NCERT - Exercise - 1.1

Find the principal values of the following :

1. 1. Determine whether each of the following relations are reflexive, symmetric and transitive:

(i) Relation R in the set. A $= \{1,2,3,\ldots\ldots\ldots,13,14\}$ defined as $R = \{(x,\ y) : 3x - y = 0\}$

SOLUTION

(i) A $= \{1,2,3,4,5,6,\cdots.,13,14\}$ is the given set R $= \{(x,y) : 3x - y = 0\} \Rightarrow$ R $= \{(1,3),(2,6)(3,9),(4,12)\}$

Reflexive

Let $x \in A$ be any element.

Since, $(x,\ x) \notin R \quad \therefore\ R\ \ is\ \ not\ \ reflexive$

Symmetric

$x,\ y \in A,\ \ (x,\ y) \in R\ \ but\ \ (y,\ x) \notin R$

$\therefore R$ is not symmetric.

Transitive

$x,\ y,\ z \in A\ \ \Rightarrow (x,\ y) \in R\ \ and\ \ (y,\ z) \in R \Rightarrow (x,\ z) \in R$

For example : (1, 3)∈R and (3, 9)∈R but (1, 9) $\notin R$,therefore, R is not transitive.

Hence, R is neither reflexive, nor symmetric and nor transitive.

(ii) Relation R in the set N of natural numbers defined as $R = \{(x,\ y) : y = x + 5\ \ and\ \ x < 4\}$

SOLUTION

(ii) N is the set of natural numbers

R $= \{(x,y) : y = x + 5$ and $x < 4\}$ is the set of natural numbers.

$R = \{(1,\ 6)(2,\ 7),\ (3,\ 8)\}$

Reflexive

Let $x \in N$ be any element.

$(x,\ x) \notin R \quad \therefore\ \ R$ is not reflexive.

Symmetric

$x,\ y \in N,\ \ (x,\ y) \in R\ \ but\ \ (y,\ x) \notin R$

$\therefore$ R is not symmetric.

Transitive

(1, 6)∈R and (6, 7) $\notin R$ and (1, 7) $\notin R$

$\therefore$ R is not transitive.

Hence, R is neither reflexive, nor symmetric and nor transitive.

(iii) Relation R in the set A$= \{1,2,3,4,5,6\}$ defined as $R = \{(x,\ y) : y$ is divisible by x$\}$

SOLUTION

(iii) A $=$ 1, 2, 3, 4, 5, 6 is the given set

R $= \{(x,y)$:y is divisible by x in A $\}$

R $= \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,2),(2,4),(2,6),(3,3),(3,6),(4,4),(5,5),(6,6)\}$

Reflexive

Let $x \in A$ be any element.

Now, $(x,\ x) \in R\ \ i.e.\ \ (1,\ 1) \in R,\ (2,\ 2) \in R,\ (3,\ 3) \in R,\ (4,\ 4) \in R,\ (5,\ 5) \in R,\ (6,\ 6) \in R$

R is reflexive.

Symmetric

$x,\ y \in A,\ \ (x,\ y) \in R\ \ \ (y,\ x) \in R$

i.e., $(1,\ 2) \in R\ \ but\ \ (2,\ 1) \notin R$

$\therefore$ R is not symmetric.

Transitive

$x,\ y,\ z \in A,\ \ (x,\ y) \in R,\ (y,\ z) \in R\ \ \Rightarrow (x,\ z) \in R$

i.e., $(1,\ 2) \in R\ \ and\ \ (2,\ 4) \in R\ \ \Rightarrow\ (1,\ 4) \in R$

Thus, R is transitive.

Hence, R is reflexive and transitive, but not symmetric.

(iv) Relation R in the set Zof all integers defined as R $= \{(x,y) : x-y$ is an integer$\}$

SOLUTION

(iv) Z is the set of all integers

R $= \{(x,y) : x-y$ is an integer$\}$

Reflexive

Let $x \in Z$, be any element, (x, x) i.e., (1, 1) = 1 - 1 = 0∈Z.

$\therefore$ R is reflexive.

Symmetric

$x,\ y \in Z,\ \ (x,\ y) \in R\ \ \ \ \Rightarrow (y,\ x) \in R$

i.e., $x-y$ is an integer $\Rightarrow (y,\ x) \in R$

i.e., $x-y$ is an integer $\Rightarrow y-x$ is also an integer.

$\therefore$ R is symmetric.

Transitive

$(x,\ y) \in R\ \ and\ \ (y,\ z) \in R$

i.e., $(x-y)$ is integer and $(y-z)$ is integer

$\Rightarrow (x-z) = (x-y+y-z) \in$ integer $\Rightarrow (x,\ z) \in R$

Hence, R is reflexive, symmetric and transitive.

(v) Relation R in the set A of human beings in town at a particular time given by

(a) $R = \{(x,\ y) : x$ and y work at the same place$\}$

(b) R $= \{(x,y)$: x and y live in the same locality$\}$

(c) R $= \{(x,y)$:x is exactly 7 cm taller than y$\}$

(d) R $= \{x,y) : x$ is wife of y $\}$

(e) R $= \{(x,y)$: x is father of y $\}$

SOLUTION

Relation R in the set A of human beings in a town at a particular time.

(a) $R = \{(x,\ y) : x\ \ and\ \ y$ work at the same place$\}$

Reflexive

$(x,\ x) \in R$ because x and x work at the same place. Thus, R is reflexive.

Symmetric

Let $(x,\ y) \in R \Rightarrow$ x and y work at the same place

$\Rightarrow$ y and x work at the same place $\Rightarrow (y,\ x) \in R$

Thus, R is symmetric.

Transitive

(x, y) R and (y, z) $\in$ R

$\Rightarrow$ x and y work at the same place and y and z work at the same place

$\Rightarrow$ x and z work at the same place $\Rightarrow$ (x, z) $\in$ R

Thus, R is transitive.

Hence, R is reflexive, symmetric and transitive

(b) R $= \{(x, y) : x$ and y live in the same locality$\}$

Reflexive

(x, x)$\in$R because x and x live in the same locality.

$\therefore$ R is reflexive.

Symmetric

Let (x, y)$\in$R $\Rightarrow$ x and y live in the same locality

$\Rightarrow$ y and x also live in the same locality $\Rightarrow$ (y, x)$\in$R

Thus, R is symmetric.

Transitive

Let (x, y)$\in$R and (y, z) $\in$ R

$\Rightarrow$ x and y live in the same locality and y and z live in the same locality

$\Rightarrow$ x and z live in the same locality (x, z)$\in$R

Thus, R is transitive.

Hence, R is reflexive, symmetric and transitive.

(c) R $= \{(x, y) : x$ is exactly 7 cm taller than y$\}$

Reflexive

x is not exactly 7 cm taller than x, so (x, x) $\in$ R, thus R is not reflexive.

Symmetric

If x is exactly 7 cm taller than y, then y is not exactly 7 cm taller than x.

So, if (x, y) $\in$ R then (y, x) $\notin$ R $\Rightarrow$ R is not symmetric.

Transitive

If x is exactly 7 cm taller thany and if y is exactly 7 cm taller than z, then it does not imply that x is exactly 7 cm taller than z.Thus, R is not transitive.

Hence, R is not reflexive, not symmetric and not transitive.

(d) R $= \{(x, y) : x$ is wife of y$\}$

Reflexive

x is not wife of x, therefore, (x, x) $\notin$ R and thus R is not reflexive.

Symmetric

If x is wife of y, then y is not wife of x.

If (x, y)$\in$R, then (y, x) $\notin$ R.

So, R is not symmetric.

Transitive

If x is the wife ofy, then y is not wife of z.

and R is transitive as transitivity is not contradicted in this case.

$(x,\ y) \in R$ and $(y,\ z) \notin R$, then $(x,\ z) \notin R$, for any z if x is wife of y, then y is a male and male cannot be wife]

Hence, R is not reflexive, not symmetric but transitive.

(e) R $= \{(x, y) : x$ is father of y$\}$

Reflexive

x is not father of x, so (x, x) $\notin$ R, so R is not reflexive.

Symmetric

If x is father of y, then y is not father of x.

If (x, y)$\in$R, then (y, x) $\notin$ R, so R is not symmetric.

Transitive

If (x, y) $\in$ R and (y, z)$\in$R, then (x, z)$\notin$ R.

i.e., x is father of y, y is father of z, then x is not father of z.

So, R is not transitive.

Hence, R is neither reflexive, nor symmetric nor transitive.

2. Show that the relation R in the set R of real numbers, defined as $A = \{(a,\ b) : a \le b^2\}$, is neither reflexive nor symmetric nor transitive.

SOLUTION

We have $R = \{(a,\ b);\ a \le b^2\}$, where $a,\ b \in R$

(i) Reflexivity

We observe that, $\frac{1}{3} \le \left(\frac{1}{3}\right)^2$ is not true.

$\therefore \left(\frac{1}{3},\ \frac{1}{3}\right) \notin R$. So, R is not reflexive.

(ii) Symmetry

We observe that, $1 \le (2)^2$ *but* $2\ 1^2$

i.e., $(1,\ 2) \in R$ but $(2,\ 1) \notin R$

So, R is not symmetric.

(iii) Transitivity

We observe that, $10 \le 4^2$ *and* $4 \le 3^2$ but $10\ (3)^2$

i.e., $(10,\ 4) \in R$ *and* $(4,\ 3) \in R$ *but* $(10,\ 3) \notin R$

So, R is not transitive.

3. Check whether the relation R defined in the set $\{1, 2, 3, 4, 5, 6\}$ as R $= \{(a, b) : b = a + 1\}$ is reflexive, symmetric or transitive.

SOLUTION

Given R $= \{(a, b) : b = a + 1\}, a, b \in \{1, 2, 3, 4, 5, 6\}$

$\Rightarrow R = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)\}$

(i) Reflexive

Consider, $a \in \{1,\ 2,\ 3,\ 4,\ 5,\ 6\} \Rightarrow a = a + 1$ which is false.

$\therefore (a,\ a) \notin R$. Thus, R is not reflexive.

(ii) Symmetric

Let $a,\ b \in \{1,\ 2,\ 3,\ 4,\ 5,\ 6\}$

Consider, (a, b)$\in$R b = a + 1

and (b, a)$\in$R $\Rightarrow$ a = b + 1 which is false.

$\therefore$ R is not symmetric.

(iii) Transitive

Let, $a,\ b,\ c \in \{1,\ 2,\ 3,\ 4,\ 5,\ 6\}$

Consider $a,\ b,\ c \in R \Rightarrow\ b = a+1,\ (b,\ c) \in R$

$\Rightarrow c = b+1$

$\Rightarrow c = a+2\ \ \Rightarrow\ (a,\ c) \notin$

$\therefore$ R is not transitive.

Hence, R is neither reflexive nor symmetric nor transitive.

4. Show that the relation R in R defined as $R = \{(a,\ b) : a \le b\}$, is reflexive and transitive but not symmetric.

SOLUTION

(i) Reflexive

Let $a \in R,\ a \le a$ which is true. $\therefore (a,\ a) \in R$

Thus, R is reflexive.

(ii) Symmetric

Let $a,\ b \in R\ \&\ (a,\ b) \in R$ Consider, $a \le b$ does not imply $b \le a \Rightarrow (a,\ b) \in R\ \ but\ \ (b,\ a) \notin R$

$\therefore$ R is not symmetric.

(iii) Transitive

Let $a,\ b,\ c \in R$ If $(a,\ b) \in R \Rightarrow a \le b\ \ and\ \ (b,\ c) \in R \Rightarrow\ b \le c \Rightarrow a \le c$

$\Rightarrow (a,\ c) \in R$

Thus, R is transitive.

Hence, R is reflexive and transitive but not symmetric.

5. Check whether the relation R in R defined by $R = \{(a,\ b) : a \le b^3\}$ is reflexive, symmetric or transitive.

SOLUTION

We have $R = \{(a,\ b);\ \ a \le b^3\}$ where $a,\ b \in R$.

(i) Reflexive : We observe that, $\frac{1}{2} \le \left(\frac{1}{2}\right)^3$ is not true. $\therefore \left(\frac{1}{2},\ \frac{1}{2}\right) \notin R$. So R is not reflexive.

(ii) Symmetric We observe that $1 \le (3)^2\ \ but\ \ 3 \not\le 1^3\ \ i.e.,\ \ (1,\ 3) \in R\ \ but\ \ (3,\ 1) \notin R$ So, R is not symmetric.

(iii) Transitive We observe that, $10 \le 3^3\ \ and\ \ 3 \le 2^3\ \ but\ \ 10 \not\le 2^3$ i.e., $(10,\ 3) \in R\ \ and\ \ (3,\ 2) \in R\ \ but\ \ (10,\ 2) \notin R$ So, R is not transitive.

R is neither reflexive nor symmetric nor transitive.

6. Show that the relation R in the set $\{1,2,3\}$ given by $\mathrm{R} = \{(1,2),(2,1)\}$ is symmetric but neither reflexive nor transitive.

SOLUTION

Given the set $\{1,2,3\}$ where $\mathrm{R} = \{(1,2),(2,1)\}$

(i) Reflexive

$1,\ 2,\ 3 \in \{1,\ 2,\ 3\},\ (1,\ 1) \notin R,\ (2,\ 2) \notin R,\ (3,\ 3) \notin R$

$\therefore$ R is not reflexive.

(ii) Symmetric

$1,\ 2 \in \{1,\ 2,\ 3\},\ (1,\ 2) \in R \Rightarrow (2,\ 1) \in R$

$\therefore$ R is symmetric.

(iii) Transitive

$1,\ 2,\ 3 \in \{1,\ 2,\ 3\}$, Consider, $(1,\ 2) \in R,\ (2,\ 3) \notin R,\ (1,\ 3) \notin R$

R is not transitive. Hence, R is symmetric but neither reflexive nor transitive.

7. Show that the relation R in the set A of all the books in a library of a college, given by $R = \{(x,\ y) : x$ and y have same number of pages $\}$ is an equivalence relation.

SOLUTION

R $= \{(x,y) : x and y$ have same number of pages$\}$

(i) Reflexive Books x and x have same number of pages.

$\therefore (x,\ x) \in R \therefore$ R is reflexive.

(ii) Symmetric

If $(x,\ y) \in R$, i.e. Books x and y have same number of pages. $\Rightarrow$ Books y and x have same number of pages.

$\Rightarrow (y,\ x) \in R \therefore$ R is symmetric.

(iii) Transitive

$If\ \ (x,\ y) \in R\ \ and\ \ (y,\ z) \in R \Rightarrow$ Books x and y have same number of pages and books y and z have same number of pages. $\Rightarrow$ Books x and z have same number of pages.

$\Rightarrow (x,\ z) \in R\ \ \ \therefore\ \ R$ is transitive. Hence, R is an equivalence relation.

8. Show that the relation R in the set A $= \{1,2,3,4,5\}$ given by $R = \{(a,\ b) : |a-b|\ is\ even\}$, is an equivalence relation. Show that all the elements of $\{1,3,5\}$ are related to each other and all the elements of $\{2,4\}$ are related to each other. But no element of $\{1,3,5\}$ is related to any element of $\{2,4\}$.

SOLUTION

We have A $= \{1,2,3,4,5\}$

$R = \{(a,\ b) : |a-b|\ \ is\ \ even\},\ a,\ b \in A$

(i) Reflexive For any a∈A, we have $|a-a| = 0$, which is even $\Rightarrow (a,\ a) \in R \forall a \in A$ So, R is reflexive..

(ii) Symmetry Let $a,\ b \in A$.

Let $(a,\ b) \in R,\ then\ \ |a-b|\ \ is\ \ even \Rightarrow\ \ |b-a|\ \ is\ even$

$\Rightarrow (b,\ a) \in R$

Thus, $(a,\ b) \in R \Rightarrow (b,\ a) \in R$

So, R is symmetric.

(iii) Transitive

Let $a,\ b,\ c \in A$. Let $(a,\ b) \in R\ \ and\ \ (b,\ c) \in R$

$\Rightarrow |a-b|$ is even and $|b-c|$ is even

$\Rightarrow$ (a and b both are even or both are odd) and (b and c both are even or both are odd)

Case I : When b is even Let $(a,\ b) \in R$ and $(b,\ c) \in R \Rightarrow |a-b|\ \ is\ even\ \ and\ \ |b-c|\ \ is\ even$

$\Rightarrow$ a is even and c is even [$\therefore$ b is even] $\Rightarrow |a-c|$ is even $\Rightarrow (a,\ c) \in R$ Case II : When b is odd

Let $(a,\ b) \in R$ and $(b,\ c) \in R \Rightarrow\ \ |a-b|$ is even and $|b-c|$ is even

$\Rightarrow$ a is odd and c is odd [b is odd]

$\Rightarrow |a-c|\ \ is\ even\ \ \Rightarrow (a,\ c) \in R$

Thus, (a, b)∈R and $(b,\ c) \in R \Rightarrow (a,\ c) \in R$

So, R is transitive.

Hence, R is an equivalence relation.

We know that thedifference of anytwo odd (even) natural numbers is always an even natural number.

$\therefore$ All the elements of set $\{1,3,5\}$ are related to each other and all the elements of $\{2,4\}$ are related to each other.

We know that the difference of an even natural number and an odd natural number is an odd number. $\therefore$ No element of $\{1,3,5\}$ is related to any element of $\{2,4\}$.

9. Show that each of the relation R in the set $A = \{x \in Z : 0 \leq x \leq 12\}$, given by

(i) $R = \{(a,\ b) : |a-b|$ is a multiple of 4$\}$

(ii) $R = \{(a,\ b) : a = b\}$

is an equivalence relation.

Find the set of all elements related to 1 in each case.

SOLUTION

(i) $A = \{x \in Z : 0 \le x \le 12\}$

$\therefore A = \{0,\ 1,\ 2,\ 3,\ ...,\ 12\}$ We have, $R = \{(a,\ b) : |a - b|\ \ is\ multiple\ of\ 4\}$

(a) Reflexive

For any $a \in A,\ |a - a| = 0$ is a multiple of 4 Thus, $(a,\ a) \in R\ \ \therefore$ R is reflexive.

(b) Symmetry

For any a, b∈A, let (a, b)∈R

$\Rightarrow |a - b|\ \ is\ \ multiple\ \ of\ \ 4\ \ \Rightarrow\ |b - a|\ \ is\ multiple\ of\ 4 \Rightarrow (b,\ a) \in R$ i.e., $(a,\ b) \in R\ \Rightarrow\ (b,\ a) \in R$

$\therefore$ R is symmetric.

(c) Transitive For any a, b,c∈A, let $(a,\ b) \in R$ and (b, c)∈R

$\Rightarrow |a - b|\ is\ \ mutiple\ \ of\ \ 4\ \ and\ \ |b - c|\ \ is\ \ multiple\ \ of\ \ 4$

$\Rightarrow |a - c|\ = \ |a - b + b - c|$

$\Rightarrow |a - c|\ = \ |4k_1 + 4k_2|\ \ where\ \ a - b = 4k_1\ \ and\ \ b - c = 4k_2$

$\Rightarrow |a - c|\ = \ 4|k_1 + k_2|$

$\Rightarrow |a - c|$ is multiple of 4

$\Rightarrow (a - c) \in R$

$\therefore$ R is transitive.

Hence, R is an equivalence relation.

(ii) R $= \{(a,b) : a = b\}$

$\Rightarrow R = \{(0,\ 0),\ (1,\ 1),\ ....,\ (12,\ 12)\}$ and $A = (0,\ 1,\ 2,\ ....,\ 12\}$

(a) Reflexive $a \in A\ \ \Rightarrow\ \ a = a\ \ \Rightarrow\ \ (a,\ a) \in R\ \ \Rightarrow R$ is reflexive.

(b) Symmetry a, b∈A Let $(a,\ b) \in R \Rightarrow a = b \Rightarrow b = a \Rightarrow (b,\ a) \in R$

$\Rightarrow$ R is symmetric.

(c) Transitive

$a,\ b,\ c \in A,\ \ \ Let\ (a,\ b) \in R\ \ \Rightarrow a = b\ (b,\ c) \in R\ \ \Rightarrow b = c \Rightarrow a = c \Rightarrow (a,\ c) \in R \Rightarrow$ R is transitive.

Hence, R is an equivalence relation.

Now set of all elements related to 1 in each case.

(i) Required set $= \{(5,1),(1,5),(9,1),(1,9)\}$

(ii) Required set $= \{(1,1)\}$

10. Give an example of a relation which is

(i) Symmetric but neither reflexive nor transitive.

(ii) Transitive but neither reflexive nor symmetric.

(iii) Reflexive and symmetric but not transitive.

(iv) Reflexive and transitive but not symmetric.

(v) Symmetric and transitive but not reflexive.

SOLUTION

(i) Relation R "is perpendicular to"

i.e., $R = \{(x,\ y) : x\ \ is\ \ perpendicular\ \ to\ y\}$

l_1 is not perpendicular to $l_1 \Rightarrow R$ is not reflexive

If $l_1 \perp l_2$, then $l_2 \perp l_1 \Rightarrow$ R is symmetric

If $l_1 \perp l_2$ and $l_2 \perp l_3$, then l_1 is not perndicular to l_3.

$\Rightarrow$ R is not transitive Clearly, R "is perpendicular to" is a symmetric but neither reflexive nor transitive.

(ii) Relation R $= \{(x,y) : x > y\}$

We know that$x > x$ is false. So, R is not reflexive. If$x > y$, then it does not imply that $y > x$. So, R is not symmetric. If $x > y, y > z$ imply $x > z$. So, R is transitive.

Thus, R is transitive but neither reflexive nor symmetric.

(iii) Relation "is friend of "R $= \{(x,y) : x$ is a friend of y$\}$ x is a friend of x. $\therefore$ R is reflexive.

If x is a friend of y, then y is a friend of x. $\therefore$ R is symmetric.

If x is a friend ofy and y is a friend of z, then x cannot be friend of z.

$\therefore$ R is reflexive and symmetric but not transitive.

(iv) R is relation “is greater or equal to” i.e.,

$R = \{(x,\ y) : x \geq y\}$

$x \geq x\ is\ true.\quad \therefore\ R\ is\ reflexive.$

If $x \geq y$ then it does not imply $y \geq x \therefore\ R\ is\ not\ symmetric$

If $x \geq y$ then it does not imply $y \geq x \therefore$ R is not symmetric

If $x \geq y,\ y \geq z \Rightarrow x \geq z \therefore$ R is transitive.

Hence, R is reflexive and transitive but not symmetric.

(v) R is relation "is brother of " i.e.

R $= \{(x,y) : x$ is a brother of y$\}$

x is not a brother of x. So, R is not reflexive If x is a brother ofy, then y is a brother ofx. So, R is symmetric

If x R y, and y Rz , i.e., x is brother ofy and y is brother of z

$\Rightarrow$ x is brother of z $\Rightarrow$ x R z $=$ R is transitive.

Hence, R is symmetric, transitive but not reflexive.

11. Show that the relation R in the set A of points in a plane given by R $= \{(P,Q) :$ distance of the point P rom the origin is same as the distance of the point Q from the origin $\}$, is an equivalence relation. Further, show that the set of all points related to a point $P \neq (0,\ 0)$ is the circle passing through P with origin as centre.

SOLUTION

R $= \{(P,Q)$: distance of the point P from the origin is same as the distance of the point Q from the origin$\}$

Let $P(x_1,\ y_1),\ Q(x_2,\ y_2)\ and\ O(0,\ 0)$.

$\therefore OP = OQ = \sqrt{x_1^2 + y_1^2} = \sqrt{x_2^2 + y_2^2} \Rightarrow x_1^2 + y_1^2 = x_2^2 + y_2^2$

(i) Reflexive

$P \in A \Rightarrow (P,\ P) \in R\ (\because\ OP = OP)$

Distance of the point P from origin is same as the distance of the point P from origin.

$\therefore$ R is reflexive.

(ii) Symmetric

$P,\ Q \in A;\ \ (P,\ Q) \in R$

$\Rightarrow$ Distance of the point P from origin is same as the distance of point Q from origin

i.e., OP $=$ OQ

$\Rightarrow$ OQ $=$ OP $\Rightarrow (Q,\ P) \in R \therefore$ R is symmetric.

(iii) Transitive $P,\ Q,\ S \in R,\ \ (P,\ Q) \in R\ \ and\ \ (Q,\ S) \in R, \Rightarrow OP = OQ$ and $OQ = OS \Rightarrow OP = OS \Rightarrow (P,\ S) \in R$

$\Rightarrow$ R is transitive. Hence, R is an equivalence relation.

We have to find the set of points related to $P \neq (0,\ 0)$

As, $x_1^2 + y_1^2 = x_2^2 + y_2^2 = r^2 \Rightarrow x^2 + y^2 = r^2$

Which represents a circle with centre (0, 0) and radius $=$ r.

12. Show that the relation R defined in the set A of all triangles as $R=\{(T_1,\ T_2):T_1$ is similar to $T_2\}$, is an equivalence relation. Consider three right angle triangles T_1 , with sides 3, 4, 5, T_2 with sides 5, 12, 13 and T_3 , with sides 6, 8, 10. Which triangles among T_1, T_2 and T_3 are related?

SOLUTION

$R=\{(T_1,\ T_2):T_1$ is similar to T_2 and T_1 , T_2 are triangles.

(i) Reflexivity We know that each triangle is similar to itself and thus $(T_1,\ T_1)\in R$. $\therefore$ R is reflexive.

(ii) Symmetry Also, two triangles are similar. Then, $T_1\sim T_2\Rightarrow\ T_2\sim T_1$. $\therefore$ R is symmetric.

(iii) Transitivity Again, if if $T_1\sim T_2$ and $T_2\sim T_3\Rightarrow T_1\sim T_3\therefore$ R is transitive. Hence, R is an equivalence relation.

We are given three right angled triangles $T_1,\ T_2\ and\ T_3$.

T_1 with sides 3, 4, 5 ; T_2 with sides 5, 12, 13 and T_3 with sides 6, 8, 10

We know that twotriangles are similar if corresponding sides are proportional. We observe that T_1 , and T_3 are similar because $\frac{3}{6}=\frac{4}{8}=\frac{5}{10}\left(=\frac{1}{2}\right)$

Hence, triangles T_1 , and T_3 are related.

13. Show that the relation R defined in the set A of all polygons as if $R=\{(P_1,\ P_2):P_1$ and P_2 have same number of sides $\}$, is an equivalence relation. What is the set of theelements in A related to the right angle triangle T with sides 3, 4 and 5 ?

SOLUTION

R $=\{(P,,P2):P$, and P2 have same number of sides$\}$.

(i) Reflexive

Let $P_1\in A$. Consider the element$(P_1,\ P_1)$. It shows that P_1 and P_1 have same number of sides. P_1 have same number of sides.

$\Rightarrow(P_1,\ P_2)\in R$. Hence, R is reflexive.

(ii) Symmetric

Let $P_1,\ P_1\in A.\ \ If\ \ (P_1,\ P_2)\in R,$

$\Rightarrow P_1$ and P_2 have same number of sides. $\Rightarrow P_2$ and P_1 have same number of sides

$\Rightarrow(P_2,\ P_1)\in R\Rightarrow R$ is symmetric.

(iii) Transitive

Let $P_1,\ P_2,\ P_3\in A,\ \ If\ \ (P_1,\ P_2)\in R$ and $(P_2,\ P_3)\in R,$

$\Rightarrow P_1\ \ and\ \ P_2$ have same number of sides and P_2 and P_3 have same number of sides $\Rightarrow P_1$ and P_3 have same number of sides $\Rightarrow$ $(P_1,\ P_3)\in R$ Thus, R is transitive.

Hence, R is an equivalence relation.

We know that, if 3, 4, 5 are the sides of a triangle, then thetriangle is right-angled. Now, the set of elements in A related to T is the set of right angled triangles.

14. Let L be the set of all lines inXY-plane and R be the relation in L defined as $R=\{(L_1,\ L_2):L_1$ is parallel to $L_2\}$. Show that R is an equivalence relation. Find the set of all lines related to the line y = 2x+ 4.

SOLUTION

$R=\{(L_1,\ L_2):L_1$ is parallel to $L_2\}$

(i) Reflexive

Let $L_1\in L\ L_1\in L\cdot L_1\parallel L_1\ \ i.e.,\ \ (L_1,\ L_1)\in R$. Thus R, is reflexive.

(ii) Symmetric $L_1,\ L_2\in L$ Let $(L_1,\ L_2)\in R\Rightarrow L_1\parallel L_2\Rightarrow L_2\parallel L_1\Rightarrow(L_2,\ L_1)\in R$

Thus, R is symmetric.

(iii) Transitive

$L_1,\ L_2,\ L_3\in L$. Let $(L_1,\ L_2)\in R\ \ and\ \ L_2,\ L_3\in R$

$\Rightarrow L_1\parallel L_2\ \ and\ \ L_2\parallel L_3\Rightarrow L_1\parallel L_3$

Thus, R is transitive. Hence, R is equivalence relation.

All lines related to the line y = 2x + 4 are y = 2x + c, where c is a real number.

$L=\{(y=2x+4,\ y=2x+c):x,\ y\in R\}$.

15. Let R be the relation in the set $\{1,2,3,4\}$ given by R $= \{(1,2),(2,2),(1,1),(4,4),(1,3),(3,3),(3,2)\}$. Choose the correct answer.
    (A) R is reflexive and symmetric but not transitive.

    (B) R is reflexive and transitive but not symmetric.

    (C) R is symmetric and transitive but not reflexive.

    (D) R is an equivalence relation.

    SOLUTION

    (B) R is reflexive for all 1, 2, 3, 4 $\in \{1,2,3,4\}$
    R is not symmetric for all 1, 2 $\in$ $\{1,\ 2,\ 3,\ 4\}\}$
    R is not symmetric and $(3,\ 2) \in R$
    $\Rightarrow$ $(1,\ 2) \in R$ for all 1, 2, 3$\in$ $\{1,2,3,4\}$

16. Let R be the relation in the set N given by $R = \{(a,\ b) : a = b-2,\ b > 6\}$. Choose the correct answer.
    (A) (2, 4) $\in$ R
    (B) (3, 8) $\in$ R
    (C) (6, 8) $\in R$
    (D) (8, 7) $\in$ R

    SOLUTION

    (C) $(a,\ b) \in R\ only\ if\ \ a = b-2\ \ and\ \ b > 6$
    $(6,\ 8) \in R\ \ as\ \ 6 = 8-2\ \ and\ \ 8 > 6.$