



### Instructions

1. All questions are compulsory .
2. The question paper consists of 29 questions into three sections A,B and C. Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section C comprises of 7 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice . However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculator is not permitted.

#### SECTION -A

1. Verify that the given functions (explicit or implicit) is a solution of the corresponding differential equation:  $y = x^2 + 2x + C$  :  
 $y' - 2x - 2 = 0$

Answer:  $y = x^2 + 2x + C$

2. Let \* be a binary operation, defined by  $a * b = 3a + 4b - 2$ . Find  $4 * 5$ .

Answer: 30

3. A random variable X has the following distribution

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	$k^2$	$2k^2$	$7k^2 + k$

Answer:  $k = \frac{1}{10}$

4. Find the values of x, y and z from the following equations:

$$\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$$

Answer:  $x = 1, y = 4, z = 3.$

5. Differentiate :  $\sin(x^2 + 5)$

Answer:  $2x \cos(x^2 + 5)$

6. Find equation of line joining (3, 1) and (9, 3) using determinants.

Answer:  $x - 3y = 0$

7. The total revenue in Rupees received from the sale of  $x$  units of a product is given by  $R(x) = 13x^2 + 2x + 15$ . Find the marginal revenue when  $x = 7$ .

Answer: Rs208 .

8. Evaluate :  $\int \frac{\sin(\tan^{-1}x)}{1+x^2} dx$

Answer:  $-\cos(\tan^{-1}x) + C$

OR

Evaluate :  $\int_0^1 \frac{\tan^{-1}x}{1+x^2} dx$

Answer:  $\frac{\pi^2}{32}$

9. Show that the line joining the origin to the point  $(2, 1, 1)$  is perpendicular to the line determined by the points  $(3, 5, -1), (4, 3, -1)$ .

10. Consider two points  $P$  and  $Q$  with position vectors  $\vec{OP} = 3\vec{a} - 2\vec{b}$  and  $\vec{OQ} = \vec{a} + \vec{b}$ . Find the position vector of a point  $R$  which divides the line joining  $P$  and  $Q$  in the ratio  $2 : 1$  externally.

Answer:  $4\vec{b} - \vec{a}$

SECTION -B

11. The probability that a student entering a university will graduate is 0.4. Find the probability that out of 3 students of the university,

- (a) none will graduate
- (b) only one will graduate
- (c) all will graduate.

Answer: 0.216, 0.432, 0.064

12. Prove that :  $\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right) = \frac{2b}{a}$ .

13. . Evaluate the integral :  $\int \frac{\sin 2x}{a^2 \sin^2 x + b^2 \cos^2 x} dx$

Answer :  $\frac{1}{a^2 - b^2} \log |a^2 \sin^2 x + b^2 \cos^2 x| + C$

OR

Evaluate the integral :  $\int \frac{\sqrt{16 + (\log x)^2}}{x} dx$

Answer :  $\frac{\log x}{2} \sqrt{16 + (\log x)^2} + 8 \log \left| \log x + \sqrt{16 + (\log x)^2} \right| + C$

14. . Evaluate :  $\int_{-5}^0 f(x) dx$  where  $f(x) = |x| + |x+3| + |x+6|$

Answer :  $\frac{73}{2}$

15. Express the vector  $\vec{a} = 5\hat{i} - 2\hat{j} + 5\hat{k}$  as sum of two vectors such that one is parallel to the vector  $\vec{b} = 3\hat{i} + 5\hat{k}$  and the other is perpendicular to  $\vec{b}$ .

Answer:  $6\hat{i} + 2\hat{k}, -\hat{i} - 2\hat{j} + 3\hat{k}$

16. Form the differential equation corresponding to  $y^2 - 2ay + x^2 = a^2$ , where  $a$  is an arbitrary constants.

Answer:  $(x^2 - y^2) \left(\frac{dy}{dx}\right)^2 - 2xy \frac{dy}{dx} = \left(x + y \frac{dy}{dx}\right)^2$

OR

- Solve the differential equation:  $(1 + x^2) \frac{dy}{dx} - 2xy = (x^2 + 2)(x^2 + 1)$ .

Answer:  $y = (x^2 + 1)(x + \tan^{-1}x + C)$

17. The surface area of a spherical bubble is increasing at the rate of  $2cm^2/s$ . Find the rate at which the volume of the bubble is increasing at the instant when its radius is 6cm. Answer:  $6cm^3/sec$

18. Find the equation of the tangent to the curve  $x^2 + 3y = 3$  which is parallel to the line  $y - 4x + 5 = 0$

Answer:  $4x - y = -13$

19. Prove that the line  $\frac{x}{a} + \frac{y}{b} = 1$  is a tangent to the curve  $y = be^{-\frac{x}{a}}$  at the point where the curve crosses y-axis.

20. If  $x = a \left(\frac{1+t^2}{1-t^2}\right)$  and  $y = \frac{2t}{1-t^2}$ . Find  $\frac{dy}{dx}$ .

Answer:  $\frac{1+t^2}{2at}$

OR

- If  $x = a(\theta + \sin \theta), y = a(1 - \cos \theta)$ . Find  $\frac{d^2y}{dx^2}$  at  $\theta = \frac{\pi}{2}$ .

Answer:  $\frac{1}{a}$

21. Using properties of determinants, solve for x. 
$$\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$$

Answer:  $x = 0, 3a$ .

22. If  $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$ , then find k such that,  $A^2 - 8A + kI = O$ .

Answer:  $k = 7$

SECTION -C

23. Find the co-ordinates of the foot of the perpendicular drawn from the point  $A(1, 8, 4)$  to the line joining the points  $B(0, -1, 3)$  and  $C(2, -3, -1)$ .

Answer:  $\left(-\frac{5}{3}, \frac{2}{3}, \frac{19}{3}\right)$

24. . An insurance company insured 2000 scooters and 3000 motorcycles. The probability of an accident involving a scooter is 0.01 and that of a motor cycle is 0.02. An insured vehicle met with an accident. Find the probability that the accidented vehicle was a motor cycle. Answer:  $\frac{3}{4}$

OR

A coin is biased so that the head is 3 times as likely to occur as a tail. If the coin is tossed twice, then find the probability distribution for the number of tails.

Answer:	X	0	1	2
	$P(X)$	$\frac{9}{16}$	$\frac{6}{16}$	$\frac{1}{16}$

25. . Two tailors A and B earn Rs. 150 and Rs. 200 per day respectively. A can stitch 6 shirts and 4 pants per day while B can stitch 10 shirts and 4 pants per day. Form a linear programming problem to minimise the labour cost to produce at least 60 shirts and 32 pants.

**Answer :**

A works for 5 days and B works for 3 days, minimum cost Rs. 1350.

26. Find the area enclosed by the parabola  $y^2 = x$  and the line  $y = x - 2$  and the x-axis.

Answer :  $\frac{9}{2}$  sq. units

27. Evaluate the integral using limits of sums :  $\int_1^2 (2x^2 + 5x) dx$

Answer :  $\frac{73}{6}$

28. Show that the area of the triangle formed by the tangent and the normal at the point  $(a, a)$  on the curve  $y^2(2a - x) = x^3$  and the line  $x = 2a$  is  $\frac{5a^2}{4}$  sq. units

29. If  $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ , find  $A^{-1}$ .  $y + 2z = 4, x + 2y + 3z = 6, 3x + y + z = 4$ .

Answer:  $A^{-1} = -\frac{1}{2} \begin{bmatrix} -1 & 1 & -1 \\ 3 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}; x = 1, y = -2, z = 3$

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