

## Instructions

1. **All** questions are compulsory .
2. The question paper consists of 29 questions into three sections A,B and C. Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section C comprises of 7 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice . However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculator is not permitted.

### SECTION -A

1. Construct a  $2 \times 3$  matrix A, whose elements are given by  $a_{ij} = \frac{(i-2j)^2}{2}$ .

Answer:  $a_{ij} = \frac{(i-2j)^2}{2}, i = 1, 2 \text{ and } j = 1, 2, 3$

2. A coin is tossed 3 times and the random variable X represents " number of heads " What values of X can take ?

Answer :X can take value 0,1,2,3

3. If A and B are two independent events such that  $P(A \cup B) = 0.5, P(A) = 0.2$  then find  $P(B)$  .

Answer:  $\frac{3}{8}$

4. If  $f(x) = e^x$  and  $g(x) = \log x$ , show that  $f \circ g = g \circ f$  given  $x > 0$

OR

If  $f(x) = \sqrt{x}, (x > 0)$  and  $g(x) = x^2 - 1$ , find whether  $f \circ g = g \circ f$ .

Answer :No.

5. Find the intervals in which the function  $f$  given by  $f(x) = 2x^2 - 3x$  is strictly decreasing

Answer :  $\left(-\infty, \frac{3}{4}\right)$

6. Differentiate :  $\sin(x^2 + 5)$

$$\text{Answer : } \cos(x^2 + 5)(2x + 0) = 2x\cos(x^2 + 5)$$

7. The points  $(3, -2), (x, 2), (8, 8)$  are collinear, find  $x$  using determinant.

$$\text{Answer : } x = 5$$

8. Evaluate :  $\int_0^2 e^{3-4x} dx$

$$\text{Answer : } \frac{-1}{4} [e^{-5} - e^3]$$

9. Evaluate  $\int \frac{x^3}{x+2} dx$

$$\text{Answer : } \frac{x^3}{3} - x^2 + 4x - 8 \log|x+2| + C$$

10. Find the principal value of  $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ .

$$\text{Answer : } \frac{1}{\sqrt{2}}$$

11. If a line makes angles  $90^\circ, 135^\circ, 45^\circ$  with the  $x, y$  and  $z$ -axes respectively, find its direction cosines.

$$\text{Answer : } \langle \cos 90^\circ, \cos 135^\circ, \cos 45^\circ \rangle \text{ or } \langle -\cos 90^\circ, -\cos 135^\circ, -\cos 45^\circ \rangle \text{ i.e., either } \langle 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle \text{ or } \langle 0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \rangle$$

SECTION -B

12. Using the properties of determinants, prove that 
$$\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2(a+b)(b+c)(c+a).$$

13. If  $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$  and  $\vec{b} = 3\hat{i} + 2\hat{j} - \hat{k}$ , then find  $(\vec{a} + 3\vec{b}) \cdot (2\vec{a} - \vec{b})$ .

$$\text{Answer : } -15$$

14. Evaluate the integral :  $\int \frac{xdx}{1+x+x^2+x^3}$

$$\text{Answer : } \frac{1}{4} \log|x^2+1| - \frac{1}{2} \log|x+1| + \frac{1}{2} \tan^{-1}x + C$$

15. Let  $f : \mathbb{N} \rightarrow \mathbb{N}$  be defined by  $f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$  for all  $n \in \mathbb{N}$  Find whether the function  $f$  is bijective.

$$\text{Answer : } f \text{ is not bijective}$$

16. Evaluate the integral :  $\int e^x \cos bx dx$

$$\text{Answer : } \frac{e^{ax}}{a^2 + b^2} (b \sin bx + a \cos bx) + C$$

OR

Evaluate the integral :  $\int_1^2 \left( \frac{x-1}{x^2} \right) e^x dx$

Answer :  $\frac{e^2 - 2e}{2}$

17. Five dice are thrown simultaneously. If the occurrence of 3,4 or 5 in a single dice is considered as a success, then find the probability of at least 3 successes.

Answer:  $\frac{1}{2}$

18. Solve the following differential equation :  $\left( y - x \frac{dy}{dx} \right) = a \left( y^2 + \frac{dy}{dx} \right)$ .

Answer :  $y = C(1 - ay)(a + x)$

19. Prove that the following function is always increasing on  $f(x) = x^3 - 6x^2 + 12x - 16$

20. Show that curves  $xy = a^2$  and  $x^2 + y^2 = 2a^2$  touch each other.

OR

Show that the function  $f(x) = 2x - |x|$  is continuous at  $x = 0$ .

OR

If the function  $f(x) = \begin{cases} 3ax + b, & \text{if } x < 1 \\ 11, & \text{if } x = 1 \\ 5ax - 2b, & \text{if } x > 1 \end{cases}$  is continuous at  $x = 1$ , find the values of a and b. Answer :  $a = 3, b = 2$

21. Find the area of a parallelogram whose adjacent sides are given by vectors  $\hat{i} - 3\hat{j} + \hat{k}$  and  $\hat{i} + \hat{j} + \hat{k}$ .

Answer :  $4\sqrt{2}$  sq. units

22. Show that,  $\sin^{-1} \left( 2x\sqrt{1-x^2} \right) = 2\sin^{-1}x$ .

SECTION -C

23. Using matrix method, solve the following system of equations:  $x + 2y + z = 7, x + 3z = 11, 2x - 3y = 1$ .

Answer :  $x = 2, y = 1, z = 3$

24. Using integration, find the area of the region bounded by the line  $2y + x = 8$ , x-axis and the lines  $x = 2$  and  $x = 4$ .

Answer : 5 sq units

25. A window is in the form of a rectangle above which there is a semi-circle. If the perimeter of the window is  $P$  cm. Show that the window will allow the maximum possible light only when the radius of the semi-circle is  $\frac{P}{\pi + 4}$  cm.

26. Find the image of the point  $(3, 5, 3)$  in the line  $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ .

Answer :  $(-1, 1, 7)$

27. In a bolt factory, three bolts machines  $A, B$  and  $C$  manufacture 25, 35 and 40 percent bolts. Of the total bolts manufactured of their output, 5, 4 and 2 percent are defective respectively. A bolt is drawn at random and is found to be defective. Find the probability that it was manufactured by either machine  $A$  or  $C$

Answer:  $\frac{41}{69}$

28. A farmer has supply of chemical fertilizer of type I which contains 10% nitrogen and 6% phosphoric acid and type II fertilizer which contains 5% nitrogen and 10% phosphoric acid. After testing the soil conditions of a field, it is found that at least 14 kg of nitrogen and 14 kg of phosphoric acid is required for a good crop. The fertilizer type I costs Rs. 2.00 per kg and type II costs Rs. 3.00 per kg. How many kilograms of each fertilizer should be used to meet the requirement and the cost will be minimum ?

**Answer :**

100 kg of type I and 80 kg of type II, minimum cost Rs. 440.

29. Evaluate the integral using limits of sums :  $\int_0^2 (x+4) dx$

Answer : 10

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