Exercise – 4.3

1. Find area of the triangle with vertices at the point given in each of the following:

(i)$(1,0),(6,0),(4,3)$

(ii)$(2,7),(1,1),(10,8)$

(iii)$(-2,-3),(3,2),(-1,-8)$

SOLUTION

(i) Area of triangle $=\frac{1}{2}\begin{vmatrix}1&0&1\\6&0&1\\4&3&1\end{vmatrix}$

$=\frac{1}{2}\left[1\begin{vmatrix}0&1\\3&1\end{vmatrix}-0+1\begin{vmatrix}6&0\\4&3\end{vmatrix}\right]$ [Expanding along R_1]

$=\frac{1}{2}[1(0-3)+1(18-0)]=\frac{15}{2}=7.5$ sq. units.

(ii) Area of triangle $=\frac{1}{2}\begin{vmatrix}2&7&1\\1&1&1\\10&8&1\end{vmatrix}$

$=\frac{1}{2}\left[2\begin{vmatrix}1&1\\8&1\end{vmatrix}-7\begin{vmatrix}1&1\\10&1\end{vmatrix}+1\begin{vmatrix}1&1\\10&8\end{vmatrix}\right]$ [Expanding along R_1]

$=\frac{1}{2}[2(1-8)-7(1-10)+1(8-10)]=\frac{1}{2}[-14+63-2]=\frac{47}{2}=23.5$ sq. units

(iii) Area of triangle $=\frac{1}{2}\begin{vmatrix}-2&-3&1\\3&2&1\\-1&-8&1\end{vmatrix}$

$=\frac{1}{2}\left[-2\begin{vmatrix}2&1\\-8&1\end{vmatrix}+3\begin{vmatrix}3&1\\-1&1\end{vmatrix}+1\begin{vmatrix}3&2\\-1&-8\end{vmatrix}\right]$ [Expanding alongR_1]

$=\frac{1}{2}[-2(2+8)+3(3+1)+1(-24+2)]=\frac{1}{2}[-20+12-22]=\frac{-30}{2}=-15\therefore$ Area $=15$ square units. [Absolute value]

2. Show that points $A(a,b+c),B(b,c+a),C(c,a+b)$ are collinear.

SOLUTION

Consider $\begin{vmatrix}a&b+c&1\\b&c+a&1\\c&a+b&1\end{vmatrix}$

Applying $C_1\to C_1+C_2$, we get $=\begin{vmatrix}a+b+c&b+c&1\\a+b+c&c+a&1\\a+b+c&a+b&1\end{vmatrix}$

Taking $(a+b+c)$ common from C_1, we get $=(a+b+c)\begin{vmatrix}1&b+c&1\\1&c+a&1\\1&a+b&1\end{vmatrix}$

$=(a+b+c)\times 0=0$. Hence, the points are collinear.

3. Find values of k if area of triangle is 4 sq. units and vertices are

(i) $(k,0),(4,0),(0,2)$

(ii) $(-2,0),(0,4),(0,k)$

SOLUTION

(i) Area of triangle $=4$ sq. units Also, area $=\frac{1}{2}\begin{vmatrix}k&0&1\\4&0&1\\0&2&1\end{vmatrix}=\frac{1}{2}[k(0-2)-0+1(8-0)]$ [Expanding alongR_1] $=\frac{1}{2}[-2k+8]=-k+4$ Now, $-k+4=\pm 4\Rightarrow -k+4=4$ or $-k+4=-4\Rightarrow k=0$ or $k=8\Rightarrow k=0,8$

(ii) Area of triangle $= 4$ sq. units Also, area $= \frac{1}{2}\begin{vmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{vmatrix}$ [Expanding alongR_1] $= \frac{1}{2}[-2(4-k)-0+1(0)] = -4+k$

Now, $-4+k = \pm 4 \Rightarrow -4+k = +4$ or $-4+k = -4 \Rightarrow k = 0$ or $k = 8 \Rightarrow k = 0, 8$

4. (i) Find equation of line joining (1, 2) and (3, 6) using determinants.

   (ii) Find equation of line joining (3, 1) and (9, 3) using determinants.

   SOLUTION

   (i) Equation of the line joining (x_1, y_1) and (x_2, y_2) is $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$

   $\Rightarrow \begin{vmatrix} x & y & 1 \\ 1 & 2 & 1 \\ 3 & 6 & 1 \end{vmatrix} = 0$

   $\Rightarrow x(2-6) - y(1-3) + 1(6-6) = 0 \Rightarrow -4x + 2y = 0 \Rightarrow 2x - y = 0$ Hence, $y = 2x$ is the required line.

   (ii) Equation of the line joining (x_1, y_1) and (x_2, y_2) is $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$

   $\Rightarrow \begin{vmatrix} x & y & 1 \\ 3 & 1 & 1 \\ 9 & 3 & 1 \end{vmatrix} = 0 \Rightarrow x(1-3) - y(3-9) + 1(9-9) = 0 \Rightarrow -2x + 6y = 0$

   $\Rightarrow -x + 3y = 0 \Rightarrow x - 3y = 0$ Hence, $x - 3y = 0$ is the required line.

5. If area of triangle is 35 sq. units with vertices (2,– 6), (5, 4) and (k, 4), then k is

   (A) 12

   (B) – 2

   (C) – 12, – 2

   (D) 12, – 2

   SOLUTION

   (D) Area of triangle $= \frac{1}{2}\begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix} = \pm 35 \Rightarrow \frac{1}{2}[2(4-4) + 6(5-k) + 1(20-4k)] = \pm 35$

   $\Rightarrow \frac{1}{2}[30 - 6k + 20 - 4k] = \pm 35 \Rightarrow \frac{1}{2}[50 - 10k] = \pm 35 \Rightarrow 25 - 5k = \pm 35 \Rightarrow 25 - 5k = 35$ or $25 - 5k = -35 \Rightarrow = 5k = 10$ or $5k = 60 \Rightarrow k = -2$ or $k = 12$