DELHI BOARD [2008 SET - I CBSE XII MATHEMATICS]

Instructions

1. **All** questions are compulsory .
2. The question paper consists of 29 questions into three sections A,B and C. Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section C comprises of 7 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice . However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculator is not permitted.

SECTION -A

1. If f(x) =x+7, and g(x) = x-7, $x \in R$ find (fog)(7)

   Answer : 7

2. Evaluate $sin[\frac{\pi}{3} - sin^{-1}(-\frac{1}{2})]$

3. Find the value of x and y is :

   $2\begin{pmatrix}1 & 3\\ 0 & x\end{pmatrix} + \begin{pmatrix}y & 0\\ 1 & 2\end{pmatrix} = \begin{pmatrix}5 & 6\\ 1 & 8\end{pmatrix}$

   Answer : x = 3, y = 3

4. Evaluate $\begin{vmatrix}a+ib & c+id\\ -c+id & a-ib\end{vmatrix}$

   Answer : $a^2+b^2+c^2+d^2$

5. Find the Co-factor of a_{12} in the following $\begin{vmatrix}2 & -3 & 5\\ 6 & 0 & 4\\ 1 & 5 & -7\end{vmatrix}$

   Answer : 46

6. Evaluate $\int \frac{x^2}{1+x^3}dx$

   Answer : $\frac{1}{3}log|1+x^3|+c$

7. Evaluate $\int_0^1 \frac{dx}{1+x^2}$

   Answer : $\frac{\pi}{4}$

8. Find a unit vector in the direction of $\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$

   Answer : $\frac{3}{7}\hat{i} - \frac{2}{7}\hat{j} + \frac{6}{7}\hat{k}$

9. Find the angle between the vectors $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$

   Answer : $cos^{-1}(-\frac{1}{3})$

10. For what value of λ are the vectors $\vec{a} = 2\hat{i} + \lambda j + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ perpendicular to each other ?

    Answer : $\frac{5}{2}$

SECTION B

11. (a) Is the binary operation * defined on set N, given by $a * b = \frac{a+b}{2}$ for all, $a, b \in Q$. commutative.

    (b) Is the above binary operation associative.

    Answer : Yes, No

12. Prove the following

$$tan^{-1}\frac{1}{3} + tan^{-1}\frac{1}{5} + tan^{-1}\frac{1}{7} + tan^{-1}\frac{1}{8} = \frac{\pi}{4}$$

13. Let A = $\begin{pmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{pmatrix}$ Express A as sum of two matrices such that one is symmetric and the other is skew symmetric.

    Answer : $\begin{pmatrix} 3 & 3 & 5/2 \\ 3 & 1 & 9/2 \\ 5/2 & 9/2 & 7 \end{pmatrix} + \begin{pmatrix} 0 & -1 & 5/2 \\ 1 & 0 & -3/2 \\ -5/2 & 3/2 & 0 \end{pmatrix}$

14. For what value of k is the following function continuous at x = 2 ?

$$f(x) = \begin{cases} 2x+1; x < 2 \\ k; x = 2 \\ 3x-1; x > 2 \end{cases}$$

    Answer : 5

15. Differentiate the following w.rt. x.

$$tan^{-1}\left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right)$$

    Answer : $\frac{1}{2\sqrt{1-x^2}}$

16. Find the equation of tangent to the curve $x = sin3t, y = cos2t$, at t = $\frac{\pi}{4}$

    Answer : $2\sqrt{2}x - 3y - 2 = 0$

17. Evaluate

$$\int_0^{\pi} \frac{xsinx}{1+cos^2 x} dx$$

    Answer : $\frac{\pi^2}{4}$

18. Solve the following differential equation

$$(x^2 - y^2)dx + 2xydy = 0$$

given that y =1, when x = 1

Answer : $x^2 + y^2 = 2x$

OR

Solve the following differential equation

$$\frac{dy}{dx} = \frac{x(2y-x)}{x(2y+x)}$$

if y =1 , when x = 1

Answer : $\frac{1}{2}log2 + \frac{3}{\sqrt{7}}tan^{-1}\frac{3}{\sqrt{7}}$

19. Solve the following differential equation

$$cos^2 x\frac{dy}{dx} + y = tanx$$

Answer : $y = (tanx - 1) + ce^{-tanx}$

20. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$ find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a}.\vec{c} = 3$

Answer : $\vec{c} = \frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$

OR

If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $|\vec{a}| = 3, |\vec{b}| = 5$ and $|\vec{c}| = 7$ show that angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$

21. Find the shortest distance between the following lines

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$$

and

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$$

Answer : $2\sqrt{29}$

OR

Find the point on the line

$$\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$$

at a distance $3\sqrt{2}$ from the point $(1,2,3)$

Answer : (-2,-1,3) , or (56/17, 43/17, 111/17)

22. A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability distribution of number of successes.

Answer :

x	0	1	2	3	4
P(x)	625 / 1296	500/1296	150 /1296	20/1296	1/1296

SECTION - C

23. Using properties of determinants prove the following

$$\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{vmatrix} = (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma)$$

24. Show that the rectangle of maximum area that can be inscribed in a circle is a square .

OR

Show that the height of the cylinder of maximum volume that can be inscribed in a cone of height h is $\frac{1}{3}h$

25. Using integration find the area of the region bounded by parabola$y^2 = 4x$ and the circle $4x^2 + 4y^2 = 9$

Answer : $\frac{\sqrt{2}}{6} + \frac{9}{4}cos^{-1}(\frac{1}{3})$ sq units

26. Evaluate

$$\int_{-a}^{a} \sqrt{\frac{a-x}{a+x}}dx$$

Answer : $a\pi$

27. Find the equation of the plane passing through the point (-1,-1,2) and perpendicular to each of the following planes :

2x+3y -3z =2 and 5x -4y +z =6

OR

Find the equation of the plane passing through the points (3,4,1) and (0,1,0) and parallel to the line $\frac{x+3}{2} = \frac{y-3}{7} = \frac{z-2}{5}$

Answer : 8x-13y +15z +13=0

28. A factory owner purchases two types of machines, A and B for his factory . The requirements and the limitations for the machines are as follows :

Machine	Area occupied	Labour force	Daily output (in units)
A	1000	12 men	60
B	1200	8 men	40

He has maximum area of 9000 m^2 available and 72 skilled labourers who can operate both the machines. How many machines of each type should he buy to maximise the daily output.

Answer : A = 6 , B = 0

29. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident involving a scooter, a car and a truck are 0.01, 0.03 , and 0.15 respectively. One of the insured persons meets with an accident . What is the probability that he is a scooter driver.

Answer : $\frac{1}{52}$