DELHI BOARD [1999 SET - I CBSE XII MATHEMATICS]

Instructions

1. **All** questions are compulsory .
2. The question paper consists of 29 questions into three sections A,B and C. Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section C comprises of 7 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice . However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculator is not permitted.

SECTION -A

1. Evaluate the inetgral : $\int \frac{dx}{50+2x^2}$

   Answer: $\frac{1}{10}\tan^{-1}\left(\frac{x}{5}\right)+C$

2. Find a unit vector perpendicular to both $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + \hat{k}$.

   Answer: $\frac{(-1-8\hat{j}-5\hat{k})}{3\sqrt{10}}$

3. Find λ, if $\vec{a} = 4\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \lambda\hat{i} - 2\hat{j} + 2\hat{k}$ are perpendicular to each other.

   Answer: $\lambda = -1$

4. Construct a 3 2 matrix whose elements in the ith row and jth column are given by $a_{ij} = \frac{(3i+j)}{2}$.

   Answer: $\begin{bmatrix} 2 & \frac{5}{2} \\ \frac{7}{2} & 4 \\ 5 & \frac{11}{2} \end{bmatrix}$

5. Find λ, if $\vec{a} = 3\hat{i} - \hat{j} + 4\hat{k}$ and $\vec{b} = -\lambda\hat{i} + 3\hat{j} + 3\hat{k}$ are perpendicular to each other.

   Answer: $\lambda^{-} = 3$

6. Find a unit vector perpendicular to both $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} - \hat{k}$.

Answer: $\frac{1}{\sqrt{3}}\hat{i} - \frac{1}{5\sqrt{3}}\hat{j} + \frac{7}{5\sqrt{3}}\hat{k}$

6. Evaluate the inetgral : $\int (2x+4)\sqrt{x^2+4x+3}dx$

Answer: $\frac{2}{3}(x^2+4x+3)^{\frac{3}{2}} + C$

7. Construct a 2 x 3 matrix whose elements in the ith row and jth column are given by $a_{ij} = \frac{3i-j}{2}$.

Answer: $a_{ij} = \frac{3i-j}{2}$, i = 1, 2 and j = 1, 2, 3

8. If $A = \begin{bmatrix} 1 & -3 & 2 \\ 2 & 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$, then find the matrix C such that $A+B+C$is a zero matrix.

Answer:$C = \begin{bmatrix} -3 & 4 & -1 \\ -3 & 0 & -1 \end{bmatrix}$

9. . If $A = \begin{bmatrix} 2 & 2 \\ -3 & 1 \\ 4 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 6 & 2 \\ 1 & 3 \\ 0 & 4 \end{bmatrix}$, then find the matrix C, such that A + B + C is a zero matrix. [Delhi 1999]

Answer:$C = \begin{bmatrix} -8 & -4 \\ 2 & -4 \\ -4 & -4 \end{bmatrix}$

10. Construct a 3 2 matrix whose elements in the ith row and jth column are given by $a_{ij} = \frac{(2i-j)}{2}$.

Answer: $\begin{bmatrix} \frac{1}{2} & 0 \\ \frac{3}{2} & 1 \\ \frac{5}{2} & 2 \end{bmatrix}$

SECTION -B

11. Two cards are drawn without replacement from a well shuffled pack of 52 cards. What is the probability that one is a red queen and the other is a king of black colour?

Answer: $\frac{2}{663}$

12. If $y = \tan^{-1}x$, then show that $(1+x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = 0$.

13. If $y = \cot x$, then show that $\frac{d^2y}{dx^2} + 2y\frac{dy}{dx} = 0$.

14. If $y = \tan x$, then show that $\frac{d^2y}{dx^2} = 2y\frac{dy}{dx}$.

15. Discuss the applicability of Rolle s theorem for the function $f(x) = x^{2/3}$ on $[-1\ ,\ 1]$,

Answer : Rolle’s theorem is not applicable

16. Prove, using the properties of determinants $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

17. Solve :$(1+x)(1+y^2)dx+(1+y)(1+x^2)dy=0$.

Answer:$\tan^{-1}\left|\frac{x+y}{1-xy}\right|+\frac{1}{2}\log\left|(1+x^2)(1+y^2)\right|=C$

18. . The slope of the tangent at any point on a curve is λ times the slope of the straight line joining the point of contact to the origin. Formulate the differential equation representing the problem and hence find the equation of the curve.

Answer:$y=x^{\lambda}C$

19. Evaluate the integral : $\int\sqrt{x^2-4x+2}dx$

Answer:$\frac{(x-2)}{2}\sqrt{x^2-4x+2}-\log\left|(x-2)+\sqrt{x^2-4x+2}\right|+C$

20. . Evaluate the integral : $\int\frac{x^2}{x^2-4x+3}dx$

Answer:$x+\frac{9}{2}\log|x-3|-\frac{1}{2}\log|x-1|+C$

OR

. Evaluate the integral : $\int\frac{x^2}{x^2+3x+3}dx$

Answer:$x-\frac{3}{2}\log\left|x^2+3x+3\right|+\sqrt{3}\tan^{-1}\left(\frac{2x+3}{\sqrt{3}}\right)+C$

21. . Evaluate the integral : $\int_0^3(|x|+|x-1|+|x-2|)dx$

Answer:$\frac{19}{2}$

22. Find the cartesian as well as vector equations of the planes through the intersection of the planes $\vec{r}\cdot(2\hat{i}+6\hat{j})+12=0$ and $\vec{r}\cdot(3\hat{i}-\hat{j}+5\hat{k})=0$ which are at unit distance from origin.

Answer:$2x+y+2z+3=0; x-2y+2z=3; \vec{r}\cdot(2\hat{i}+\hat{j}+2\hat{k})+3=0; \vec{r}\cdot(\hat{i}-2\hat{j}+2\hat{k})=3$

OR

Find the cartesian as well as vector equations of the planes through the intersection of the planes $\vec{r}\cdot(2\hat{i}+6\hat{j})+12=0$ and $\vec{r}\cdot(3\hat{i}-\hat{j}+5\hat{k})=0$ which are at unit distance from origin.

Answer:$2x+y+2z+3=0; x-2y+2z=3; \vec{r}\cdot(2\hat{i}+\hat{j}+2\hat{k})+3=0; \vec{r}\cdot(\hat{i}-2\hat{j}+2\hat{k})=3$

SECTION -C

23. An open tank with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water. Show that the cost of the material will be least when the depth of the tank is half of its width.

24. Find the foot of perpendicular from $(0,2,7)$ on the line $\frac{x+2}{-1}=\frac{y-1}{3}=\frac{z-3}{-2}$. [Delhi 1999] Answer: $\left(\frac{-3}{2},\frac{-1}{2},4\right)$

25. . Three balls are drawn without replacement from a bag containing 5 white and 4 red balls. Find the probabilty distribution ofthe number of red balls drawn.

Answer: Now, $P(X=0)=P(\text{No red ball})=P(WWW)=\frac{5}{9}\times\frac{4}{8}\times\frac{3}{7}=\frac{5}{42}$ $P(X=1)=P(\text{ one red and two white balls})=P(RWW)+P(W$

26. Solve the following linear programming problem graphically also Maximize and Minimize $Z = 3x + 5y$

subject to the following constraints.

$3x - 4y + 12 \geq 0$

$2x - y + 2 \geq 0$

$2x + 3y - 12 \geq 0$

$0 \leq x \leq 4$

$y \geq 2$

Answer : Z assumes its minimum value 19 at $x = 3$ and $y = 2$. The maximum value of Z is 42 at $x = 4$ and $y = 6$.

27. Sketch the region common to the circle $x^2 + y^2 = 16$ and the Parabola $x^2 = 6y$. Also, find the area of region using integration.

OR

Using integration, find the area of the region $\{(x,\ y) : x^2 + y^2 \leq 16, x^2 \leq 6y\}$ [Delhi 2010 C]

Answer : $\frac{4}{3}(4\pi + \sqrt{3})$ sq. units

28. .Evaluate the integral : $\int_1^4 (|x-1| + |x-2| + |x-3|)dx$

Answer: $\frac{19}{2}$

OR

.Evaluate the integral : $\int_2^5 |x-2| + |x-3| + |x-4| dx$

Answer: $\frac{19}{2}$

29. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find $A^{-1}$and hence prove that $A^2 - 4A - 5I = 0$.

Our Mathematics E-Books

1. J.E.E. (Join Entrance Exam)
   ★ Chapter Tests (Full Syllabus- Fully Solved)
   ★ Twenty Mock Tests (Full Length - Fully Solved)
2. B.I.T.S.A.T. Twenty Mock Tests (Fully Solved)
3. C.BS.E.
   ★ Work-Book Class XII (Fully Solved)
   ★ Objective Type Questions Bank C.B.S.E. Class XII (Fully Solved)
   ★Chapter Test Papers Class XII (Fully Solved)
   ★ Past Fifteen Years Topicwise Questions (Fully Solved)
   ★ Sample Papers Class XII (Twenty Papers Fully Solved- includes 2020 solved paper)
   ★ Sample Papers Class X (Twenty Papers Fully Solved -includes 2020 solved paper)
4. I.C.S.E. & I.S.C.
   ★Work-Book Class XII (Fully Solved)
   ★ Chapter Test Papers Class XII (Fully Solved)
   ★ Sample Papers Class XII (Twenty Papers Fully Solved -includes 2020 solved paper)
   ★ Sample Papers Class X (Twenty Papers Fully Solved -includes 2020 solved paper)
5. Practice Papers for SAT -I Mathematics (15 Papers - Fully Solved)
6. SAT - II Subject Mathematics (15 Papers - Fully Solved)