C.B.S.E. Class XII

Instructions

1. **All** questions are compulsory .

2. The question paper consists of 29 questions into three sections A,B and C. Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section C comprises of 7 questions of six marks each.

3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

4. There is no overall choice . However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

5. Use of calculator is not permitted.

SECTION -A

1. Write the principal value of $\tan^{-1}(1) + \cos^{-1}\left(-\dfrac{1}{2}\right)$.

Answer: $\dfrac{11\pi}{12}$

2. Find the value of a if $\begin{pmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{pmatrix} = \begin{pmatrix} -1 & 5 \\ 0 & 13 \end{pmatrix}$

Answer:a = 1

3. Write the value of $\tan\left(2\tan^{-1}\dfrac{1}{5}\right)$.

Answer: $\dfrac{5}{12}$

4. If $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$, then write the value of x.

Answer:$x = 2$

5. Write the degree of the differential equation $\left(\dfrac{dy}{dx}\right)^4 + 3x\dfrac{d^2y}{dx^2} = 0$.

Answer: 1

6. If $\begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} = A + \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$, then find the matrix A.

Answer: $\begin{bmatrix} 8 & -3 & 5 \\ -2 & -3 & -6 \end{bmatrix}$

7. If a unit vector $\vec{a}$ makes angles $\dfrac{\pi}{3}$ with $\hat{i}$, $\dfrac{\pi}{4}$ with $\hat{j}$ and an acute angle θ with $\hat{k}$, then find the value of θ.

Answer: $\dfrac{\pi}{3}$

8. Find the Cartesian equation of the line which passes through the point $(-2, 3, -5)$ and is parallel to the line $\dfrac{x+3}{3} = \dfrac{4-y}{5} = \dfrac{z+8}{6}$.

Answer: $\dfrac{x+2}{3} = \dfrac{y-3}{-5} = \dfrac{z+5}{6}$

9. If $\vec{a} = x\hat{i} + 2\hat{j} - z\hat{k}$ and $\vec{b} = 3\hat{i} - y\hat{j} + \hat{k}$ are two equal vectors, then write the value of $x + y + z$.

Answer: 0

10. The amount of pollution content added in air in a city due to x-diesel vehicles is given by $P(x) = 0.005x^3 + 0.02x^2 + 30x$. Find the marginal increase in pollution content when 3 diesel vehicles are added and write which value is indicated in the above question.

Answer: 30.255

SECTION B

11. Differentiate the following function with respect to x : $(\log x)^x + x^{\log x}$.

Answer: $= (\log x)^x \left\{ \dfrac{1}{\log x} + \log(\log x) \right\} + 2 \left(\dfrac{\log x}{x} \right) x^{\log x}$

12. A speaks truth in 60% of the cases and B in 90% of the cases. In what percentage of cases are they likely to contradict each other in stating the same fact?

Answer: 42% cases A and B are likely to contradict each other.

13. If $y = \log\left[x + \sqrt{x^2 + a^2} \right]$, show that $(x^2 + a^2)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} = 0$.

14. Find the Cartesian equation of the line which passes through the point $(-2, 3, -5)$ and is parallel to the line $\dfrac{x+3}{3} = \dfrac{4-y}{5} = \dfrac{z+8}{6}$.

Answer : $\dfrac{x+2}{3} = \dfrac{y-4}{-5} = \dfrac{z+5}{6}$

15. Find the value of the following : $\tan\dfrac{1}{2}\left[\sin^{-1}\dfrac{2x}{1+x^2} + \cos^{-1}\dfrac{1-y^2}{1+y^2} \right]$, $|x| < 1$, $y > 0$ and $xy < 1$

Answer: $\dfrac{x+y}{1-xy}$

OR

Prove that : $\tan^{-1}\left(\dfrac{1}{2} \right) + \tan^{-1}\left(\dfrac{1}{5} \right) + \tan^{-1}\left(\dfrac{1}{8} \right) = \dfrac{\pi}{4}$

16. Using properties of determinants, prove the following : $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = ab + bc + ca + abc$

OR

Using properties of determinants, prove the following : $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2$.

17. If $x = a\sin t$ and $y = a(\cos t + \log\tan(t/2))$, find $\dfrac{d^2y}{dx^2}$.

> Answer: $\dfrac{-1}{a}\cos ec^2 t \sec t$

18. Evaluate : $\int \dfrac{x^2}{(x^2+4)(x^2+25)}dx$

> Answer: $= -\dfrac{2}{21}\tan^{-1}\left(\dfrac{x}{2}\right) + \dfrac{5}{21}\tan^{-1}\left(\dfrac{x}{5}\right) + C$

19. Evaluate : $\int \dfrac{\sin(x-a)}{\sin(x+a)}dx$

> Answer: $= x\cos 2a - \sin 2a \log|\sin(x+a) + C|$

OR

Evaluate : $\int \dfrac{5x-2}{1+2x+3x^2}dx$

> Answer: $\dfrac{5}{6}\log(3x^2+2x+1) - \dfrac{11}{3\sqrt{2}}\tan^{-1}\left(\dfrac{3x+1}{\sqrt{2}}\right) + C$

20. Evaluate : $\int\limits_{1}^{3} [|x-1| + |x-2| + |x-3|]dx$

> Answer: 5

21. Show that the function f in $A = R - \left\{\dfrac{2}{3}\right\}$ defined as $f(x) = \dfrac{4x+3}{6x-4}$ is one-one and onto. Hence find f^{-1} .

22. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, find a vector $\vec{c}$, such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a}.\vec{c} = 3$.

> Answer : $\dfrac{5}{3}\hat{i} + \dfrac{2}{3}\hat{j} - \dfrac{2}{3}\hat{k}$

SECTION - C

23. A school wants to award its students for the value of Honesty, Regularity and Hard work with a total cash award of Rs. 6,000. Three times the award money for Hard work added to that given for honesty amounts to Rs. 11,000. The award money given for Honesty and Hard work together is double the one given for Regularity. Represent the above situation algebraically and find the award money for each value, using matrix method. Apart from these values, namely, Honesty, Regularity and Hard work, suggest one more value which the school must include for awards.

> Answer: the award for Honesty = Rs. 500, the award for Regularity = Rs. 2000, and the award for Hard work = Rs. 3500

24. Assume that the chances of a patient having a heart attack is 40%. Assuming that a meditation and yoga course reduces the risk of heart attack by 30% and prescription of certain drug reduces its chance by 25%. At a time a patient can choose any one of the two options-with equal probabilities. It is given that after going through one of the two options, the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga. Interpret the result and state which of the above stated methods is more beneficial for the patient.

> Answer: $\dfrac{15}{29}$

25. Find the coordinates of the point, where the line $\dfrac{x-2}{3} = \dfrac{y+1}{4} = \dfrac{z-2}{2}$ intersects the plane $x - y + z - 5 = 0$. Also find the angle between the line and the plane.

> Answer :$(2, -1, 2), \sin^{-1}\left(\dfrac{1}{\sqrt{87}}\right)$

OR

Find the vector equation of the plane which contains the line of intersection of the planes $\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) - 4 = 0$ and $\vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) + 5 = 0$ and which is perpendicular to the plane $\vec{r} \cdot (5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0$

> Answer:$\vec{r} \cdot (33\hat{i} + 45\hat{j} + 50\hat{k}) = 41$

26. A cooperative society of farmers has 50 hectares of land to grow two crops A and B. The profits from crops A and B per hectare are estimated as Rs.10,500 and Rs.9,000 respectively. To control weeds a liquid herbicide has to be used for crops A and B at the rate of 20 litres and 10 litres per hectare, respectively. Further not more than 800 litres of herbicide should be used in order to protect fish and wildlife using a pond which collects drainage from this land. Keeping in mind that the protection of fish and other wildlife is more important then earning profit, how much land should be allocated to each crop so as to maximize the total profit ?. Form an LPP from the above and solve it graphically. Do you agree with the message that the protection of wildlife is utmost necessary to preserve the balance in environment?.

> **Answer :** Society will allocate 30 hectares of land to crop A and 20 hectares of land to crop B to maximize the total profit.

27. Show that the differential equation $x\dfrac{dy}{dx}\sin\left(\dfrac{y}{x}\right) + x - y\sin\left(\dfrac{y}{x}\right) = 0$ is homogeneous. Find the particular solution of this differential equation, given that $x = 1$ when $y = \dfrac{\pi}{2}$.

28. Show that height of the cylinder of maximum volume, that can be inscribed in a sphere of radius R is $\dfrac{2R}{\sqrt{3}}$. Also find the maximum volume.

OR

Find the equation of the normal at a point on the curve $x^2 = 4y$ which passes through the point $(1, 2)$ .Also, find the equation of the corresponding tangent.

> Answer:Equation of normal : $(y - 1) = -1(x - 2) \Rightarrow x + y - 3 = 0$ and Equation of tangent :$(y - 2) = 1(x - 1) \Rightarrow x - y = -1$.

29. Using integration, find the area of the region enclosed between the two circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$.

> Answer:$2\left[\dfrac{-3\sqrt{3} + 4\pi}{3}\right]$ sq. units

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