CBSE CLASS XII MATHEMATICS - 2006

Instructions

1. **All** questions are compulsory .
2. The question paper consists of 29 questions into three sections A,B and C. Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section C comprises of 7 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice . However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculator is not permitted.

SECTION -A

1. Evaluate : $\int \frac{\sin x}{1+\sin x} dx$

   Answer : $= \sec x - \tan x + x + C$

2. . Evaluate : $\int_{-1}^{1} 5x^4 \sqrt{x^5+1} dx$

   Answer : $\frac{4\sqrt{2}}{3}$

3. The total cost $C(x)$ in rupees associated with the production of x units of an item is given by $C(x) = 0.007x^3 - 0.003x^2 + 15x + 4000$. Find the marginal cost when 17 units are produced.

   Answer : $= Rs 20.967.$

4. The mean and variance of a binomial distribution are 12 and 3 . Find the distribution.

   Answer : $(\frac{1}{4} + \frac{3}{4})^{16}$

5. Find $\frac{dy}{dx}$ if $2x + 3y = sinx$

   Answer : $\frac{\cos x - 2}{3}$

6. Let A $=\{1,2,3\}$ and relation R in A is given by $R = \{(1,1),(1,2),(2,1),(2,3)\}$ Is R a symmetric relation ? Give reasons.

   Answer : No

7. A matrix A of order 3 $\times$3 has determinant 5. What is the value of $|3A|$?

   Answer :135.

8. Form a differential equation representing the given family of curves by eliminating arbitrary constants a and b. $y^2 = a(b^2 - x^2)$

   Answer :$yy_1 = x(y_1^2 + yy_2)$

9. Construct a 2 2 matrix, $A = [a_{ij}]$, whose elements are given by : $a_{ij} = \frac{(i+2j)^2}{2}$

   Answer : $A = \begin{bmatrix} \frac{9}{2} & \frac{25}{2} \\ 8 & 18 \end{bmatrix}$

10. Show that the three lines with direction cosines $\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13}; \frac{4}{13}, \frac{12}{13}, \frac{3}{13}; \frac{3}{13}, \frac{-4}{13}, \frac{12}{13}$ are mutually perpendicular.

SECTION -B

11. If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} + \hat{k}$, $\vec{c} = \hat{i} + \hat{j} + \hat{k}$, then verify that $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$

12. A and B toss a coin alternately one of them tosses a head and wins the game.

    Answer:$\frac{2}{3}$

13. Solve for x : $\tan^{-1}\left(\frac{1-x}{1+x}\right) - \frac{1}{2}\tan^{-1}x = 0,\ x > 0.$

    Answer :$x = \frac{1}{\sqrt{3}}$

14. Find the equation of the plane passing through the points $(1,2,3)$ and $(0,-1,0)$ and parallel to the line $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z}{-3}$.

    Answer :$6x - 3y + z - 3 = 0$

15. Evaluate the inetgral : $\int \frac{x+3}{x^2+4x+3} dx$

    Answer :$\log|x+1| + C$

16. . Evaluate the integral : $\int \frac{\sin(x-\alpha)}{\sin(x+\alpha)} dx$

    Answer : $x\cos 2\alpha - \sin 2\alpha \log|\sin(x+\alpha)| + C$

17. . Evaluate the integral : $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} [\sin|x| - \cos|x|]\, dx$

    Answer : 0

18. If $x = a\sin 2t(1+\cos 2t)$ and $y = b\cos 2t(1-\cos 2t)$, then show that $\left(\frac{dy}{dx}\right)_{at\ t=\frac{\pi}{4}} = \frac{b}{a}$.

OR

If $y \cdot \sqrt{x^2+1} = \log\left[\sqrt{x^2+1} - x\right]$, then show that $(x^2+1)\frac{dy}{dx} + xy + 1 = 0$.

19. Find the point on the curve $y^2 = 4x$ which is nearest to the point $(2, -8)$.

Answer: $(4,4), (4,-4)$

20. The volume of a cube is increasing at the rate of 7cubic centimetres per second. How fast is the surface area of the cube increasing when length of an edge is12 centimetres?

Answer: $\frac{7}{3} cm^2/\sec$

21. Prove that $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ yz & zx & xy \end{vmatrix} = (y-z)(z-x)(x-y)$

OR

Using the properties of determinants, prove that $\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2(a+b)(b+c)(c+a).$

22. Express the following matrix as the sum of a symmetric and skew symmetric matrix $A = \begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$

Answer: $\frac{1}{2}\begin{bmatrix} 2 & -3 & 1 \\ -3 & 16 & 9 \\ 1 & 9 & 10 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & -2 & 5 \\ 2 & 0 & -3 \\ -5 & 3 & 0 \end{bmatrix}$

SECTION -C

23. . Solve the differential equation : $2xy\,dx + (x^2 + 2y^2)dy = 0.$

Answer: $3x^2y + 2y^3 = C$

OR

. Solve the differential equation : $\sin x \frac{dy}{dx} + y\cos x = \cos x \cdot \sin^2 x.$

Answer : $y = \frac{\sin^2 x}{3} + \frac{C}{\sin x}$

24. Five dice are thrown simultaneously. If the occurrence of $3, 4$ or 5 in a single dice is considered as a success, then find the probability of at least 3 successes.

Answer: $\frac{1}{2}$

25. . A furniture dealer, deals only in two items - tables and chairs. He has Rs. 10,000 to invest and a space to store at most 60 pieces. A table costs him Rs. 500 and chair Rs. 200. He can sell a table at profit of Rs. 50' and a chair at a profit of Rs. 15. Assume that he can sell all items that he buys. Using linear programming, formulate the problem for maximum profit and solve it graphically.

Answer :
Tables : 60, Chairs : 0, maximum profit Rs. 1000.

26. Find the area of the region bounded by $y^2 = 4x, x = 1, x = 4$ and x-axis in the first quadrant.

Answer : $\frac{28}{3}$ sq. units

27. A window is in the form of a rectangle surmounted by a semi-circular opening. If the perimeter of the window is20, find the dimension of the window so that the maximum possible light is admitted through the whole opening.

Answer: $\frac{40}{\pi+4}, \frac{20}{\pi+4}$

28. Find the vector equation of a line passing through the point with position vector $(2\hat{i}-3\hat{j}-5\hat{k})$ and perpendicular to the plane $\vec{r}\cdot(6\hat{i}-3\hat{j}+5\hat{k})+2=0$. Also, find the point of intersection of this line and the plane.

Answer : $\vec{r}=2\hat{i}-3\hat{j}-5\hat{k}+\lambda(6\hat{i}-3\hat{j}+5\hat{k}); \left(\frac{76}{35}, \frac{-108}{35}, \frac{-170}{35}\right)$

29. Evaluate the integral using limits of sums : $\int_0^2 (x^2+x+1)\,dx$

Answer : $\frac{20}{3}$

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