Exercise – 4.4

Write Minors and Cofactors of the elements of following determinants:.

1. (i) $\begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$

(ii) $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$

SOLUTION

(i) Let $P = \begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$

Minor of the element a_{ij} is M_{ij},. Here, $M_{11} = 3, M_{12} = 0, M_{21} = -4, M_{22} = 2$

For cofactors, we know that $P_{ij} = (-1)^{i+j} M_{ij} \therefore P_{11} = 3, P_{12} = -0 = 0, P_{21} = 4, P_{22} = 3$

(ii) Let $P = \begin{vmatrix} a & c \\ b & d \end{vmatrix}$, Minor of the element a_{ij} is M_{ij}, Here $M_{11} = d, M_{12} = b, M_{21} = c, M_{22} = a$

For cofactors, we know that $P_{ij} = (-1)^{i+j} M_{ij} \therefore P_{11} = d, P_{12} = -b, P_{21} = -c, P_{22} = a$

2. (i) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

(ii) $\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$

SOLUTION

(i) Let $P = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$, we have

$M_{11} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1, M_{12} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0, M_{13} = \begin{vmatrix} 0 & 1 \\ 0 & 1 \end{vmatrix} = 0$

$M_{21} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0, M_{22} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1, M_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 0$

$M_{31} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0, M_{32} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 0, M_{33} = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = 1$

For cofactors, we know that $P_{ij} = (-1)^{i+j} M_{ij}$ $P_{11} = 1, P_{12} = 0, P_{13} = 0, P_{21} = 0, P_{22} = 1, P_{23} = 0, P_{31} = 0, P_{32} = 0, P_{33} = 1$

(ii) Let $P = \begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$, we have $M_{11} = \begin{vmatrix} 5 & -1 \\ 1 & 2 \end{vmatrix} = 10 + 1 = 11, M_{12} = \begin{vmatrix} 3 & -1 \\ 0 & 2 \end{vmatrix} = 6, M_{13} = \begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = 3$

$M_{21} = \begin{vmatrix} 0 & 4 \\ 1 & 2 \end{vmatrix} = -4, M_{22} = \begin{vmatrix} 1 & 4 \\ 0 & 2 \end{vmatrix} = 2, M_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

$M_{31} = \begin{vmatrix} 0 & 4 \\ 5 & -1 \end{vmatrix} = -20, M_{32} = \begin{vmatrix} 1 & 4 \\ 3 & -1 \end{vmatrix} = -13$

$M_{33} = \begin{vmatrix} 1 & 0 \\ 3 & 5 \end{vmatrix} = 5$ For cofactors, we know that $P_{ij} = (-1)^{i+j} M_{ij}$,

$P_{11} = 11, P_{12} = -6, P_{13} = 3, P_{21} = 4, P_{22} = 2, P_{23} = -1, P_{31} = -20, P_{32} = 13, P_{33} = 5$

3. Using Cofactors of elements of second row, evaluate $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$.

SOLUTION

$\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$ Cofactors of elements of second row are $A_{21} = (-1)^{2+1} \begin{vmatrix} 3 & 8 \\ 2 & 3 \end{vmatrix} = -(9-16) = 7$

$A_{22} = (-1)^{2+2} \begin{vmatrix} 5 & 8 \\ 1 & 3 \end{vmatrix} = 15-8 = 7$ $A_{23} = (-1)^{2+3} \begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} = -(10-3) = -7$ Now, $\Delta = a_{21}A_{21} + a_{22}A_{22} + a_{33}A_{33} + 2\times 7 + 0\times 7 + 1\times(-7)$

$= 14+0-7 = 7$

4. Using Cofactors of elements of third column, evaluate $\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$

. SOLUTION

Let $\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$, Cofactors of elements of third column are

$A_{13} = (-1)^{1+3} \begin{vmatrix} 1 & y \\ 1 & z \end{vmatrix} = z-y, A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & x \\ 1 & z \end{vmatrix} = -(z-x)$ $A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & x \\ 1 & y \end{vmatrix} = y-x = -(x-y)$

Now, $\Delta = a_{13}A_{13} + a_{23}A_{23} + a_{33}A_{33}$ $\Delta = -yz(y-z) - zx(z-x) - xy(x-y)$

$= zy(z-y) + zx(x-z) + xy(y-x)$

$= yz^2 - y^2z + zx^2 - z^2x + xy^2 - x^2y$

$= x^2z - x^2y + xy^2 - xz^2 + yz^2 - y^2z$

$= x^2(z-y) + x(y^2-z^2) + yz(z-y)$

$= (z-y)[x^2 - x(y+z) + yz] = (z-y)\{x(x-y) - z(x-y)\}$

$= (z-y)(x-y)(x-z) = (x-y)(y-z)(z-x)$

5. . If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ and A_{ij} is Cofactors of a_{ij}, then value of Δ is given by

(A) $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33}$

(B) $a_{11}A_{11} + a_{12}A_{21} + a_{13}A_{31}$

(C) $a_{21}A_{11} + a_{22}A_{12} + a_{23}A_{13}$

(D)$a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

SOLUTION

(D) We know that the value of determinant is given by :

$a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

[where A_{ij} is Cofactors of a_{ij},]