Exercise – 4.6

Examine the consistency of the system of equations in Exercises 1 to 6. .

1. $x+2y=2, 2x+3y=3.$

SOLUTION

The system of equations can be written in the form $AX=B$, where $A=\begin{bmatrix}1 & 2\\2 & 3\end{bmatrix}, X=\begin{bmatrix}x\\y\end{bmatrix}, B=\begin{bmatrix}2\\3\end{bmatrix}$ Now, $|A|=3-4=-1\neq 0$ Hence, system of equations is consistent.

2. $2x-y=5, x+y=4.$

SOLUTION

The system of equations can be written in the form $AX=B$, where $A=\begin{bmatrix}2 & -1\\1 & 1\end{bmatrix}, X=\begin{bmatrix}x\\y\end{bmatrix}, B=\begin{bmatrix}5\\4\end{bmatrix}$ Now, $|A|=\begin{vmatrix}2 & -1\\1 & 1\end{vmatrix}=2+1=3\neq 0$ Hence, system of equations is consistent.

3. $x+3y=5, 2x+6y=8.$

SOLUTION

The system of equations can be written in the form $AX=B$, where $A=\begin{bmatrix}1 & 3\\2 & 6\end{bmatrix}, X=\begin{bmatrix}x\\y\end{bmatrix}, B=\begin{bmatrix}5\\8\end{bmatrix}$ Now, $|A|=\begin{vmatrix}1 & 3\\2 & 6\end{vmatrix}=6-6=0$ Hence, A is singular matrix. So, we calculate (adj A) B. $adj\,A=\begin{bmatrix}6 & -3\\-2 & 1\end{bmatrix}$ Now, (adj A)B $=\begin{bmatrix}6 & -3\\-2 & 1\end{bmatrix}\begin{bmatrix}5\\8\end{bmatrix}=\begin{bmatrix}30-24\\-10+8\end{bmatrix}=\begin{bmatrix}6\\-2\end{bmatrix}\neq\begin{bmatrix}0\\0\end{bmatrix}$ Hence, equations are inconsistent with no solution.

4. $x+y+z=1, 2x+3y+2z=2, ax+ay+2az=4.$

SOLUTION

The system of equations can be written in the form $AX=B$, where $A=\begin{bmatrix}1 & 1 & 1\\2 & 3 & 2\\a & a & 2a\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix}, B=\begin{bmatrix}1\\2\\4\end{bmatrix}$ Now, $|A|=1(6a-2a)-1(4a-2a)+1(2a-3a)=4a-3a=a\neq 0$ Two conditions arise: > I : If $a\neq 0$, then $|A|\neq 0$, hence the system of equations is consistent and has a unique solution. II : If $a=0$, then $|A|=0$. So, we need to calculate adj A. Cofactors of elements of A are given by $A_{11}=(-1)^{1+1}\begin{vmatrix}3 & 2\\a & 2a\end{vmatrix}=4a$, $A_{12}=(-1)^{1+2}\begin{vmatrix}2 & 2\\a & 2a\end{vmatrix}=2a$, $A_{13}=(-1)^{1+3}\begin{vmatrix}2 & 3\\a & a\end{vmatrix}=-a$ $A_{21}=(-1)^{2+1}\begin{vmatrix}1 & 1\\a & 2a\end{vmatrix}=-(-4+4)=0$ $A_{22}=(-1)^{2+2}\begin{vmatrix}1 & 1\\a & 2a\end{vmatrix}=a$ $A_{23}=(-1)^{2+3}\begin{vmatrix}1 & 1\\a & a\end{vmatrix}=0$ $A_{31}=(-1)^{3+1}\begin{vmatrix}1 & 1\\3 & 2\end{vmatrix}=-1$ $A_{32}=(-1)^{3+2}\begin{vmatrix}1 & 1\\2 & 2\end{vmatrix}=0$ $A_{33}=(-1)^{3+3}\begin{vmatrix}1 & 1\\2 & 3\end{vmatrix}=1$

Now, $(adj\,A)\cdot B=\begin{bmatrix}4a & -a & -1\\-2a & a & 0\\-a & 0 & 1\end{bmatrix}\begin{bmatrix}1\\2\\4\end{bmatrix}=\begin{bmatrix}2a-4\\0\\-a+4\end{bmatrix}=\begin{bmatrix}-4\\0\\4\end{bmatrix}\neq 0$] Hence, equations are inconsistent with no solution because if $a=0$, then the third system of equations is not possible.

5. $3x-y-2z=2, 2y-z=-1, 3x-5y=3.$

SOLUTION

The system of equations can be written in the form $AX=B$, where $A=\begin{bmatrix}3 & -1 & -2\\0 & 2 & -1\\3 & -5 & 0\end{bmatrix}, X\begin{bmatrix}x\\y\\z\end{bmatrix}$ and $B=\begin{bmatrix}2\\-1\\3\end{bmatrix}$ Now, $|A|=3(0-5)+1(0+3)-2(0-6)=-15+3+12=0$ Here, A is a singular matrix, so we will compute (adj A)B. For adj A, cofactors of elements of A are given by $A_{11}=(-1)^{1+1}\begin{vmatrix}3 & -1\\-5 & 0\end{vmatrix}=-5$, $A_{12}=(-1)^{1+2}\begin{vmatrix}0 & -1\\3 & 0\end{vmatrix}=-3$, $A_{13}=(-1)^{1+3}\begin{vmatrix}0 & 2\\3 & -5\end{vmatrix}=-6$, $A_{21}=(-1)^{2+1}\begin{vmatrix}-1 & -2\\-5 & 0\end{vmatrix}=-(-10)=10$, $A_{22}=(-1)^{2+2}\begin{vmatrix}3 & -2\\3 & 0\end{vmatrix}=6, 1$ $A_{23}=(-1)^{2+3}\begin{vmatrix}3 & -1\\3 & -5\end{vmatrix}=-(-15+3)=$

$12, A_{31} = (-1)^{3+1}\begin{vmatrix} -1 & -2 \\ 2 & -1 \end{vmatrix} = 1+4 = 5, A_{32} = (-1)^{3+2}\begin{vmatrix} 3 & -2 \\ 0 & -1 \end{vmatrix} = 3, A_{33} = (-1)^{3+3}\begin{vmatrix} 3 & -1 \\ 0 & 2 \end{vmatrix} = 6 \therefore$ Now, $(adj\,A)\cdot B =$
$\begin{bmatrix} -5 & 10 & 5 \\ -3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -10-10+15 \\ -6-6+9 \\ -12-12+18 \end{bmatrix} = \begin{bmatrix} -5 \\ -3 \\ -6 \end{bmatrix} \neq 0$

Hence, system of equations is inconsistent with no solution.

6. $5x - y + 4z = 5, 2x + 3y + 5z = 2, 5x - 2y + 6z = -1.$

SOLUTION

The system of equations can be written in the form$AX = B$, where $A = \begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ *and* $B = \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$ Now,
$|A| = \begin{vmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 5 \end{vmatrix} = 5(18+10) + 11(12-25) + 4(-4-15) = 140 - 13 - 76 = 51 \neq 0$ Hence, equations are consistent with a unique solution.

Solve system of linear equations, using matrix method, in questions 7 to 14.

7. $5x + 2y = 4, 7x + 3y = 5.$

SOLUTION

The given system of equations can be written in the form $AX = B$, where $A = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ Now,
$|A| = \begin{vmatrix} 5 & 2 \\ 7 & 3 \end{vmatrix} = 15 - 14 = 1 \neq 0 \Rightarrow$ A is non-singular and so given system has a unique solution. Cofactors of elements of A are given by $A_{11} = (-1)^{1+1}(3) = 3, A_{12} = (-1)^{1+2}(7) = -7, A_{21} = (-1)^{2+1}(2) = -2, A_{22} = (-1)^{2+2}(5) = 5$
and $A^{-1} = \frac{1}{|A|}(adj\,A) = \frac{1}{2}\begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}$ Solution of given system is given by $X = A^{-1}B. \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}\begin{bmatrix} 4 \\ 5 \end{bmatrix} =$
$\begin{bmatrix} 12 & -10 \\ -28 & +25 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$ Hence; $x = 2, y = -3$

8. $2x - y = -2, 3x + 4y = 3.$

SOLUTION

The given system of equations can be written in the form$AX = B$, where $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$ Now,
$|A| = \begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix} = 8 + 3 = 11 \neq 0 \Rightarrow$ A is non-singular and so given system has a unique solution. Cofactors of elements of A are given by $A_{11} = (-1)^{1+1}(4) = 4, A_{12} = (-1)^{1+2}(3) = -3, A_{21} = (-1)^{2+1}(-1) = 1, A_{22} = (-1)^{2+2}(2) = 2$
and $A^{-1} = \frac{1}{|A|}(adj\,A) = \frac{1}{11}\begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}$ Solution of given system is given by $X = A^{-1}B. \Rightarrow \begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} -2 \\ 3 \end{bmatrix} =$
$\frac{1}{11}\begin{bmatrix} -8+3 \\ 6+6 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} -5 \\ 12 \end{bmatrix} = \begin{bmatrix} \frac{-5}{11} \\ \frac{12}{11} \end{bmatrix} \therefore x = -\frac{5}{11}, y = \frac{12}{11}$

9. $4x - 3y = 3, 3x - 5y = 7.$

SOLUTION

The given system of equations can be written in the form $AX = B$, where $A = \begin{bmatrix} 4 & -3 \\ 3 & -5 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$ Now,
$|A| = \begin{vmatrix} 4 & -3 \\ 3 & -5 \end{vmatrix} = -20 + 9 = -11 \neq 0 \Rightarrow$ A is a non-singular matrix and so the given system has a unique solution. Cofactors of elements of A are given by $A_{11} = (-1)^{1+1}(-5) = -5, A_{12} = (-1)^{1+2}(3) = -3, A_{21} = (-1)^{2+1}(-3) = 3, A_{22} = (-1)^{2+2}(4) = 4$
and $A^{-1} = \frac{1}{|A|}(adj\,A) = \frac{1}{-11}\begin{bmatrix} -5 & 3 \\ -3 & 4 \end{bmatrix}$ Solution of given system is given by $X = A^{-1}B. \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{11}\begin{bmatrix} -5 & 3 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 3 \\ 7 \end{bmatrix}$
$= -\frac{1}{11}\begin{bmatrix} -15+21 \\ -9+28 \end{bmatrix} = \frac{1}{-11}\begin{bmatrix} 6 \\ 19 \end{bmatrix} = \begin{bmatrix} \frac{-6}{11} \\ \frac{-19}{11} \end{bmatrix}$ Hence, $x = \frac{-6}{11}, y = \frac{-19}{11}$

10. . $5x+2y=3, 3x+2y=5$.

SOLUTION

The given system of equations can be written in the form $AX=B$, where $A=\begin{bmatrix}5 & 2\\ 3 & 2\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix}$ and $B=\begin{bmatrix}3\\ 5\end{bmatrix}$ Now, $|A|=\begin{vmatrix}5 & 2\\ 3 & 2\end{vmatrix}=10-6=4\neq 0 \Rightarrow$ A is a non-singular matrix and so the given system has a unique solution. Cofactors of elements of A are given by $A_{11}=(-1)^{1+1}(2)=2, A_{12}=(-1)^{1+2}(3)=-3, A_{21}=(-1)^{2+1}(2)=-2, A_{22}=(-1)^{2+2}(5)=5$ and $A^{-1}=\frac{1}{|A|}(adj A)=\frac{1}{4}\begin{bmatrix}2 & -2\\ -3 & 5\end{bmatrix}$ Solution of given system is given by$X=A^{-1}B$. $\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}2 & -2\\ -3 & 5\end{bmatrix}\begin{bmatrix}3\\ 5\end{bmatrix}; \begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}6-10\\ -9+25\end{bmatrix}=\frac{1}{4}\begin{bmatrix}-4\\ 16\end{bmatrix}=\begin{bmatrix}-1\\ 4\end{bmatrix}$ Hence, $x=-1, y=4$

11. $2x+y+z=1, x-2y-z=3/2, 3y-5z=9$.

SOLUTION

The given system of equations can be written in the form$AX=B$, where $A=\begin{bmatrix}2 & 1 & 1\\ 1 & -2 & -1\\ 0 & 3 & -5\end{bmatrix}, X=\begin{bmatrix}X\\ Y\\ Z\end{bmatrix}$ and $B=\begin{bmatrix}1\\ \frac{3}{2}\\ 9\end{bmatrix}$ Now,

$|A|=\begin{vmatrix}2 & 1 & 1\\ 1 & -2 & -1\\ 0 & 3 & -5\end{vmatrix}=2\begin{vmatrix}-2 & -1\\ 3 & -5\end{vmatrix}-1\begin{vmatrix}1 & -1\\ 0 & -5\end{vmatrix}+1\begin{vmatrix}1 & -2\\ 0 & 3\end{vmatrix}$ $|A|=2(10+3)-1(-5)+1(3)\Rightarrow |A|=26+8=34\neq 0$.

$\Rightarrow$ A is a non-singular matrix and so the given equations have a unique solution. Here, cofactors of elements of A are : $A_{11}=(-1)^{1+1}\begin{vmatrix}-2 & -1\\ 3 & -5\end{vmatrix}=(10+3)=13, A_{12}=(-1)^{1+2}\begin{vmatrix}1 & -1\\ 0 & -5\end{vmatrix}=(-5+0)=5, A_{13}=(-1)^{1+3}\begin{vmatrix}1 & -2\\ 0 & 3\end{vmatrix}=(3-0)=3, A_{21}=(-1)^{2+1}\begin{vmatrix}1 & 1\\ 3 & -5\end{vmatrix}=-(-5-3)=8, A_{22}=(-1)^{2+2}\begin{vmatrix}2 & 1\\ 0 & -5\end{vmatrix}=(-10-0)=-10, A_{23}=(-1)^{2+3}\begin{vmatrix}2 & 1\\ 0 & 3\end{vmatrix}=-(6-0)=-6,, A_{31}=(-1)^{3+1}\begin{vmatrix}1 & 1\\ -2 & -1\end{vmatrix}=(-1+2)=1, A_{32}=(-1)^{3+2}\begin{vmatrix}2 & 1\\ 1 & -1\end{vmatrix}=-(-2-1)=3, A_{33}=(-1)^{3+3}\begin{vmatrix}2 & 1\\ 1 & -2\end{vmatrix}=-4-1=-5$

and $A^{-1}=\frac{1}{|A|}(adj A)=\frac{1}{34}\begin{bmatrix}13 & 8 & 1\\ 5 & -10 & 3\\ 3 & -6 & -5\end{bmatrix}$ Solution of given system is given by $X=A^{-1}B.\Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\frac{1}{34}\begin{bmatrix}13 & 8 & 1\\ 5 & -10 & 3\\ 3 & -6 & -5\end{bmatrix}\begin{bmatrix}3/\end{bmatrix}$

$=\frac{1}{34}\begin{bmatrix}13+12+9\\ 5-15+27\\ 3-9-45\end{bmatrix}=\frac{1}{34}\begin{bmatrix}34\\ 17\\ -51\end{bmatrix}=\begin{bmatrix}1\\ 1/2\\ -3/2\end{bmatrix}$ Hence, $x=1, y=\frac{1}{2}$ and $z=\frac{-3}{2}$.

12. . $x-y+z=4, 2x+y-3z=0, x+y+z=2$.

SOLUTION

The given system of equations can be written in the form $AX=B$, where $A=\begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix}, X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$ and $B=\begin{bmatrix}4\\ 0\\ 2\end{bmatrix}$

Now, $|A|=\begin{vmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{vmatrix}=1\begin{vmatrix}1 & -3\\ 1 & 1\end{vmatrix}+1\begin{vmatrix}2 & -3\\ 1 & 1\end{vmatrix}+\begin{vmatrix}2 & 1\\ 1 & 1\end{vmatrix}=1(1+3)+1(2+3)+1(2-1)=4+5+1=10\neq 0\Rightarrow$ A is non-singular and so the given equations have a unique solution. Here, cofactors of element of A are : $A_{11}=(-1)^{1+1}\begin{vmatrix}1 & -3\\ 1 & 1\end{vmatrix}=+(1+3)=4,\ A_{12}=(-1)^{1+2}\begin{vmatrix}2 & -3\\ 1 & 1\end{vmatrix}=-(2+3)=-5,\ A_{13}=(-1)^{1+3}\begin{vmatrix}2 & 1\\ 1 & 1\end{vmatrix}=+(2-1)=1,$ $A_{21}=(-1)^{2+1}\begin{vmatrix}-1 & 1\\ 1 & 1\end{vmatrix}=-(-1-1)=2,\ A_{22}=(-1)^{2+2}\begin{vmatrix}1 & 1\\ 1 & 1\end{vmatrix}=+(1-1)=0,\ A_{23}=(-1)^{2+3}\begin{vmatrix}1 & -1\\ 1 & 1\end{vmatrix}=-(1+1)=-2,,\ A_{31}=(-1)^{3+1}\begin{vmatrix}-1 & 1\\ 1 & -3\end{vmatrix}=+(3-1)=2,\ A_{32}=(-1)^{3+2}\begin{vmatrix}1 & 1\\ 2 & -3\end{vmatrix}=-(-3-2)=5,\ A_{33}=(-1)^{3+3}\begin{vmatrix}1 & -1\\ 2 & 1\end{vmatrix}=+(1+2)=3,$

and $A^{-1}=\frac{1}{|A|}(adj A)=\frac{1}{10}\begin{bmatrix}4 & 2 & 2\\ -5 & 0 & 5\\ 1 & -2 & 3\end{bmatrix}$ Solution of the given system-of equations is given by $X=A^{-1}B.\Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix}=$

$$\frac{1}{10}\begin{bmatrix}4&2&2\\-5&0&5\\1&-2&3\end{bmatrix}\begin{bmatrix}4\\0\\2\end{bmatrix}=\frac{1}{10}\begin{bmatrix}16+0+4\\-20+0+10\\4-0+6\end{bmatrix}\Rightarrow\begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{10}\begin{bmatrix}20\\-10\\10\end{bmatrix}=\begin{bmatrix}2\\-1\\1\end{bmatrix}\Rightarrow x=2, y=-1 \text{ and } z=1.$$

13. $2x+3y+3z=5, x-2y+z=-4, 3x-y-2z=3.$

SOLUTION

The given system of equations can be written in the form $AX=B$, where $A=\begin{bmatrix}2&3&3\\1&-2&1\\3&-1&-2\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix}$ and $B=\begin{bmatrix}5\\-4\\3\end{bmatrix}$

Now, $|A|=\begin{vmatrix}2&3&3\\1&-2&1\\3&-1&-2\end{vmatrix}=2\begin{vmatrix}-2&1\\-1&-2\end{vmatrix}-3\begin{vmatrix}1&1\\3&-2\end{vmatrix}+3\begin{vmatrix}1&-2\\3&-1\end{vmatrix}=2(4+1)-3(-2-3)+3(-1+6)=10+15+15=40\neq 0\Rightarrow$ A is a non-singular matrix and so the given equations have a unique solution. Here, cofactors of elements of A are : $A_{11}=(-1)^{1+1}\begin{vmatrix}-2&1\\-1&-2\end{vmatrix}=(4+1)=5, A_{12}=(-1)^{1+2}\begin{vmatrix}1&1\\3&-2\end{vmatrix}=-(-2-3)=5, A_{13}=(-1)^{1+3}\begin{vmatrix}1&-2\\3&-1\end{vmatrix}=(-1+6)=5$, $A_{21}=(-1)^{2+1}\begin{vmatrix}3&3\\-1&-2\end{vmatrix}=-(-6+3)=3, A_{22}=(-1)^{2+2}\begin{vmatrix}2&3\\3&-2\end{vmatrix}=(-4-9)=-13, A_{23}=(-1)^{2+3}\begin{vmatrix}2&3\\3&-1\end{vmatrix}=-(-2-9)=11,, A_{31}=(-1)^{3+1}\begin{vmatrix}3&3\\-2&1\end{vmatrix}=(3+6)=9, A_{32}=(-1)^{3+2}\begin{vmatrix}2&3\\1&1\end{vmatrix}=-(2-3)=1, A_{33}=(-1)^{3+3}\begin{vmatrix}2&3\\1&-2\end{vmatrix}=(-4-3)=-7,$

Hence, $A^{-1}=\frac{1}{|A|}(adj A)=\frac{1}{40}\begin{bmatrix}5&3&9\\5&-13&1\\5&11&-7\end{bmatrix}$ Solution of given system is given by $X=A^{-1}B.\Rightarrow\begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{40}\begin{bmatrix}5&3&9\\5&-13&1\\5&11&-7\end{bmatrix}$

$=\frac{1}{40}\begin{bmatrix}25-12+27\\25+52+3\\25-44-21\end{bmatrix}=\frac{1}{40}\begin{bmatrix}40\\80\\-40\end{bmatrix}=\begin{bmatrix}1\\2\\-1\end{bmatrix}\Rightarrow$ x =1, y=2 and z =-1 .

14. $x-y+2z=7, 3x+4y-5z=-5, 2x-y+3z=12$

SOLUTION

The given system of equations can be written in the form $AX=B$, where $A=\begin{bmatrix}1&-1&2\\3&4&-5\\2&-1&3\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix}$ and $B=\begin{bmatrix}7\\-5\\12\end{bmatrix}$

Now, $|A|=\begin{vmatrix}1&-1&2\\3&4&-5\\2&-1&3\end{vmatrix}=1\begin{vmatrix}4&-5\\-1&3\end{vmatrix}+1\begin{vmatrix}3&-5\\2&3\end{vmatrix}+2\begin{vmatrix}3&4\\2&-1\end{vmatrix}\Rightarrow|A|=1(12-5)+1(9+10)+2(-3-8)\Rightarrow |A|=7+19-22=4\neq 0\Rightarrow$ A is a non-singular matrix and so the given equations have a unique solution. Here, cofactors of elements of A are : $A_{11}=(-1)^{1+1}\begin{vmatrix}4&-5\\-1&3\end{vmatrix}=(12-5)=7, A_{12}=(-1)^{1+2}\begin{vmatrix}3&-5\\2&3\end{vmatrix}=-(9+10)=-19, A_{13}=(-1)^{1+3}\begin{vmatrix}3&4\\2&-1\end{vmatrix}=(-3-8)=-11, A_{21}=(-1)^{2+1}\begin{vmatrix}-1&2\\-1&3\end{vmatrix}=-(-3+2)=1, A_{22}=(-1)^{2+2}\begin{vmatrix}1&2\\2&3\end{vmatrix}=(3-4)=-1,$ $A_{23}=(-1)^{2+3}\begin{vmatrix}1&-1\\2&-1\end{vmatrix}=-(-1+2)=-1,, A_{31}=(-1)^{3+1}\begin{vmatrix}-1&2\\4&-5\end{vmatrix}=(5-8)=-3, A_{32}=(-1)^{3+2}\begin{vmatrix}1&2\\3&-5\end{vmatrix}=-(5-6)=11, A_{33}=(-1)^{3+3}\begin{vmatrix}1&-1\\3&4\end{vmatrix}=(4+3)=7,$

Hence, $A^{-1}=\frac{1}{|A|}(adj A)=\frac{1}{4}\begin{bmatrix}7&1&-3\\-19&-1&11\\-11&-1&7\end{bmatrix}$ Solution of given system is given by $X=A^{-1}B.\Rightarrow\begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{4}\begin{bmatrix}7&1&-3\\-19&-1&11\\-11&-1&7\end{bmatrix}$

$=\frac{1}{4}\begin{bmatrix}49-5-36\\-133+5+132\\-77+5+84\end{bmatrix}=\frac{1}{4}\begin{bmatrix}8\\4\\12\end{bmatrix}=\begin{bmatrix}2\\1\\3\end{bmatrix}\Rightarrow x=2, y=1$ and $z=3$.

15. If $A=\begin{bmatrix}2&-3&5\\3&2&-4\\1&1&-2\end{bmatrix}$, find A^{-1}.Using A^{-1}, solve the system of equations $2x-3y+5z=11, 3x+2y-4z=-5, x+y-2z=-3$.

SOLUTION

We have, $A=\begin{bmatrix}2&-3&5\\3&2&-4\\1&1&-2\end{bmatrix}$ $\therefore |A| = 2(-4+4)+3(-6+4)+5(3-2) = 0-6+5 = -1 \neq 0$. So, A is invertible. Co-factors of elements of A are : $A_{11}=(-1)^{1+1}\begin{vmatrix}2&-4\\1&-2\end{vmatrix}=(-4+4)=0$, $A_{12}=(-1)^{1+2}\begin{vmatrix}3&-4\\1&-2\end{vmatrix}=-(-6+4)=2$, $A_{13}=(-1)^{1+3}\begin{vmatrix}3&2\\1&1\end{vmatrix}=(3-2)=1$, $A_{21}=(-1)^{2+1}\begin{vmatrix}-3&5\\1&-2\end{vmatrix}=-(6-5)=-1$, $A_{22}=(-1)^{2+2}\begin{vmatrix}2&5\\1&-2\end{vmatrix}=(-4-5)=-9$, $A_{23}=(-1)^{2+3}\begin{vmatrix}2&-3\\1&1\end{vmatrix}=-(2+3)=-5$,, $A_{31}=(-1)^{3+1}\begin{vmatrix}-3&5\\2&-4\end{vmatrix}=(12-10)=2$, $A_{32}=(-1)^{3+2}\begin{vmatrix}2&5\\3&-4\end{vmatrix}=-(-8-15)=23$, $A_{33}=(-1)^{3+3}\begin{vmatrix}2&-3\\3&2\end{vmatrix}=(4+9)=13$,

Hence, $A^{-1}=\frac{1}{|A|}(adj\ A)=\frac{1}{-1}\begin{bmatrix}0&-1&2\\2&-9&23\\1&-5&13\end{bmatrix}=\begin{bmatrix}0&1&-2\\-2&9&-23\\-1&5&-13\end{bmatrix}$ Now, the given system of equations can be written in form $AX=B$, where $A=\begin{bmatrix}2&-3&5\\3&2&-4\\1&1&-2\end{bmatrix}$, $X=\begin{bmatrix}x\\y\\z\end{bmatrix}$ and $B=\begin{bmatrix}11\\-5\\-3\end{bmatrix}$ As, $|A|=-1\neq 0$, so given system of equations has a unique solution given by $X=A^{-1}B$. $\Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0&1&-2\\-2&9&-23\\-1&5&-13\end{bmatrix}\begin{bmatrix}11\\-5\\-3\end{bmatrix}=\begin{bmatrix}0-5+6\\-22-45+69\\-11-25+39\end{bmatrix}=\begin{bmatrix}1\\2\\3\end{bmatrix}$ Hence, $x=1, y=2$ and $z=3$.

16. The cost of 4 kg onion, 3 kg wheat and 2 kg rice is Rs.60. The cost of 2 kg onion, 4 kg wheat and 6 kg rice is Rs. 90. The cost of 6 kg onion, 2 kg wheat and 3 kg rice is Rs. 70. Find cost of the each item per kg by matrix method.

SOLUTION

Let cost of 1 kg onion = Rs. x, cost of 1 kg wheat = Rs. y and cost of 1 kg rice = Rs. z $\therefore$ According to question, wo have $4x+3y+2z=60$ $2x+4y+6z=90$ $6x+2y+3z=70$ This system can be written as AX=B, where $A=\begin{bmatrix}4&3&2\\2&4&6\\6&2&3\end{bmatrix}$, $X=\begin{bmatrix}x\\y\\z\end{bmatrix}$ and $B=\begin{bmatrix}60\\90\\70\end{bmatrix}$ Now, $|A|=\begin{vmatrix}4&3&2\\2&4&6\\6&2&3\end{vmatrix}=4(12-12)-3(6-36)+2(4-24)90-40=50\neq 0$ $\therefore$ A is a non-singular matrix and system has a unique solution. Cofactors of elements of A are : $A_{11}=(-1)^{1+1}\begin{vmatrix}4&6\\2&3\end{vmatrix}=0$, $A_{12}=(-1)^{1+2}\begin{vmatrix}2&6\\6&3\end{vmatrix}=30$, $A_{13}=(-1)^{1+3}\begin{vmatrix}2&4\\6&2\end{vmatrix}=-20$, $A_{21}=(-1)^{2+1}\begin{vmatrix}3&2\\2&3\end{vmatrix}=-5$, $A_{22}=(-1)^{2+2}\begin{vmatrix}4&2\\6&3\end{vmatrix}=0$, $A_{23}=(-1)^{2+3}\begin{vmatrix}4&3\\6&2\end{vmatrix}=10$,, $A_{31}=(-1)^{3+1}\begin{vmatrix}3&2\\4&6\end{vmatrix}=10$, $A_{32}=(-1)^{3+2}\begin{vmatrix}4&2\\2&6\end{vmatrix}=-20$, $A_{33}=(-1)^{3+3}\begin{vmatrix}4&3\\2&4\end{vmatrix}=10$ $\therefore adj A=\begin{bmatrix}0&30&-20\\-5&0&10\\10&-20&10\end{bmatrix}'=\begin{bmatrix}0&-5&10\\30&0&-20\\-20&10&10\end{bmatrix}$ Hence, $A^{-1}=\frac{1}{|A|}(adj A)=\frac{1}{50}\begin{bmatrix}0&-5&10\\30&0&-20\\-20&10&10\end{bmatrix}$ As, $AX=B\Rightarrow X=A^{-1}B$ $\begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{50}\begin{bmatrix}0&-5&10\\30&0&-20\\-20&10&10\end{bmatrix}\begin{bmatrix}60\\90\\70\end{bmatrix}=\frac{1}{5}\begin{bmatrix}0&-5&10\\30&0&-20\\-20&10&10\end{bmatrix}\begin{bmatrix}6\\9\\7\end{bmatrix}=\frac{1}{5}\begin{bmatrix}0-45+70\\180+0-140\\-120+90+70\end{bmatrix}=\frac{1}{5}\begin{bmatrix}25\\40\\40\end{bmatrix}=\begin{bmatrix}5\\8\\8\end{bmatrix}$ $\therefore x=5, y=8, z=8$ Hence, cost of 1 kg of onion=Rs. 5, cost of 1 kg of wheat = Rs. 8 and cost of 1 kg of rice = Rs. 8

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