$ being a polynomial, is continuous and derivable on $R$ Differentiating (i) w.r.t. $x$, we get $f\prime(x) = -9-2x$ For increasing, $f\prime(x) > 0 \Rightarrow -9-2x > 0 \Rightarrow -2x > 9 \Rightarrow x < -\frac{9}{2} \Rightarrow x \in \left(-\infty, -\frac{9}{2}\right)$ $\therefore f$ is strictly decreasing on $\left(-\frac{9}{2}, \infty\right)$.

(e) We have, $f\,(x) = (x+1)^3(x-3)^3$ $f\,(x)$ being a polynomial, is continuous and derivable on $R$

Differentiating (i) w.r.t. $x$, we get $f\prime(x) = (x+1)^3 \cdot 3(x-3)^2 \cdot 1 + (x-3)^3 \cdot 3(x+1)^2 \cdot 1$

$= (x-3)^2(x+1)^2\{3(x+1)+3(x-3)\}$

$(x-3)^2(x+1)^2\{6x-6\} = 6(x-3)^2(x+1)^2(x-1)$

For critical points, putting $f\prime(x) = 0$ $6(x-3)^2(x+1)^2(x-1) = 0 \Rightarrow x = 3, x = -1, x = 1$

The above points divide the real line into following disjoint intervals $(-\infty,-1), (-1,1), (1,3)$ and $(3,\infty)$ Interval Sign of $f\prime(x) = 6(x-3)^2(x+1)^2(x-1)$

Nature of function $f$ $(-\infty,-1)$ $(+ve)(+ve)(+ve)(-ve) = -ve$ $f$ is strictly decreasing

$(-1,1)$ $(+ve)(+ve)(+ve)(-ve) = -ve$

$f$ is strictly decreasing $(1,3)$ $(+ve)(+ve)(+ve)(+ve) = +ve$

$f$ is strictly increasing $(3,\infty)$ $(+ve)(+ve)(+ve)(+ve) = +ve$

$f$ is strictly increasing

Thus, $f$ is strictly increasing on $(1,3)\cup(3,\infty)$ and strictly decreasing on $(-\infty,-1)\cup(-1,1)$

7. Show that $y = \log(1+x) - \frac{2x}{2+x}, x > -1$, is an increasing function of $x$ throughout its domain.

SOLUTION

We have, $y = \log(1+x) - \frac{2x}{2+x}, x > -1$ (i)

The domain of $y$ is $(-1,\infty)$

Differentiating (i), w.r.t. $x$, we get $\frac{dy}{dx} = \frac{1}{1+x} - 2\frac{d}{dx}\left(\frac{x}{2+x}\right) = \frac{1}{1+x} - 2\left\{\frac{(2+x)-x(0+1)}{(2+x)^2}\right\}$

$\frac{1}{1+x} - \frac{4}{(2+x)^2} = \frac{(2+x)^2-4(1+x)}{(1+x)(2+x)^2}$

$= \frac{x^2}{(1+x)(2+x)^2} \forall x > -1$

Now $x^2 \geq 0$, $(2+x)^2 \geq 0$ (Being perfect squares) and $(1+x) > 0 \,\forall\, x > -1 \Rightarrow \frac{dy}{dx} \geq 0$ for all $x > -1$

Hence, $y$ is an increasing function of $x$ throughout its domain.

8. Find the values of $x$ for which $y = [x(x-2)]^2$ is an increasing function.

SOLUTION

We have, $y = (x(x-2))^2, x \in R \Rightarrow y = \left(x^2-2x\right)^2$ (i)

Differentiating (i) w.r.t. $x$, we get $\frac{dy}{dx} = 2\left(x^2-2x\right)\frac{d}{dx}\left(x^2-2x\right) = 2\left(x^2-2x\right)(2x-2)$

$= 4x(x-2)(x-1) = 4x(x-1)(x-2)$

For critical points, putting $\frac{dy}{dx} = 0$, we get $\Rightarrow x(x-1)(x-2) = 0 \Rightarrow x = 0,1,2$

The above points divide the real line into following disjoint intervals $(-\infty,0), (0,1), (1,2)$ and $(2,\infty)$

Interval Sign of $\frac{dy}{dx} = 4x(x-1)(x-2)$

Nature of function $f$ $(-\infty,0)$ $(-ve)(-ve)(-ve) = -ve$

Decreasing $(0,1)$ $(+ve)(-ve)(-ve) = +ve$

Increasing $(1,2)$ $(+ve)(+ve)(-ve) = -ve$

Decreasing $(2,\infty)$ $(+ve)(+ve)(+ve) = +ve$

Increasing

$\therefore$ $y$ is an increasing function in $(0,1)\cup(2,\infty)$

9. Prove that $y = \dfrac{4\sin\theta}{(2+\cos\theta)} - \theta$ is an increasing function of $\theta$ in $\left[0, \dfrac{\pi}{2}\right]$.

SOLUTION

We have, $y = \dfrac{4\sin\theta}{2+\cos\theta} - \theta, \theta \in \left[0, \dfrac{\pi}{2}\right] \Rightarrow \dfrac{dy}{d\theta} = 4\left\{\dfrac{(2+\cos\theta)\cos\theta - \sin\theta(-\sin\theta)}{(2+\cos\theta)^2}\right\} - 1$

$= \dfrac{4(2\cos\theta+1)}{(2+\cos\theta)^2} - 1 = \dfrac{8\cos\theta+4-(2+\cos\theta)^2}{(2+\cos\theta)^2} = \dfrac{4\cos\theta - \cos^2\theta}{(2+\cos\theta)^2} = \dfrac{\cos\theta(4-\cos\theta)}{(2+\cos\theta)^2}$

Now $\cos\theta > 0$ in $\left[0, \dfrac{\pi}{2}\right]$ ; $4-\cos\theta > 0$ in $\left[0, \dfrac{\pi}{2}\right]$

and $(2+\cos\theta)^2 > 0$ in $\left[0, \dfrac{\pi}{2}\right]$ [Being a perfect square] $\therefore \dfrac{dy}{d\theta} > 0$ for all $\theta \in \left[0, \dfrac{\pi}{2}\right]$

Hence, $y$ is strictly increasing function in $\left[0, \dfrac{\pi}{2}\right]$

10. Prove that the logarithmic function is strictly increasing on $(0, \infty)$

SOLUTION

We have, $f(x) = \log x$ (i) (Note that, $\log x$ is defined only for $x > 0$)

Domain of $f(x)$ is $(0, \infty)$ Now, $f'(x) = \dfrac{1}{x} > 0$ for all $x \in (0, \infty)$

$\Rightarrow f'(x) > 0$ for all $x \in (0, \infty)$ $\therefore f$ is strictly increasing on $(0, \infty)$

11. Prove that the function $f$ given by $f(x) = x^2 - x + 1$ is neither strictly increasing nor strictly decreasing on $(-1, 1)$ .

SOLUTION

We have, $f(x) = x^2 - x + 1 \forall x \in (-1, 1)$ (i)

Differentiating (i) w.r.t. $x$, we get $f(x) = 2x - 1$

For increasing, $f'(x) > 0 \Rightarrow 2x - 1 > 0 \Rightarrow x > \dfrac{1}{2}$

For decreasing, $f'(x) < 0 \Rightarrow 2x - 1 < 0 \Rightarrow x < \dfrac{1}{2} \Rightarrow f'(x) < 0$ for all $x \in \left(-1, \dfrac{1}{2}\right)$ and $f'(x) > 0$ for all $x \in \left(\dfrac{1}{2}, 1\right)$

Hence, $f$ is neither increasing nor decreasing on $(-1, 1)$

12. Which of the following functions are strictly decreasing on $\left(0, \dfrac{\pi}{2}\right)$? (A) $\cos x$ (B) $\cos 2x$ (C) $\cos 3x$ (D) $\tan x$

SOLUTION

Let us see each option one-by-one.

(A) Let $f(x) = \cos x$, then $f'(x) = -\sin x < 0$ for all $x \in \left(0, \dfrac{\pi}{2}\right)$

$\Rightarrow f$ is strictly decreasing on $\left(0, \dfrac{\pi}{2}\right)$

(B) Let $f(x) = \cos 2x$ , then $f'(x) = -2\sin 2x < 0$ for all $x \in \left(0, \dfrac{\pi}{2}\right)$ ( in $(0, \pi) \Rightarrow \sin 2x > 0$ in $(0, \pi/2)) \Rightarrow f$ is strictly decreasing on $\left(0, \dfrac{\pi}{2}\right)$

(C) Let $f(x)=\cos 3x$ , then $f'(x)=-3\sin 3x$, which assumes $+ve$ as well as $-ve$ values in $\left(0, \frac{\pi}{2}\right)$. $\left[\text{if} 0<x<\frac{\pi}{2} \Rightarrow 0<3x<\frac{3\pi}{2} \Rightarrow \sin 3x >\right.$ in $\left(0, \frac{\pi}{3}\right)$ and $\sin 3x<0$ in

$\left.\left(\frac{\pi}{3}, \frac{\pi}{2}\right)\right]$ $\therefore f$ is neither increasing nor decreasing on $\left(0, \frac{\pi}{2}\right)$

(D) Let $f(x)=\tan x$, then $f'(x)=\sec^2 x>0$ for all $x \in\left(0, \frac{\pi}{2}\right) \Rightarrow f$ is strictly increasing on $\left(0, \frac{\pi}{2}\right)$

13. On which of the following intervals is the function $f$ given by $f(x)=x^{100}+\sin x-1$ strictly decreasing?
(A) $(0,1)$
(B) $\left(\frac{\pi}{2}, \pi\right)$
(C) $\left(0, \frac{\pi}{2}\right)$
(D) None of these

SOLUTION

(D) We have, $f(x)=x^{100}+\sin x-1$ (i) Differentiating (i) w.r.t. $x$, we get $f'(x)=100x^{99}+\cos x$

(A $f'(x)$ assumes only $+ve$ values in $(0,1)$ $\therefore f$ is strictly increasing in $(0,1)$ (B) $f'(x)>0$ for all $x \in\left(\frac{\pi}{2}, \pi\right)$, therefore $f$ is strictly increasing in $x \in\left(\frac{\pi}{2}, \pi\right)$

(C) $f'(x)>0$ for all $x \in\left(0, \frac{\pi}{2}\right)$, therefore $f$ is strictly increasing in $\left(0, \frac{\pi}{2}\right)$.

14. Find the least value of $a$ such that the function $f$ given by $f(x)=x^2+ax+1$ is strictly increasing on $(1,\ 2)$.

SOLUTION

We have, $f(x)=x^2+nx+1$ (i) $\Rightarrow f'(x)=2x+a$ If $1<x<2 \Rightarrow 2<2x<4 \Rightarrow 2+a<2x+a<4+a \Rightarrow 2+a<f'(x)<4+a$
Now, $f(x)$ is strictly increasing on $(1,\ 2)$ only if $f'(x)>0$
$\Rightarrow 2+a \geq 0 \Rightarrow a \geq -2$ $\therefore$ Required least value of $a$ is $-2$.

15. Let $I$ be any interval disjoint from $(-1,1)$. Prove that the function $f$ given by $f(x)=x+\frac{1}{x}$ is strictly increasing on $I$.

SOLUTION

We have, $f(x)=x+\frac{1}{x}, x \in I$ (i)

Differentiating (i) w.r.t. $x$, we get $f'(x)=1-\frac{1}{x^2}=\frac{x^2-1}{x^2}$

As $x^2>0$ and in $I, x^2-1>0 \Rightarrow x^2>1 \Rightarrow x<-1$ or $x>1 \Rightarrow x \in(-\infty,\ -1)$ or $x \in(1,\ \infty) \Rightarrow x \in(-\infty,\ -1) \cup(1,\ \infty) \Rightarrow x \in R-(-1,1) f(x)$ is strictly increasing on $I$ [$I$ is an interval which is a subset of $R-(-1,1)$]

16. Prove that the function $f$ given by $f(x)=\log \sin x$ is strictly increasing on $\left(0, \frac{\pi}{2}\right)$ and strictly decreasing on $\left(\frac{\pi}{2}, \pi\right)$.

SOLUTION

We have, $f(x)=\log(\sin x)$ (i)

Differentiating (i) w.r.t. $x$, we get $f'(x)=\frac{1}{\sin x}(\cos x)=\cot x$

As $\cot x > 0$ for all $x \in \left(0, \dfrac{\pi}{2}\right)$ and $\cot x < 0$ for all $x \in \left(\dfrac{\pi}{2}, \pi\right)$

therefore, $f(x)$ is strictly increasing on $\left(0, \dfrac{\pi}{2}\right)$ and strictly decreasing on $\left(\dfrac{\pi}{2}, \pi\right)$.

17. Prove that the function $f$ given by $f(x) = \log\cos x$ is strictly decreasing on $\left(0, \dfrac{\pi}{2}\right)$ and strictly increasing on $\left(\dfrac{\pi}{2}, \pi\right)$.

SOLUTION

We have, $f(x) = \log(\cos x)$ (i) Differentiating (i) wr.t. $x$, we get $f'(x) = \dfrac{1}{\cos x}(-\sin x) = -\tan x$

As $\tan x > 0, -\tan x < 0$ for all $x \in \left(0, \dfrac{\pi}{2}\right)$ and $\tan x < 0 \Rightarrow -\tan x > 0$ for all $x \in \left(\dfrac{\pi}{2}, \pi\right)$

Therefore, $f(x) < 0$ for all $x \in \left(0, \dfrac{\pi}{2}\right)$ and $f(x) > 0$ for all $x \in \left(\dfrac{\pi}{2}, \pi\right)$.

Hence, $f(x)$ is strictly decreasing on $\left(0, \dfrac{\pi}{2}\right)$ and strictly increasing on $\left(\dfrac{\pi}{2}, \pi\right)$

18. Prove that the function given by $f(x) = x^3 - 3x^2 + 3x - 100$ is increasing in $R$.

SOLUTION

We have, $f(x) = x^3 - 3x^2 + 3x - 100$ (i)
Differentiating (i) w.r.t. $x$, we get $f'(x) = 3x^2 - 6x + 3 = 3(x^2 - 2x + 1) = 3(x-1)^2 \geq 0$ for all $x \in R$
$[3 > 0, (x-1)^2 \geq 0$ (being perfect square)]
$\Rightarrow f(x) \geq 0 \Rightarrow f(x)$ is increasing on $R$

19. The interval in which $y = x^2 e^{-x}$ is increasing, is
(A) $(-\infty, \infty)$
(B) $(-2, 0)$
(C) $(2, \infty)$
(D) $(0, 2)$

SOLUTION

(D) We have, $y = x^2 e^{-x}$ (i)

Differentiating (i) w.r.t. $x$, we get $\dfrac{dy}{dx} = x^2 e^{-x}(-1) + e^{-x}(2x) = xe^{-x}(-x+2) = x(2-x)e^{-x}$

For critical points, putting $\dfrac{dy}{dx} = 0 \Rightarrow x(2-x)e^{-x} = 0 \Rightarrow x = 0, x = 2$
The above points divide the real line into following disjoint intervals $(-\infty,\ 0), (0,2), (2,\ \infty)$

Interval Sign of $\dfrac{dy}{dx} = x\,(2-x)\,e^{-x}$
Nature of function $y$ $(-\infty, 0)$ $(-ve)\,(+ve)\,(+ve) = -ve$
Decreasing $(0,2)$ $(+ve)\,(+ve)\,(+ve) = +ve$
Increasing $(2, \infty)$ $(+ve)\,(-ve)\,(+ve) = -ve$
Decreasing
Thus, $y$ is increasing function on $(0,2)$