DELHI BOARD [2011 CBSE XII MATHEMATICS]

Instructions

1. **All** questions are compulsory .
2. The question paper consists of 29 questions into three sections A,B and C. Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section C comprises of 7 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice . However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculator is not permitted.

SECTION -A

1. Construct a 2 2 matrix, $A=[a_{ij}]$, whose elements are given by : $a_{ij}=\frac{i}{j}$

   Answer: $A=\begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{bmatrix}$

2. For what value of x, the matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular ?

   Answer: x = 3

3. Write A^{-1} for $A=\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

   Answer: $A^{-1}=\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$

4. Write the direction cosines of the vector $-2\hat{i}+\hat{j}-5\hat{k}$.

   Answer : $\left(\frac{2}{\sqrt{30}},-\frac{1}{\sqrt{30}},-\frac{-5}{\sqrt{30}}\right)$

5. Evaluate the inetgral : $\int \frac{dx}{x^2+16}$

   Answer : $\frac{1}{4}\tan^{-1}\frac{x}{4}+C$

6. State the reason for the relation R in the set $\{1,2,3\}$ given by $R = \{(1,2),(2,1)\}$ is not to be transitive.

SOLUTION

7. For a 2 2 matrix, $A = [a_{ij}]$whose elements are given by $a_{ij} = \frac{i}{j}$,write the value of a_{12}.

Answer: $\frac{1}{2}$

8. For what value of $'a'$, the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $a\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear.

Answer: $a = -4$

9. Write the principal value of $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$.

Answer : $\frac{5\pi}{6}$

10. Write the intercept cut off by the plane $2x + y - z = 5$ on x-axis.

Answer : $\frac{5}{2}$

SECTION B

11. Find the angle between the following pair of lines: $\frac{-x+2}{-2} = \frac{y-1}{7} = \frac{z+3}{-3}$ and $\frac{x+2}{-1} = \frac{2y-8}{4} = \frac{z-5}{4}$ and check whether the lines are parallel or perpendicular.

Answer : $\frac{\pi}{2}$

12. . Evaluate the integral : $\int_0^{\frac{\pi}{2}} \frac{x\sin x\cos x}{\sin^4 x + \cos^4 x}dx$

Answer : $\frac{\pi^2}{16}$

13. If $x = a(\theta - \sin\theta), y = a(1 + \cos\theta)$, find $\frac{d^2y}{dx^2}$.

Answer: $\frac{1}{4a}\cos ec\frac{\theta}{2}$

14. For what value of a is the function f defined by $f(x) = \begin{cases} a\sin\frac{\pi}{2}(x+1), & x \le 0 \\ \frac{\tan x - \sin x}{x^3}, & x > 0 \end{cases}$ is continuous at x= 0 ?

Answer: . $a = \frac{1}{2}$

15. Using properties of determinants, prove that $\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = xyz(x-y)(y-z)(z-x)$

16. Solve the following differential equation : $e^x \tan y dx + (1-e^x)\sec^2 y dy = 0$.

Answer:$\tan y = C(1-e^x)$, which is required solution. $\left[C = \frac{C_1}{C_2}\right]$

17. Form the differential equation of the family of parabolas having vertex at origin and axis along positive y-axis.

Answer: $xy_1 - 2y = 0$

18. Probabilities of solving a specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, then find the probability that

(i) the problem is solved

(ii) exactly one of them solves the problem. Answer: $\frac{2}{3}, \frac{1}{2}$

19. Sand is pouring from a pipe at the rate of $12cm^3/sec$. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?

Answer: $\frac{1}{48\pi}$ cm/sec.

OR

Find the points on the curve $x^2 + y^2 - 2x - 3 = 0$ at which the tangents are parallel to x-axis.

Answer: $(1,2), (1,-2)$

20. Find a unit vector perpendicular to each of the vectors $\vec{a}+\vec{b}$ and $\vec{a}+\vec{b}$, where $\vec{a} = 3\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{b} = \hat{i}+2\hat{j}-2\hat{k}$.

Answer: $\frac{2\hat{i}}{3} - \frac{2\hat{j}}{3} - \frac{\hat{k}}{3}$

21. Evaluate the integral : $\int \frac{5x+3}{\sqrt{x^2+4x+10}}dx$

Answer: $5\sqrt{x^2+4x+10} - 7\log\left|x+2+\sqrt{x^2+4x+10}\right| + C \left[C = \frac{5}{2}C_1 - 7C_2\right]$

22. Prove that : $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = \frac{x}{2}, x \in \left(0, \frac{\pi}{4}\right)$

SECTION - C

23. $\int_0^{\frac{\pi}{2}} 2\sin x \cos x \tan^{-1}(\sin x)\, dx$

Answer: $\frac{\pi}{2} - 1$

OR

$\int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$

Answer: $\frac{\pi^2}{16}$

24. Solve the following system of equations, using matrices : $\frac{2}{x}+\frac{3}{y}+\frac{10}{z} = 4, \frac{4}{x}-\frac{6}{y}+\frac{5}{z} = 1, \frac{6}{x}+\frac{9}{y}-\frac{20}{z} = 2.$

Answer: x = 2 , y = 3 , z=5

25. A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftman's time in its making while a cricket bat takes 3 hours of machine time and 1 hour of craftman's time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftsman's time.

(a) What number of rackets and bats must be made if the factory is to work at full capacity ?

(b) If the profit on a racket and on a bat is Rs. 20 and Rs. 10 respectively, then find the maximum profit of the factory when it works at full capacity.

Answer :
(i)$Z_{\max} = 16$ i.e. 4 tennis rackets and 12 cricket bats must be made so that the factory works at full capacity.
(ii)$Z_{\max} = Rs.200$ when 4 tennis rackets and 12 cricket bats are made.

26. Find the equation of the plane which contains the line of intersection of the planes $\vec{r}\cdot(\hat{i}+2\hat{j}+3\hat{k})-4=0;\vec{r}\cdot(2\hat{i}+\hat{j}-\hat{k})+5=0$ and which is perpendicular to the plane $\vec{r}\cdot(5\hat{i}+3\hat{j}-6\hat{k})+8=0$.

Answer:$33x+45y+50z-41=0,$

27. Using integration, find the area of the triangular region whose sides have equations $y=2x+1, y=3x+1$ and $x=4$.

Answer:8 sq. units

28. Show that of all the rectangles with a given perimeter, the square has the largest area.

29. Suppose 5% of men and 0.25% of women have grey hair. A grey haired person is selected at random. What is the probability of this person being male? Assume that there are equal number of males and females.

Answer: $\frac{20}{21}$

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