Exercise – 4.5

Find adjoint of each of the matrices in Exercises 1 and 2.

1. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

SOLUTION

Let $P = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Let A_{ij} be cofactors of a_{ij} in P. Then, the cofactors of elements of P are given by $A_{11} = (-1)^{1+1}(4) = 4$, $A_{12} = (-1)^{1+2}(3) = -3$, $A_{21} = (-1)^{2+1}(2) = -2$ $A_{22} = (-1)^{2+2}(1) = 1$

$\therefore$

$= \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

2. $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$

SOLUTION

Let $P = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$. Let A_{ij} be cofactors of a_{ij} in A. Then,

$A_{11} = (-1)^{1+1}\begin{bmatrix} 3 & 5 \\ 0 & 1 \end{bmatrix} = 3-0 = 3$ $A_{12} = (-1)^{1+2}\begin{bmatrix} 2 & 5 \\ -2 & 1 \end{bmatrix} = -(2+10) = -12$

$A_{13} = (-1)^{1+3}\begin{bmatrix} 2 & 3 \\ -2 & 0 \end{bmatrix} = 0+6 = 6$ $A_{21} = (-1)^{2+1}\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} = -(-1-0) = 1$

$A_{22} = (-1)^{2+2}\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} = 1+4 = 5$ $A_{23} = (-1)^{2+3}\begin{bmatrix} 1 & -1 \\ -2 & 0 \end{bmatrix} = -(0-2) = 2$ $A_{31} = (-1)^{3+1}\begin{bmatrix} -1 & 2 \\ 3 & 5 \end{bmatrix} = -5-6 = -11$

$A_{32} = (-1)^{3+2}\begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix} = -(5-4) = -1$ $A_{33} = (-1)^{3+3}\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = 3+2 = 5$

$\therefore adj\,P$

$= \begin{bmatrix} 3 & 1 & -11 \\ -12 & 5 & -1 \\ 6 & 2 & 5 \end{bmatrix}$

Verify : $A(adjA) = (adjA)A = |A|I$ in Exercises 3 and 4.

3. . $\begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$

SOLUTION

Let $A = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$ $|A| = -12+12 \Rightarrow |A| = 0$

Let A_{ij} be co-factors of a_{ij} in A. Then, the co-factors of elements of A are given by $A_{11} = (-1)^{1+1}(-6) = -6, A_{12} = (-1)^{1+2}(-4) = 4$

$A_{21} = (-1)^{2+1}(3) = -3, A_{22} = (-1)^{2+2}(2) = 2$

$\Rightarrow adjA =$

$= \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}$

$\therefore A(adjA) = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}\begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$(adjA)/A = \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ and $|A|I = O$ as $|A| = 0$ Hence, $A(adjA) = (adjA)A = |A|I$

4. $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$

SOLUTION

Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$ Let A_{ij} be cofactor of a_{ij} in A. Then, the cofactors of elements of A are given by

$A_{11} = (-1)^{1+1}\begin{bmatrix} 0 & -2 \\ 0 & 3 \end{bmatrix} = 0\ A_{12} = (-1)^{1+2}\begin{bmatrix} 3 & -2 \\ 1 & 3 \end{bmatrix} = -(9+2) = -11$

$A_{13} = (-1)^{1+3}\begin{bmatrix} 3 & 0 \\ 1 & 0 \end{bmatrix} = 0\ A_{21} = (-1)^{2+1}\begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix} = -(-3-0) = 3$

$A_{22} = (-1)^{2+2}\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} = 3-2 = 1\ A_{23} = (-1)^{2+3}\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} = -(0+1) = -1\ A_{31} = (-1)^{3+1}\begin{bmatrix} -1 & 2 \\ 0 & -2 \end{bmatrix} = 2$

$A_{32} = (-1)^{3+2}\begin{bmatrix} 1 & 2 \\ 3 & -2 \end{bmatrix} = -(-2-6) = 8\ A_{33} = (-1)^{3+3}\begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix} = 0+3 = 3$

$\therefore adjA = \begin{bmatrix} 0 & -11 & 0 \\ 3 & 1 & -1 \\ 2 & 8 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$ Now, $A(adjA) = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}\begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 8 \end{bmatrix}$

$= \begin{bmatrix} 0+11+0 & 3-1-2 & 2-8+6 \\ 0+0+0 & 9+0+2 & 6+0-6 \\ 0+0+0 & 3+0-3 & 2+0+9 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix} = 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 11I$

$(AdjA)A = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$

$= \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix} = 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 11I$

Also, $|A| = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{vmatrix} = 1(0)+1(9+2)+2(0) = 11$

Hence, $A(AdjA) = (AAdjA)A = 11I = |A|I$.

Find the inverse of each of the matrices (if it exists) given in Exercises 5 to 11. .

5. $\begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$

SOLUTION

Let A$= \begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$ Then, $|A| = \begin{vmatrix} 2 & -2 \\ 4 & 3 \end{vmatrix} = 6+8 = 14 \neq 0.$

So, A is a non-singular matrix and therefore, it is invertible. Let A_{ij} be cofactor of a_{ij}in A. Then, the cofactors of elements of A are given by $A_{11} = (-1)^{1+1}(3) = 3, A_{12} = (-1)^{1+2}(4) = -4,$

$A_{21} = (-1)^{2+1}(-2) = 2, A_{22} = (-1)^{2+2}(2) = 2$

$\therefore adj\,A$

$= \begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix}$ Hence, $A^{-1} = \frac{1}{|A|}adj\,A$

$= \frac{1}{14}\begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix}$

6. . $\begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$

SOLUTION

Let $A = \begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$ Then, $|A| = \begin{vmatrix} -1 & 5 \\ -3 & 2 \end{vmatrix} = -2+15 = 13 \neq 0.$

So, A is a non-singular matrix and therefore, it is invertible. Let A_{ij} be cofactor of a_{ij} in A. Then, the cofactors of elements of A are given by

$A_{11} = (-1)^{1+1}(2) = 2, A_{12} = (-1)^{1+2}(-3) = 3\ A_{21} = (-1)^{2+1}(5) = -5, A_{22} = (-1)^{2+2}(-1) = -1$

$\therefore adjA$

$= \begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$

Hence, $A^{-1} = \frac{1}{|A|}(adj\ A) = \frac{1}{13}\begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$

7. $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$

SOLUTION

Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$

Then, $|A| = 10 \neq 0$. $A_{11} = (-1)^{1+1}\begin{bmatrix} 2 & 4 \\ 0 & 5 \end{bmatrix} = 10\ A_{12} = (-1)^{1+2}\begin{bmatrix} 0 & 4 \\ 0 & 5 \end{bmatrix} = 0$

$A_{13} = (-1)^{1+3}\begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix} = 0\ A_{21} = (-1)^{2+1}\begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix} = -10\ A_{22} = (-1)^{2+2}\begin{bmatrix} 1 & 3 \\ 0 & 5 \end{bmatrix} = 5$

$A_{23} = (-1)^{2+3}\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} = 0\ A_{31} = (-1)^{3+1}\begin{bmatrix} 2 & 3 \\ 2 & 4 \end{bmatrix} = 2$

$A_{32} = (-1)^{3+2}\begin{bmatrix} 1 & 3 \\ 0 & 4 \end{bmatrix} = -4\ A_{33} = (-1)^{3+3}\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix} = 2$

$\therefore$ adj A

$= \begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$ Hence, $A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{10}\begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$

8. $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$

SOLUTION

Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$

Then, $|A| = \begin{vmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{vmatrix} = 1\begin{vmatrix} 3 & 0 \\ 2 & -1 \end{vmatrix} = -3-0 = -3 \neq 0$

So, A is non-singular matrix and therefore, it is invertible. Let A_{ij} be cofactors of a_{ij} in A. Then, the cofactor of elements of A are given by $A_{11} = (-1)^{1+1}\begin{bmatrix} 3 & 0 \\ 2 & -1 \end{bmatrix} = (-3-0) = -3$

$A_{12} = (-1)^{1+2}\begin{bmatrix} 3 & 0 \\ 5 & -1 \end{bmatrix} = -(-3-0) = 3\ A_{13} = (-1)^{1+3}\begin{bmatrix} 3 & 3 \\ 5 & 2 \end{bmatrix} = 6-15 = -9\ A_{21} = (-1)^{2+1}\begin{bmatrix} 0 & 0 \\ 2 & -1 \end{bmatrix} = 0\ A_{22} = (-1)^{2+2}\begin{bmatrix} 1 & 0 \\ 5 & -1 \end{bmatrix} = -1$

$A_{23} = (-1)^{2+3}\begin{bmatrix} 1 & 0 \\ 5 & 2 \end{bmatrix} = -2\ A_{31} = (-1)^{3+1}\begin{bmatrix} 0 & 0 \\ 3 & 0 \end{bmatrix} = 0$

$A_{32} = (-1)^{3+2}\begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix} = 0\ A_{33} = (-1)^{3+3}\begin{bmatrix} 1 & 0 \\ 3 & 3 \end{bmatrix} = 3$

$\therefore adjA$

$= \begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$ Hence, $A^{-1} = \frac{1}{|A|}(adjA) = -\frac{1}{3}\begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$

9. $\begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix}$

SOLUTION

Let $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix}$

Then, $|A| = \begin{vmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{vmatrix} = 2(-1-0)-1(4-0)+3(8-7) = -2-4+3 = -3 \neq 0.$

So, A is a non-singular matrix and therefore, it is invertible. Let A_{ij} be cofactor of a_{ij} in A. Then, the cofactors of elements of A are given by $A_{11} = (-1)^{1+1}\begin{bmatrix} -1 & 0 \\ 2 & -1 \end{bmatrix} = -1$

$A_{12} = (-1)^{1+2}\begin{bmatrix} 4 & 0 \\ -7 & 1 \end{bmatrix} = -4\ A_{13} = (-1)^{1+3}\begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix} = 8-7 = 1$

$A_{21} = (-1)^{2+1}\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix} = -(1-6) = 5\ A_{22} = (-1)^{2+2}\begin{bmatrix} 2 & 3 \\ -7 & 1 \end{bmatrix} = 2+21 = 23$

$A_{23} = (-1)^{2+3}\begin{bmatrix} 2 & 1 \\ -7 & 2 \end{bmatrix} = -(4+7) = -11\ A_{31} = (-1)^{3+1}\begin{bmatrix} 1 & 3 \\ -1 & 0 \end{bmatrix} = 3\ A_{32} = (-1)^{3+2}\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} = 12$

$A_{33} = (-1)^{3+3}\begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix} = -2-4 = -6 \therefore adjA = \begin{bmatrix} -1 & -4 & 1 \\ 5 & 23 & -11 \\ 3 & 12 & -6 \end{bmatrix} = \begin{bmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{bmatrix}$

Hence, $A^{-1} = \frac{1}{|A|}(adjA) = -\frac{1}{3}\begin{bmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{bmatrix}$

10. $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$

SOLUTION

Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \therefore |A| = 1(8-6)+1(0+9)+2(0-6) = 2+9-12 = -1 \neq 0$

$A_{11} = (-1)^{1+1}\begin{bmatrix} 2 & -3 \\ -2 & 4 \end{bmatrix} = 8-6 = 2\ A_{12} = (-1)^{1+2}\begin{bmatrix} 0 & -3 \\ 3 & 4 \end{bmatrix} = -(0+9) = -9$

$A_{13} = (-1)^{1+3}\begin{bmatrix} 0 & 2 \\ 3 & -2 \end{bmatrix} = 0-6 = -6\ A_{21} = (-1)^{2+1}\begin{bmatrix} -1 & 2 \\ -2 & 4 \end{bmatrix} = -(-4+4) = 0$

$A_{22} = (-1)^{2+2}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = 4-6 = -2\ A_{23} = (-1)^{2+3}\begin{bmatrix} 1 & -1 \\ 3 & -2 \end{bmatrix} = -(-2+3) = -1\ A_{31} = (-1)^{3+1}\begin{bmatrix} -1 & 2 \\ 2 & -3 \end{bmatrix} = 3-4 = -1$

$A_{32} = (-1)^{3+2}\begin{bmatrix} 1 & 2 \\ 0 & -3 \end{bmatrix} = -(-3-0) = 3\ A_{33} = (-1)^{3+3}\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 2+0 = 2 \therefore adj\,A$

$= \begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix}$

Hence, $A^{-1} = \frac{adjA}{|A|} = \frac{1}{-1}\begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$

11. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$

SOLUTION

Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$, then $|A| = 1(-\cos^2\alpha - \sin^2\alpha) = -1 \neq 0. \therefore A^{-1} exists.$

So, A is non-singular matrix and therefore, A is invertible. Let A_{ij}be the cofactors of a_{ij} in A. Then the cofactors of elements of A are given by For adjoint $A, A_{ij} = (-1)^{i+j} M_{ij}$

$A_{11} = (-1)^{1+1} \begin{vmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & -\cos\alpha \end{vmatrix} = -\cos^2\alpha - \sin^2\alpha = -1$

$A_{12} = (-1)^{1+2} \begin{vmatrix} 0 & \sin\alpha \\ 0 & -\cos\alpha \end{vmatrix} = 0, A_{13} = (-1)^{1+3} \begin{vmatrix} 0 & \cos\alpha \\ 0 & \sin\alpha \end{vmatrix} = 0$

$A_{21} = (-1)^{2+1} \begin{vmatrix} 0 & 0 \\ \sin\alpha & -\cos\alpha \end{vmatrix} = 0\ A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 0 \\ 0 & -\cos\alpha \end{vmatrix} = -\cos\alpha\ A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 0 \\ 0 & \sin\alpha \end{vmatrix} = -\sin\alpha$

$A_{31} = (-1)^{3+1} \begin{vmatrix} 0 & 0 \\ \cos\alpha & \sin\alpha \end{vmatrix} = 0\ A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 0 \\ 0 & \sin\alpha \end{vmatrix} = -\sin\alpha$

$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 0 \\ 0 & \cos\alpha \end{vmatrix} = \cos\alpha \therefore adj\,A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$

$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$ Hence, $A^{-1} = \frac{1}{|A|} adj A = \frac{1}{-1} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$

12. Let $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$. Verify that $(AB)^{-1} = B^{-1}A^{-1}$.

SOLUTION

We have, $AB = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix} = \begin{bmatrix} 67 & 87 \\ 47 & 61 \end{bmatrix}$ Since, $|AB| = 67 \times 61 - 47 \times 87 = -2 \neq 0$

So, AB is non-singular matrix and therefore, $(AB)^{-1}$ exists and is given by $(AB)^{-1} = \frac{1}{|AB|} adj(AB) = -\frac{1}{2} \begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -61 & 87 \\ 47 & -67 \end{bmatrix}$

Further, $|A| = 15 - 14 = 1 \neq 0$ and $|B| = 54 - 56 = -2 \neq 0$. So, A and B are both non-singular matrices and therefore, $A^{-1}$and B^{-1}both exist and are given by

$A^{-1} = 1 \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}, B^{-1} = -\frac{1}{2} \begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix} \therefore B^{-1}A^{-1} = -\frac{1}{2} \begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix} \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$

$= -\frac{1}{2} \begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -61 & 87 \\ 47 & -67 \end{bmatrix}$ Hence, $(AB)^{-1} = B^{-1}A^{-1}$.

13. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = O$. Hence, find A^{-1}.

SOLUTION

$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ L.H.S. $= A^2 - 5A + 7I = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} 8 - 15 + 7 & 5 - 5 + 0 \\ -5 + 5 + 0 & 3 - 10 + 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O \Rightarrow A^2 - 5A + 7I = O$

Hence, proved. Now, multiplying by A^{-1} on both sides, we get $(A^{-1}A)A - 5AA^{-1} - 7IA^{-1} = O \Rightarrow IA - 5I + 7A^{-1} = O \Rightarrow A - 5I + 7A^{-1} = O \Rightarrow 7A^{-1} = 5I - A$

$\Rightarrow 7A^{-1} = 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \Rightarrow 7A^{-1} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$\Rightarrow 7A^{-1} = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

14. . For the matrix $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, find the numbers a and b such that $A^2 + aA + bI = 0$.

SOLUTION

We are given that, $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$

Now, $A^2 + aA + bI = O \Rightarrow \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + a\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + b\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = O$

$\Rightarrow \begin{bmatrix} 9+2 & 6+2 \\ 3+1 & 2+1 \end{bmatrix} + \begin{bmatrix} 3a & 2a \\ a & a \end{bmatrix} + \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix} = O$

$\Rightarrow \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 3a+b & 2a \\ a & a+b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 11+3a+b & 8+2a \\ 4+a & 3+a+b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow 4 + a = 0 \Rightarrow a = -4$ Also, $3 + a + b = 0 \Rightarrow b = -3 + 4 \Rightarrow b = 1$ Hence,$a = -4, b = 1$

15. For the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$, show that $A^3 - 6A^2 + 5A + 11I = O$. Hence, find A^{-1} .

SOLUTION

We have $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$

$\therefore A^2 = AA = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1\cdot1+1\cdot1+1\cdot2 & 1\cdot1+1\cdot2+1\cdot(-1) & 1\cdot1+1\cdot(-3)+1\cdot3 \\ 1\cdot1+2\cdot1+(-3)\cdot2 & 1\cdot1+2\cdot2+(-3)\cdot(-1) & 1\cdot1+2\cdot(-3)+(-3)\cdot(-3) \\ 2\cdot1+(-1)\cdot1+3\cdot2 & 2\cdot1+(-1)\cdot2+3\cdot(-1) & 2\cdot1+(-1)\cdot(-3)+3\cdot3 \end{bmatrix}$

$= \begin{bmatrix} 1+1+2 & 1+2-1 & 1-3+3 \\ 1+2-6 & 1+4+3 & 1-6-9 \\ 2-1+6 & 2-2-3 & 2+3+9 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$ and $A^3 = A^2A = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 4\cdot1+2\cdot1+1\cdot2 & 4\cdot1+2\cdot2+1\cdot(-1) & 4\cdot1+2\cdot(-3)+1\cdot3 \\ -3\cdot1+8\cdot1+(-14)\cdot2 & -3\cdot1+8\cdot2+(-14)\cdot(-1) & -3\cdot1+8\cdot(-3)+(-14)\cdot3 \\ 7\cdot1+(-3)\cdot1+14\cdot2 & \cdot1+(-1)\cdot2+3\cdot(-1) & 7\cdot1+(-3)\cdot(-3)+14\cdot3 \end{bmatrix}$

$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix}$ Now, L.H.S. $= A^3 - 6A^2 + 5A + 11I$

$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - 6\begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} + 5\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} + 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} + \begin{bmatrix} -24 & -12 & -6 \\ 18 & -48 & 84 \\ -42 & 18 & -84 \end{bmatrix} + \begin{bmatrix} 5 & 5 & 5 \\ 5 & 10 & -15 \\ 10 & -5 & 15 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$

$= \begin{bmatrix} 8-24+5+11 & 7-12+5+0 & 1-6+5+0 \\ -23+18+5+0 & 27-48+10+11 & -69+84-15+0 \\ 32-42+10+0 & -13+18-5+0 & 58-84+15+11 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O \Rightarrow A^3 - 6A^2 + 5A + 11I = O$ Hence, proved. Now, $A^3 - 6A^2 + 5A + 11I = O$

$\Rightarrow 11I = -A^3 + 6A^2 - 5A \ldots$ (i) Multiplying (i) by A^{-1}, we get $11A^{-1}I = -A^{-1}A^3 + 6A^{-1}A^2 - 5A^{-1}A \Rightarrow 11A^{-1} = -A^2 + 6A - 5I$

$\Rightarrow A^{-1} = -\frac{1}{11}A^2 + \frac{6}{11}A - \frac{5}{11}I \Rightarrow = -\frac{1}{11}\begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} + \frac{6}{11}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} - \frac{5}{11}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \frac{1}{11}\begin{bmatrix} -4+6-5 & -2+6+0 & -1+6+0 \\ 3+6+0 & -8+12-5 & 14-18+0 \\ -7+12+0 & 3-6+0 & -14+18-5 \end{bmatrix}$

$= \frac{1}{11}\begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$

16. If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, verify that $A^3 - 6A^2 + 9A - 4I = O$ and hence, find A^{-1}.

SOLUTION

We have $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \therefore A^2 = AA = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 4+1+1 & -2-2-1 & 2+1+2 \\ -2-2-1 & 1+4+1 & -1-2-2 \\ 2+1+2 & -1-2-2 & 1+1+4 \end{bmatrix} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$ and $A^3 = A^2A = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 12+5+5 & -6-10-5 & 6+5+10 \\ -10-6-5 & 5+12+5 & -5-6-10 \\ 10+5+6 & -5-10-6 & 5+5+12 \end{bmatrix} = \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$ Now, $A^3 - 6A^2 + 9A - 4I$

$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - 6\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} + 9\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 22-36+18-4 & -21+30-9+0 & 21-30+9+0 \\ -21+30-9-0 & 22-36+18-4 & -21+30-9+0 \\ 21-30+9+0 & -21+30-9+0 & 22-36+18-4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$

Hence, $A^3 - 6A^2 + 9A - 4I = O$ Now, $A^3 - 6A^2 + 9A - 4I = O \Rightarrow 4I = A^3 - 6A^2 + 9A$ Multiplying both sides by A^{-1}, we get $\Rightarrow$ $A^{-1} = \frac{1}{4}A^2 - \frac{6}{4}A + \frac{9}{4}I$

$= \frac{1}{4}\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - \frac{6}{4}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} + \frac{9}{4}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \frac{1}{4}\begin{bmatrix} 6-12+9 & -5+6+0 & 5-6+0 \\ -5+6+0 & 6-12+9 & -5+6+0 \\ 5-6+0 & -5+6+0 & 6-12+9 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$

17. Let A be a non-singular square matrix of order 3×3. Then, $|adj\,A|$ is equal to

(A) $|A|$

(B)$|A|^2$

(C) $|A|^3$

(D) $3|A|$

SOLUTION

(B) : For any n × n matrix A, $\det(adj\,A) = |A|^{n-1}$ (It holds for singular and non-singular matrices.)

18. If A is an invertible matrix of order 2, then det (A^{-1}) is equal to

(A) det (A)

(B) $\dfrac{1}{\det(A)}$

(C) 1

(D) 0

SOLUTION

(B) When A is an invertible matrix of order 2, $AA^{-1} = I_2 = A^{-1}A$, where I_2 is identity matrix of order 2. $\Rightarrow \det(AA^{-1}) = \det I \Rightarrow \det A . \det(A^{-1}) = 1 \Rightarrow \det(A^{-1}) = \dfrac{1}{\det A}, \det A \neq 0$

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