

Instructions

1. **All** questions are compulsory .
2. The question paper consists of 29 questions into three sections A,B and C. Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section C comprises of 7 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice . However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculator is not permitted.

SECTION -A

1. What is the value of the determinant ?

$$\begin{vmatrix} 0 & 2 & 0 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{vmatrix}$$

Answer: 8

2. . Find the order and degree of the differential equations $5x\left(\frac{dy}{dx}\right)^2 - \frac{d^2y}{dx^2} - 6y = \log x$.

Answer: order = 2 , degree = 1

3. . Evaluate the inetgral : $\frac{(\log x)^2}{2} + C$

Answer : $\int \frac{\log x}{x} dx$

4. What is the cosine of the angle which the vector $\sqrt{2}\hat{i} + \hat{j} + \hat{k}$ makes with y-axis?

Answer : $\frac{1}{2}$

5. . What is the principal value of $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)$?

Answer : $\frac{-\pi}{3}$

6. If $\sin^{-1}(x) + \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$, then find x.

Answer : $\frac{1}{2}$

7. If $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ k & 23 \end{bmatrix}$, then write the value of k.

Answer: k = 17

8. What is the range of the function $f(x) = \frac{|x-1|}{(x-1)}$?

Answer : $\{-1, 1\}$

9. Write a vector of magnitude 15 units in the direction of vector $\hat{i} - 2\hat{j} + 2\hat{k}$.

Answer : $5\hat{i} - 10\hat{j} + 10\hat{k}$

10. Write the vector equation of the following line $\frac{x-5}{3} = \frac{y+4}{7} = \frac{6-z}{2}$.

Answer: $\vec{r} = 5\hat{i} - 4\hat{j} + 6\hat{k} + \lambda(3\hat{i} + 7\hat{j} - 2\hat{k})$.

SECTION B

11. Find the points on the curve $y = x^3$ at which the slope of the tangent is equal to y-coordinate of the point.

Answer: (0,0) and (3, 27).

12. Using elementary row transformations, find the inverse of the following matrix. $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

Answer: $A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$

13. Let Z be the set of all integers and R be the relation on Z defined as $R = \{(a,b) : a, b \in Z \text{ and } (a-b) \text{ is divisible by } 3\}$. Prove that R is an equivalence relation.

OR

Show that the relation S in the set R of real numbers defined as $S = (a, b) : a, b \in R \text{ and } a \leq b^3$ is neither reflexive, nor symmetric, nor transitive.

14. If $A = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$, then for what value of α is A an identity matrix?

Answer: $\alpha = 0^\circ$

15. Solve for x : $\tan^{-1}\left(\frac{x-1}{x+2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$.

Answer : $\pm \frac{1}{\sqrt{2}}$

OR

Solve for x : $\tan^{-1}\left(\frac{1-x}{1+x}\right) - \frac{1}{2}\tan^{-1}x = 0, x > 0$.

Answer : $x = \frac{1}{\sqrt{3}}$

16. . Using the properties of determinants, prove that $\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2(a+b)(b+c)(c+a).$

17. . Evaluate the integral : $\int e^x \left(\frac{\sin 4x - 4}{1 - \cos 4x} \right) dx$

Answer : $e^x \cot 2x + C$

18. . In a multiple choice examination with three possible answers (out of which only one is correct) for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing?

Answer: $\frac{11}{243}$

19. . Evaluate the integral : $\int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$

Answer : $\frac{\pi^2}{16}$

20. Find the cartesian equation of the plane passing through the points $A(0,0,0)$ and $B(3,-1,2)$ and parallel to the line $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$

Answer : $x - 19y - 11z = 0$

21. Find the interval in which the function is increasing and decreasing : $f(x) = (x+2)e^{-x}$.

Answer : $(-\infty, -1)$ $(-1, \infty)$

OR

Show that the function defined as follows, is continuous at $x = 2$, but not differentiable. $f(x) = \begin{cases} 3x-2, & 0 < x < 1 \\ 2x^2-x, & 1 < x \leq 2 \\ 5x-4, & x > 2 \end{cases}$

22. . Find $\frac{dy}{dx}$ if $y = \sin^{-1} [x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}]$.

Answer: $\frac{1}{\sqrt{1-x^2}} - \frac{1}{2\sqrt{x-x^2}}$

SECTION - C

23. Find the image of the point $(1, 6, 3)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Also, write the equation of the line joining the given point and its image and find the length of the segment joining the given point and its image.

Answer : $(1, 0, 7); 2\sqrt{13}$ units 8.

24. A small firm manufactures gold rings and chains. The total number of rings and chains manufactured per day is at most 24. It takes 1 hour to make a ring and 30 minutes to make a chain. The maximum number of hours available per day is 16. If the profit on a ring is Rs. 300 and that on a chain is Rs.190, then find the number of rings and chains should be manufactured per day so as to earn the maximum profit. Form it as an LPP and solve it graphically.

Answer :
8 gold rings and 16 chains, maximum profit Rs. 5,440.

25. Show that a right circular cylinder, which is open at the top and has a given surface area, will have the greatest volume, if its height is equal to the radius of its base.

26. Evaluate the integral using limits of sums : $\int_1^3 (2x^2 + 3) dx$

Answer : $\frac{70}{3}$

27. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn at random and are found to be both clubs. Find the probability of the lost card being of club.

Answer: $\frac{11}{50}$

28. The points $A(4, 5, 10)$, $B(2, 3, 4)$ and $C(1, 2, -1)$ are three vertices of a parallelogram ABCD. Find the vector equations of sides AB and BC and also find the coordinates of point D.

29. Using integration, find the area of the following region : $\{(x, y) : |x - 1| \leq y \leq \sqrt{5 - x^2}\}$

Answer : $\left(\frac{5\pi}{4} - \frac{1}{2}\right)$ sq. units

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