CBSE CLASS XII MATHEMATICS - 2003

Instructions

1. **All** questions are compulsory .
2. The question paper consists of 29 questions into three sections A,B and C. Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section C comprises of 7 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice . However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculator is not permitted.

SECTION -A

1. .Evaluate : $\int_{-\pi}^{\pi} x^{10}\sin^7 x dx$

   Answer :0

2. The position vectors of points A, B, C, D are $\vec{a}, \vec{b}$, $2\vec{a}+3\vec{b}$ and $\vec{a}-2\vec{b}$ respectively. Show that $\overrightarrow{DB} = 3\vec{b}-\vec{a}$ and$\overrightarrow{AC} = \vec{a}+3\vec{b}$.

3. Find the direction cosines of a line which makes equal angles with the coordinate axes.

   Answer :$< \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} >$ or $< -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} >$

4. . If f : R $\to$ R is defined by f(x) $= (3-x^3)^{\frac{1}{3}}$, then find fof(x).

   Answer: $fog(x) = x$

5. Evaluate : $\int \frac{\sin x}{1+\sin x} dx$

   Answer :$= \sec x - \tan x + x + C$

6. If A and B are mutually exclusive events, find $P(A/B)$

   Answer :$P(A/B) = 0$

7. Find values of k if area of triangle is 4 sq. units and vertices are $(-2,0), (0,4), (0,k)$

   Answer :$k = 0, 8$

8. .Differentiate : $\sec(\tan(\sqrt{x}))$

   Answer := $\sec(\tan\sqrt{x})\cdot\tan(\tan\sqrt{x})\cdot\sec^2\sqrt{x}\cdot\dfrac{1}{2\sqrt{x}}$

9. Verify that the given functions (explicit or implicit) is a solution of the corresponding differential equation: $y=\cos x+C : y'+\sin x=0$

   Answer :$y=\cos x+C$ is a solution of the given differential equation.

10. Construct a 2 2 matrix, A = $[a_{ij}]$, whose elements are given by $a_{ij}=\dfrac{(i+j)^2}{2}$.

    Answer : $\begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}$

SECTION -B

11. Let f : R $\to$ R be defined by f(x)=3x$-$7. Show that f is invertible and hence find f^{-1}.

    Answer : $f^{-1}(x)=\dfrac{x+7}{3}$

12. The dot products of a vector with the vectors $\hat{i}+\hat{j}-3\hat{k},\hat{i}+3\hat{j}-2\hat{k}$and $2\hat{i}+\hat{j}+4\hat{k}$ are $0,5$ and 8 respectively. Find the vector.

    Answer :$\hat{i}+2\hat{j}+\hat{k}$

13. A bag contains 5red, 6white and 7 b1ack balls.Two balls are drawn at random. What is the probability that both balls are red or both are black?

    Answer:$\dfrac{31}{153}$

OR

Events A and B are given to be independent. Find $P(B)$, if it is given that $P(A)=0.35, P(A\cup B)=0.60$. Answer:$\dfrac{5}{13}$

14. Find the value of λ, so that the two vectors $2\hat{i}+3\hat{j}-\hat{k}$and $4\hat{i}+6\hat{j}-\lambda\hat{k}$ are (i) parallel (ii) perpendicular to each other.

    Answer :(i) $\lambda=-2$ (ii)$\lambda=26$

15. Prove that $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix}=x^2(x+a+b+c)$.

OR

Using properties of determinants, evaluate the following $\begin{vmatrix} 0 & ab^2 & ac^2 \\ a^2b & 0 & bc^2 \\ a^2c & cb^2 & 0 \end{vmatrix}$

Answer :$2a^3b^3c^3$

16. . Evaluate the integral : $\int\tan^{-1}\sqrt{\dfrac{1-\sin x}{1+\sin x}}dx$

    Answer :$\dfrac{\pi x}{4}-\dfrac{x^2}{4}+C$

17. . Evaluate the integral : $\int x\sqrt{x^4-1}dx$

Answer : $\frac{1}{4}\left[x^2\sqrt{x^4-1}-\log\left|x^2+\sqrt{x^4-1}\right|\right]+C$

18. Evaluate the integral : $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos 2x \cdot \log \sin x\, dx$

Answer : $\frac{1}{4}(\log 2+1)-\frac{\pi}{8}$

19. Solve the following differential equation : $x^2\frac{dy}{dx}=2xy+y^2$

Answer : $y=Cx(x+y)$

OR

Form the differential equation of the following family of curves: $xy=Ae^x+Be^{-x}+x^2$.

Answer : $x\frac{d^2y}{dx^2}+2\frac{dy}{dx}=xy-x^2+2$

20. . Differentiate $\log\left(x+\sqrt{1+x^2}\right)$ w.r.t. x.

Answer : $\frac{1}{\sqrt{1+x^2}}$

OR

If $y=\log\sqrt{\frac{1-\cos x}{1+\cos x}}$, then show that $\frac{dy}{dx}=\cos ecx$.

21. . Differentiate $\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]$ w.r.t. x.

Answer : $\frac{1}{2\sqrt{1-x^2}}$

22. . Using elementary row transformations, find the inverse of the matrix $A=\begin{bmatrix}4 & 5\\ 3 & 4\end{bmatrix}$

Answer: $A^{-1}=\begin{bmatrix}4 & -5\\ -3 & 4\end{bmatrix}$

SECTION -C

23. Find the foot of the perpendicular from the point $(0,2,3)$ on the line $\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}$. Also, find the length of the perpendicular.

Answer : $(2,3,-1);\sqrt{21}$ units

24. *A* and B throw a dice alternately till one of them gets a 6" and wins the game. Find their respective probability of winning, if A starts the game.

Answer: $\frac{6}{11}$

25. Prove that : $\int_0^{\frac{\pi}{2}} \left[\sqrt{\tan x} + \sqrt{\cot x}\right] dx = \sqrt{2}\pi$

OR

Prove that : $\int_{-a}^{a} \sqrt{\frac{a-x}{a+x}} dx = a\pi$

26. An open tank with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water. Show that the cost of the material will be least when the depth of the tank is half of its width.

27. Using integration, find the area of region bounded between the line $x = 2$ and the parabola $y^2 = 8x$.

Answer : $\frac{32}{2}$ sq. units

28. A company manufactures two articles A and B. There are two departments through which these articles are processed :

(i) assembly and

(ii) finishing departments.

The maximum capacity of 1st department is 60 hours in a week and that of the other department is 48 hours in a week. The production of each article A requires 4 hours in assembling and 2 hours in finishing and that of each unit of B requires 2 hours in assembling and 4 hours in finishing. If the profit is Rs 6 for each unit of A and Rs. 8 for each unit of B, then find the number of units of A and B to be produced per week in order to have maximum profit.

Answer : Articles A: 12units, Articles5: 6 units, maximum profit Rs. 120.

29. If $A = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$, $a \neq 1$, then prove that $A^n = \begin{bmatrix} a^n & \frac{b(a^n-1)}{a-1} \\ 0 & 1 \end{bmatrix}$ $n \in N$.

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