

NCERT - Exercise - 4.1

Evaluate the determinants in Exercises 1 and 2.

1. $\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix}$

SOLUTION

$$\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix} = 2(-1) - (-5)(4) = -2 + 20 = 18$$

2. $\begin{vmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{vmatrix}$

SOLUTION

$$\begin{vmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{vmatrix} = \cos \theta \times \cos \theta - (\sin \theta) \times (-\sin \theta) = \cos^2 \theta + \sin^2 \theta = 1$$

3. $\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix}$

SOLUTION

$$\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix} = (x^2 - x + 1)(x + 1)(x + 1)(x - 1) = x^3 + 1 - (x^2 - 1) = x^3 + 1 - x^2 + 1 = x^3 - x^2 + 2$$

4. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then show that $|2A| = 4|A|$.

SOLUTION

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix} \therefore 2A = \begin{bmatrix} 2 & 4 \\ 8 & 4 \end{bmatrix}$$

$$\text{L.H.S.} = |2A| = \begin{vmatrix} 2 & 4 \\ 8 & 4 \end{vmatrix} = 8 - 32 = -24$$

$$\text{R.H.S.} = 4|A| = 4 \begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix} = 4(2 - 8) = -24 \text{ Hence, } |2A| = 4|A|$$

5. If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$, then show that $|3A| = 27|A|$.

SOLUTION

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix} \Rightarrow 3A = 3 \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{bmatrix}$$

$$\text{L.H.S.} = |3A| = \begin{vmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{vmatrix} = 3 \begin{vmatrix} 3 & 6 \\ 0 & 12 \end{vmatrix} - 0 \begin{vmatrix} 0 & 6 \\ 0 & 12 \end{vmatrix} + 3 \begin{vmatrix} 0 & 3 \\ 0 & 0 \end{vmatrix}$$

$$= 3 \times 36 - 0 + 3(0) = 108$$

$$\text{R.H.S.} = 27|A| = 27 \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{vmatrix} = 24 \left[1 \begin{vmatrix} 1 & 2 \\ 0 & 4 \end{vmatrix} - 0 \begin{vmatrix} 0 & 2 \\ 0 & 4 \end{vmatrix} + 1 \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} \right]$$

$$= 24[1(4) - 0 + 0] = 108 \text{ Hence, } |3A| = 27|A|$$

6. Evaluate the determinants

Determinants

$$(a) \begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$$

SOLUTION

$$= 3(0 - 5) + 1(0 + 3) - 2 \times 0 = -15 + 3 = -12.$$

$$(b) \begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$$

SOLUTION

$$\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix} = 3 \begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} + 4 \begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} + 5 \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = 3(1 + 6) + 4(1 + 4) + 5(3 - 2) = 21 + 20 + 5 = 46.$$

$$(c) \begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix}$$

SOLUTION

$$\begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix} = 0 \begin{vmatrix} 0 & -3 \\ 3 & 0 \end{vmatrix} - 1 \begin{vmatrix} -1 & -3 \\ -2 & 0 \end{vmatrix} + 2 \begin{vmatrix} -1 & 0 \\ -2 & 3 \end{vmatrix} = 0 - 1(0 - 6) + 2(-3) = 6 - 6 = 0$$

$$(d) \begin{vmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix}$$

SOLUTION

$$\begin{vmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix} = 2 \begin{vmatrix} 2 & -1 \\ -5 & 0 \end{vmatrix} + 1 \begin{vmatrix} 0 & -1 \\ 3 & 0 \end{vmatrix} - 2 \begin{vmatrix} 0 & 2 \\ 3 & -5 \end{vmatrix} = 2(0 - 5) + 1(0 + 3) - 2(0 - 6) = -10 + 3 + 12 = 5.$$

7. If $A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$, find $|A|$.

SOLUTION

$$A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix} \Rightarrow |A| = \begin{vmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{vmatrix} = 1 \begin{vmatrix} 1 & -3 \\ 4 & -9 \end{vmatrix} - 1 \begin{vmatrix} 2 & -3 \\ 5 & -9 \end{vmatrix} - 2 \begin{vmatrix} 2 & 1 \\ 5 & 4 \end{vmatrix}$$

$$= 1(-9 + 12) - 1(18 + 15) - 2(8 - 5) = 3 + 3 - 6 = 0.$$

8. Find the values of x , if

$$(i) \begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$$

SOLUTION

$$\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix} \Rightarrow 2 - 20 = 2x^2 - 24 \Rightarrow -18 = 2x^2 - 24 \Rightarrow 2x^2 = 24 - 18 \Rightarrow 2x^2 = 6 \Rightarrow x^2 = 3 \Rightarrow x = \pm\sqrt{3}.$$

$$(ii) \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$$

SOLUTION

$$\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix} \Rightarrow 2 \times 5 - 4 \times 3 = 5 \times x - 2 \times 3 \Rightarrow 10 - 12 = 5x - 6x \Rightarrow -2 = -x \Rightarrow x = 2.$$

8. If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$, then x is equal to

- (A) 6
- (B) ± 6
- (C) -6

(D) 0

SOLUTION

$$(B) \begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix} \Rightarrow x^2 - 36 = 36 - 36 \Rightarrow x^2 = 36 \Rightarrow x = \pm 6$$



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