Exercise – 4.2

Using the property of determinants and without expanding in Exercises 1 to 5, prove that :.

1. $\begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix} = 0$

SOLUTION

L.H.S. $= \begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix} = \begin{vmatrix} x & a & x \\ y & b & y \\ z & c & z \end{vmatrix} + \begin{vmatrix} x & a & a \\ y & b & b \\ z & c & c \end{vmatrix} = 0$ [If any two rows or columns of a determinant are identical (all corresponding elements are same), the value of determinant is zero].

2. $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$

SOLUTION

L.H.S. $= \begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$ Applying $C_1 \to C_1 + C_2 + C_3$, we get $\begin{vmatrix} a-b+b-c+c-a & b-c & c-a \\ b-c+c-a+a-b & c-a & a-b \\ c-a+a-b+b-c & a-b & b-c \end{vmatrix} = \begin{vmatrix} 0 & b-c & c-a \\ 0 & c-a & a-b \\ 0 & a-b & b-c \end{vmatrix} =$ 0 [as $C_1 = 0$]

3. $\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix} = 0$

SOLUTION

L.H.S. $= \begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix}$ Applying $C_1 \to C_1 + 9C_2$, we get $\begin{vmatrix} 65 & 7 & 65 \\ 75 & 8 & 75 \\ 86 & 9 & 86 \end{vmatrix} = 0$ [If any two rows or columns of a determinant are identical (all corresponding elements are same), the value of determinant is zero].

4. $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0$

SOLUTION

L.H.S. $= \begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = \begin{vmatrix} 1 & bc & ab+ac \\ 1 & ca & bc+ba \\ 1 & ab & ca+cb \end{vmatrix} = 0$

Applying $C_3 \to C_3 + C_2$, we get $\begin{vmatrix} 1 & bc & ab+bc+ac \\ 1 & ca & bc+ca+ab \\ 1 & ab & ca+cb+ab \end{vmatrix} = (ab+bc+ca) \begin{vmatrix} 1 & bc & 1 \\ 1 & ca & 1 \\ 1 & ab & 1 \end{vmatrix}$

$= (ab+bc+ca) \times 0 = 0$ [as $C_1 \sim C_3$]

5. $\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2 \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$

SOLUTION

L.H.S. $= \begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix}$ Applying $R_1 \to R_1 + R_2 + R_3$, we get

L.H.S$= \begin{vmatrix} 2(a+b+c) & 2(p+q+r) & 2(x+y+z) \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2 \begin{vmatrix} a+b+c & p+q+r & x+y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix}$

Applying $R_1 \to R_1 - R_2$, we get L.H.S. $= 2\begin{vmatrix} b & q & y \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix}$

Applying $R_3 \to R_3 - R_1$, we get L.H.S. $= 2\begin{vmatrix} b & q & y \\ c+a & r+p & z+x \\ a & p & x \end{vmatrix}$

Applying$R_2 \to R_2 - R_3$, we get L.H.S. $=2\begin{vmatrix} b & q & y \\ c & r & z \\ a & p & x \end{vmatrix}$

Interchanging $R_1 \leftrightarrow R_2$, we get L.H.S. $=-2\begin{vmatrix} a & p & x \\ c & r & z \\ b & q & y \end{vmatrix}$

Interchanging $R_2 \leftrightarrow R_3$, we get LH.S. $= 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$ = R.H.S. By using properties of determinants, in Exercises 6 to 14 show that:

6. $\begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix} = 0$

SOLUTION

Applying $R_1 \leftrightarrow R_2, R_2 \leftrightarrow R_3$, we get $\begin{vmatrix} -a & 0 & -c \\ b & c & 0 \\ 0 & a & -b \end{vmatrix}$ Applying $R_1 \leftrightarrow \dfrac{-R_1}{a}$, we get $\begin{vmatrix} 1 & 0 & c/a \\ b & c & 0 \\ 0 & a & -b \end{vmatrix}$

Applying $R_2 \to R_2 - bR_1$, we get $\begin{vmatrix} 1 & 0 & c/a \\ 0 & c & -bc/a \\ 0 & a & -b \end{vmatrix}$

Applying$R_2 \to \dfrac{R_2}{c}$, we get $\begin{vmatrix} 1 & 0 & c/a \\ 0 & 1 & -b/a \\ 0 & a & -b \end{vmatrix}$

Applying $R_3 \to R_3 - aR_2$, we get $\begin{vmatrix} 1 & 0 & c/a \\ 0 & 1 & -b/a \\ 0 & 0 & 0 \end{vmatrix} = 0$ (since each element of R_3 is 0)

7. $\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4a^2b^2c^2$

SOLUTION

$\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix}$

Taking a, b and c common from R_1, R_2 and R_3respectively, we get $abc\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix}$

Applying $R_1 \to R_1 + R_3$,we get $abc\begin{vmatrix} 0 & 2b & 0 \\ a & -b & c \\ a & b & -c \end{vmatrix}$

Expanding along R_1, we get $(abc)(-2b)\begin{vmatrix} a & c \\ a & -c \end{vmatrix} = (abc)(-2b)[-ac-ac] = (abc)(4abc) = 4a^2b^2c^2$ Hence, proved.

8. (i) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a).$

SOLUTION

L.H.S. $= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

Applying $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$, we get $\begin{vmatrix} 0 & a-b & a^2-b^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix} = \begin{vmatrix} 0 & (a-b) & (a-b)(a+b) \\ 0 & (b-c) & (b-c)(b+c) \\ 1 & c & c^2 \end{vmatrix}$

Taking out $(a-b)$ and $(b-c)$ common fromR_1 andR_2 respectively, we get $(a-b)(b-c)\begin{vmatrix} 0 & 1 & a+b \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$

Applying $R_1 \to R_1 - R_2$, we get $(a-b)(b-c)\begin{vmatrix} 0 & 0 & a-c \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$ Taking out $(a-c)$ common from R_1, we get $(a-b)(b-c)(a-c)\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$

Expanding along R_1, we get $(a-b)(b-c)(a-c)1(0-1) = (a-b)(b-c)(c-a) =$ R.H.S. Hence, proved.

(ii) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

SOLUTION

L.H.S. $= \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$ Applying $C_1 \to C_1 - C_2$ and $C_2 \to C_2 - C_3$, we get $\begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ a^3-b^3 & b^3-c^3 & c^3 \end{vmatrix}$

Taking out $(a-b)$ and $(b-c)$ common from $C_1$and C_2 respectively, we get $(a-b)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & c \\ a^2+ab+b^2 & b^2+c^2+bc & c^3 \end{vmatrix}$

Applying $C_1 \to C_1 - C_2$, we get $(a-b)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & c \\ a^2+ab+b^2-b^2-c^2-bc & b^2+c^2+bc & c^3 \end{vmatrix} = (a-b)(b-c)\begin{vmatrix} 0 \\ 0 \\ a^2-c^2+b(a-c) & b^2+ \end{vmatrix}$

$= (a-b)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & c \\ (a-c)(a+b+c) & b^2+c^2+bc & c^3 \end{vmatrix}$

Taking out $(a-c) \times (a+b+c)$ common from C_1, we get $(a-b)(b-c)(a-c)(a+b+c)\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & c \\ 1 & b^2+c^2+bc & c^3 \end{vmatrix}$

Expanding along R_1, we get $(a-b)(b-c)(a-c)(a+b+c)(-1) = (a-b)(b-c)(c-a)(a+b+c) =$ R.H.S. Hence, proved.

9. $\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$

SOLUTION

Let L.H.S. $\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix}$

Multiplying R_1, R_2 and R_3 by x, y and z respectively, we get $\Delta = \frac{1}{xyz}\begin{vmatrix} x^2 & x^3 & xyz \\ y^2 & y^3 & xyz \\ z^2 & z^3 & xyz \end{vmatrix} = \frac{xyz}{xyz}\begin{vmatrix} x^2 & x^3 & 1 \\ y^2 & y^3 & 1 \\ z^2 & z^3 & 1 \end{vmatrix}$ [Taking xyz common from C_3]

Applying $C_2 \leftrightarrow C_3$,we get $-\begin{vmatrix} x^2 & 1 & x^3 \\ y^2 & 1 & y^3 \\ z^2 & 1 & z^3 \end{vmatrix}$

Applying $C_1 \leftrightarrow C_2$, we get $\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix}$

Applying $R_1 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$, we get $\begin{vmatrix} 1 & x^2 & x^3 \\ 0 & y^2 - x^2 & y^3 - x^3 \\ 0 & z^2 - x^2 & z^3 - x^3 \end{vmatrix}$

Taking $(y-x)$ and $(z-x)$ common from R_2 and R_3respectively, we get $(y-x)(z-x)\begin{vmatrix} 1 & x^2 & x^3 \\ 0 & y+x & y^2+yx+x^2 \\ 0 & z+x & z^2+zx+x^2 \end{vmatrix}$

Applying $R_2 \rightarrow R_2 - R_3$, we get $(y-x)(z-x)\begin{vmatrix} 1 & x^2 & x^3 \\ 0 & y-z & y^2-z^2+x(y-z) \\ 0 & z+x & z^2+zx+x^2 \end{vmatrix}$

Taking $(y-z)$ common from R_2 , we get $(y-x)(z-x)(y-z)\begin{vmatrix} 1 & x^2 & x^3 \\ 0 & 1 & x+y+z \\ 0 & z+x & z^2+zx+x^2 \end{vmatrix}$

Expanding along C_1,we get $= (y-x)(z-x)(y-z)\left[\left(z^2+zx+x^2\right)-(z+x)(x+y+z)\right] = (y-x)(z-x)(y-z)\left[z^2+zx+x^2-zx-zy-z^2 -\right.$
$= (y-x)(z-x)(y-z)[-xy-xz-zy] = (x-y)(y-z)(z-x)(xy+yz+zx) = R.H.S.$ Hence, proved.

10. (i) $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$

SOLUTION

L.H.S. $= \begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$, we get $\begin{vmatrix} 5x+4 & 2x & 2x \\ 5x+4 & x+4 & 2x \\ 5x+4 & 2x & x+4 \end{vmatrix} = (5x+4)\begin{vmatrix} 1 & 2x & 2x \\ 1 & x+4 & 2x \\ 1 & 2x & x+4 \end{vmatrix}$

Applying $R_1 \rightarrow R_1 - R_2$, we get $(5x+4)\begin{vmatrix} 0 & x-4 & 0 \\ 1 & x+4 & 2x \\ 1 & 2x & x+4 \end{vmatrix}$

Expanding along R_1, we get $(5x+4)\left[-(x-4)\begin{vmatrix} 1 & 2x \\ 1 & x+4 \end{vmatrix}\right] = (5x+4)[-(x-4)(x+4-2x)]$

$= (5x+4)[-(x-4)(-x+4)] = (5x+4)(4-x)(4-x) = (5x+4)(4-x)^2 = R.H.S.$ Hence, proved.

(ii) $\begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix} = k^2(3y+k)$

SOLUTION

L.H.S.$\begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix}$ Applying $C_1 \rightarrow C_1 + C_2 + C_3$,we get $\begin{vmatrix} 3y+k & y & y \\ 3y+k & y+k & y \\ 3y+k & y & y+k \end{vmatrix} = (3y+k)\begin{vmatrix} 1 & y & y \\ 1 & y+k & y \\ 1 & y & y+k \end{vmatrix}$ Applying $R_1 \rightarrow R_1 - R_2$, we get $(3y+k)\begin{vmatrix} 0 & -k & 0 \\ 1 & y+k & y \\ 1 & y & y+k \end{vmatrix} = (3y+k)\left[k\begin{vmatrix} 1 & y \\ 1 & y+k \end{vmatrix}\right] = (3y+k)k[y+k-y] = (3y+k)k^2 =$ R.H.S.
Hence, proved.

11. (i) $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

SOLUTION

L.H.S. $= \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$ Applying $R_1 \to R_1+R_2+R_3$, we get $\begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$ Taking $(a+b+c)$ common from R_1, we get $(a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$ Applying $C_1 \to C_1 - C_2$, we get $(a+b+c)\begin{vmatrix} 0 \\ b+c+a & b-c- \\ 0 \end{vmatrix}$

Taking $(a+b+c)$ common from C_1, we get $(a+b+c)^2\begin{vmatrix} 0 & 1 & 1 \\ 1 & b-c-a & 2b \\ 0 & 2c & c-a-b \end{vmatrix}$ Applying $C_2 \to C_2 - C_3$, we get $(a+b+c)^2\begin{vmatrix} 0 \\ 1 & -(a- \\ 0 & (a- \end{vmatrix}$

Taking $(a+b+c)$ common from C_2, we get $(a+b+c)^3\begin{vmatrix} 0 & 0 & 1 \\ 1 & -1 & 2b \\ 0 & 1 & c-a-b \end{vmatrix}$ Expanding along R_1, we get $(a+b+c)^3 =$ R.H.S.

Hence, proved.

(ii) $\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = 2(x+y+z)^3$

SOLUTION

L.H.S.$= \begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix}$

Applying $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$, we get $\begin{vmatrix} (x+y+z) & -(x+y+z) & 0 \\ 0 & x+y+z & -(x+y+z) \\ z & x & z+x+2y \end{vmatrix}$

Taking $(x+y+z)$ common from R_1 and R_2, we get $(x+y+z)^2\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ z & x & z+x+2y \end{vmatrix}$

Applying $C_2 \to C_1 + C_2$, we get $(x+y+z)^2\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ z & x+z & z+x+2y \end{vmatrix}$

Applying $C_3 \to C_2 + C_3$, we get $(x+y+z)^2\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ z & x+z & 2(x+y+z) \end{vmatrix}$

$= 2(x+y+z)^2 \cdot (x+y+z)$

[Determinant of triangular matrix is product of its diagonal elements] $= 2(x+y+z)^3 =$ R.H.S. Hence, proved.

12. $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2.$

SOLUTION

L.H.S. : $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$

Applying $C_1 \to C_1 + C_2 + C_3$, we get $= \begin{vmatrix} 1+x+x^2 & x & x^2 \\ 1+x+x^2 & 1 & x \\ 1+x+x^2 & x^2 & 1 \end{vmatrix}$

Taking $(1+x+x^2)$ common from C_1, we get $= (1+x+x^2)\begin{vmatrix} 1 & x & x^2 \\ 1 & 1 & x \\ 1 & x^2 & 1 \end{vmatrix}$

Applying $C_1 \to C_1 - C_2$, we get $= (1+x+x^2)\begin{vmatrix} 1-x & x & x^2 \\ 0 & 1 & x \\ 1-x^2 & x^2 & 1 \end{vmatrix}$

Taking $(1-x)$ common from C_1, we get $= (1+x+x^2)(1-x)\begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & x \\ 1+x & x^2 & 1 \end{vmatrix}$

Expanding along C_1, we get $= (1+x+x^2)(1-x)\left[1\begin{vmatrix} 1 & x \\ x^2 & 1 \end{vmatrix} + (1+x)\begin{vmatrix} x & x^2 \\ 1 & x \end{vmatrix}\right] = (1-x^3)[(1-x^3)+(1+x)(x^2-x^2)] = (1-x^3)^2$ =R.H.S.

Hence, proved.

13. $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^2$

SOLUTION

L.H.S. $= \begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$

Applying $C_1 \to C_1 - bC_3$ and$C_2 \to C_2 + aC_3$, we get $= \begin{vmatrix} 1+a^2+b^2 & 0 & -2b \\ 0 & 1+a^2+b^2 & 2a \\ b(1+a^2+b^2) & -a(1+a^2+b^2) & 1-a^2-b^2 \end{vmatrix}$

Taking $(1+a^2+b^2)$common from C_1 andC_2, we get $= (1+a^2+b^2)^2\begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ b & -a & 1-a^2-b^2 \end{vmatrix}$

Applying $R_3 \to R_3 - bR_1$, we get $= (1+a^2+b^2)^2\begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ 0 & -a & 1-a^2+b^2 \end{vmatrix}$

Expanding along C_1, we get $= (1+a^2+b^2)^2[1((1-a^2+b^2+2a^2)] = (1+a^2+b^2)^2(1+a^2+b^2) = (1+a^2+b^2)^3$ = R.H.S.
Hence, proved.

14. $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$

$= 1+a^2+b^2+c^2.$

SOLUTION

L.H.S. = $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$ Applying $C_1 \to C_1+C_2+C_3$, we get

$L.H.S. = \begin{vmatrix} 1+a(a+b+c) & ab & ac \\ 1+b(a+b+c) & b^2+1 & bc \\ 1+c(a+b+c) & cb & c^2+1 \end{vmatrix}$

By property 5 and taking $(a+b+c)$common from C_1 in determinant II, we get

L.H.S.: $= \begin{vmatrix} 1 & ab & ac \\ 1 & b^2+1 & bc \\ 1 & cb & c^2+1 \end{vmatrix} + (a+b+c)\begin{vmatrix} a & ab & ac \\ b & b^2+1 & bc \\ c & cb & c^2+1 \end{vmatrix}$

Changing rows into columns, we have L.H.S.: $= \begin{vmatrix} 1 & 1 & 1 \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} + (a+b+c)\begin{vmatrix} a & b & c \\ ab & b^2+1 & cb \\ ac & bc & c^2+1 \end{vmatrix}$

For determinant I we apply, $C_1 \to C_1 - C_2, C_2 \to C_2 - C_3$ and for determinant II we take out a common from C_1, we get

L.H.S.: $\begin{vmatrix} 0 & 0 & 1 \\ ab-b^2-1 & b^2+1-cb & bc \\ ac-bc & bc-c^2-1 & c^2+1 \end{vmatrix} + a(a+b+c)\begin{vmatrix} 1 & b & c \\ b & b^2+1 & bc \\ c & bc & c^2+1 \end{vmatrix}$

Applying $C_2 \to C_2 - bC_1$ and $C_3 \to C_3 - cC_1$ in determinant II, we get

$\begin{vmatrix} 0 & 0 & 1 \\ ab-b^2-1 & b^2+1-cb & bc \\ ac-bc & bc-c^2-1 & c^2+1 \end{vmatrix} + a(a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ b & 1 & 0 \\ c & 0 & 1 \end{vmatrix}$ Expanding along R_1, we have

L.H.S. $= 1[(ab-b^2-1)(bc-c^2-1)-(ac-bc)(b^2+1-cb)]+a(a+b+c)$
$= [(ab^2c-abc^2-ab-b^3c+b^2c^2+b^2-bc+c^2+1-(ab^2c+ac-abc^2-b^3c-bc+b^2c^2)]+a(a+b+c)$

$= ab^2c - abc^2 - ab - b^3c + b^2c^2 + b^2 - bc + c^2 + 1 - ab^2c - ac + abc^2 + b^3c + bc - b^2c^2 + a(a+b+c)$

$= -ab + b^2 + c^2 + 1 - ac + a^2 + ab + ac = 1 + a^2 + b^2 + c^2 =$ R.H.S.

Hence, proved.

Choose the correct answer in Exercise 15 and 16.

15. Let A be a square matrix of order 3×3, then $|kA|$ equal to

(A) $k|A|$

(B) $k^2|A|$

(C) $k^3|A|$

(D) $3k|A|$

SOLUTION

(C) If A is a square matrix of order n, then $|kA| = k^n|A|$, where k is scalar.

16. Which of the following is correct ? (A) Determinant is a square matrix.

(B) Determinant is a number associated to a matrix.

(C) Determinant is a number associated to a square matrix.

(D) None of these

SOLUTION

(C) Determinant is a number associated to a square matrix.

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   ★ Sample Papers Class XII (Twenty Papers Fully Solved- includes 2020 solved paper)
   ★ Sample Papers Class X (Twenty Papers Fully Solved -includes 2020 solved paper)
4. I.C.S.E. & I.S.C.
   ★Work-Book Class XII (Fully Solved)
   ★ Chapter Test Papers Class XII (Fully Solved)
   ★ Sample Papers Class XII (Twenty Papers Fully Solved -includes 2020 solved paper)
   ★ Sample Papers Class X (Twenty Papers Fully Solved -includes 2020 solved paper)
5. Practice Papers for SAT -I Mathematics (15 Papers - Fully Solved)
6. SAT - II Subject Mathematics (15 Papers - Fully Solved)