

## Instructions

1. **All** questions are compulsory .
2. The question paper consists of 29 questions into three sections A,B and C. Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section C comprises of 7 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice . However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculator is not permitted.

### SECTION -A

1. . Let \* be a binary operation on  $\mathbb{N}$  given by  $a * b = \text{L.C.M.}(a, b)$  for all  $a, b \in \mathbb{N}$ . Find  $5 * 7$ .

Answer: 35

2. . Write the principal value of  $\cos^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\left(-\frac{1}{2}\right)$  [Delhi 2012]

Answer:  $\frac{2\pi}{3}$

3. Find the value of  $x + y$  from the following  $2 \begin{pmatrix} x & 5 \\ 7 & y-3 \end{pmatrix} + \begin{pmatrix} 3 & -4 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 7 & 6 \\ 15 & 14 \end{pmatrix}$

Answer:  $x=2, y=9$

4. If  $A^T = \begin{pmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{pmatrix}$  and  $B = \begin{pmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{pmatrix}$  find  $A^T - B^T$

Answer:  $\begin{pmatrix} 4 & 3 \\ -3 & 6 \\ -1 & -2 \end{pmatrix}$

5. Let A be a square matrix of order  $3 \times 3$  . Write the value of  $|2A|$  where  $|A| = 4$

Answer:32

6. Evaluate  $\int_0^2 \sqrt{4-x^2} dx$

Answer: $\pi$

7. Given  $\int e^x(\tan x + 1)\sec x dx = e^x f(x) + c$  Write  $f(x)$  satisfying above.

Answer:  $e^x \sec x + c$

8. Write the value of  $(\hat{i} \times \hat{j}) \cdot \hat{k} + \hat{i} \cdot \hat{j}$

Answer: 1

9. Find the scalar component of vector  $\vec{AB}$  with initial point  $A(2,1)$  and terminal point  $B(-5,7)$ .

Answer:  $= -7\hat{i} + 6\hat{j}$

10. Find the distance of the plane  $3x - 4y + 12z = 3$  from the origin.

Answer:  $\frac{3}{13}$

**SECTION B**

11. Evaluate  $\int_{-1}^2 (|x^3 - x|) dx$

Answer:  $\frac{11}{4}$

12. Let  $A = R \rightarrow \{3\}$  and  $B = R \rightarrow \{1\}$ . Consider the function  $f : A \rightarrow B$  defined by  $f(x) = \left(\frac{x-2}{x-3}\right)$ . Show that  $f$  is one-one and onto and hence find  $f^{-1}$ .

13. Prove that  $\sin^{-1}\left(\frac{8}{17}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \cos^{-1}\left(\frac{36}{85}\right)$

14. Using differentials, find the approximate value of  $\sqrt{495}$ .

Answer: 7.0357

15. If  $\sin y = x \sin(a+y)$ , prove that  $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$ .

16. Simplify :  $\cos \theta \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} + \sin \theta \begin{bmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{bmatrix}$

Answer:  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

17. If  $y = \sin^{-1}x$ , show that  $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$ .

18. Using properties of determinants, prove the following :  $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = ab + bc + ca + abc$

19. Evaluate :  $\int \frac{2}{(1-x)(1+x^2)} dx$

Answer :  $-\log(x-1) + \frac{1}{2}\log(1+x^2) + \tan^{-1}x + C$

20. Solve the following differential equation :  $2x^2\frac{dy}{dx} - 2xy + y^2 = 0$

Answer :  $\frac{2x}{y} = \log x + c$ .

OR

Find the particular solution of the following differential equation :  $xy \frac{dy}{dx} = (x+2)(y+2); y = -1$  when  $x = 1$ .

Answer:  $y - x + 2 = 2 \log x(y + 2)$ .

21. Find the equation of a line passing through the point  $P(2, -1, 3)$  and perpendicular to the lines  $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$  and  $\vec{r} = (2\hat{i} - \hat{j} - 3\hat{k}) + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$

Answer :  $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(-6\hat{i} - 3\hat{j} + 6\hat{k})$

22. If  $\vec{a}, \vec{b}, \vec{c}$  are three vectors such that  $|\vec{a}| = 5, |\vec{b}| = 12$  and  $|\vec{c}| = 13$  and  $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ , find the value of  $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$

Answer :  $-169$

SECTION - C

23. Evaluate :  $\int_1^3 (2x^2 + 5x) dx$  as a limit of a sum.

Answer:  $\frac{112}{3}$

24. Find the equation of the plane determined by the points  $A(3, -1, 2), B(5, 2, 4)$  and  $C(-1, -1, 6)$  and hence find the distance between the plane and the point  $P(6, 5, 9)$ .

Answer :  $3x + 4y + 3z - 19 = 0, \frac{25}{\sqrt{34}}$  units

25. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of Rs. 17.50 per package on nuts and Rs. 7 per package of bolts. How many packages of each should be produced each day so as to maximize his profits if he operates his machines for at the most 12 hours a day Rs. Form the above as a linear programming problem and solve it graphically.

**Answer :** The profit is maximum equal to Rs. 73.50 when 3 packages of each of nuts and bolts are manufactured

26. Show that the height of a closed right circular cylinder of given surface and maximum volume, is equal to the diameter of its base.

27. Using the method of integration, find the area of the region bounded by the following lines :  $5x - 2y - 10 = 0, x + y - 9 = 0, 2x - 5y - 4 = 0$

Answer :  $\frac{21}{2}$  sq. units

28. Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. Two balls are transferred at random from Bag I to Bag II and then a ball is drawn from Bag. The ball so drawn is found to be red in colour. Find the probability that the transferred balls were both black.

Answer:  $\frac{8}{27}$

29. Using elementary operations, find the inverse of the following matrix :  $\begin{pmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$

Answer:  $A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix}$



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