MULTIPLE CHOICE TYPE QUESTIONS

1. . $\int x^2 e^{x^3} dx$ equals

(a) $\frac{1}{3}e^{x^3}+C$

(b) $\frac{1}{3}e^{x^2}+C$

(c) $\frac{1}{2}e^{x^3}+C$

(d) $\frac{1}{2}e^{x^2}+C$

SOLUTION

:(a) Let $I=\int x^2 e^{x^3} dx$ Put $x^3=t \Rightarrow 3x^2 dx=dt \therefore I=\frac{1}{3}\int e^t dt=\frac{1}{3}e^t+C=\frac{1}{3}e^{x^3}+C$

2. . $\int e^x \sec x\,(1+\tan x)\,dx$ equals

(a) $e^x \cos x+C$

(b) $e^x \sec x+C$

(c) $e^x \sin x+C$

(d) $e^x \tan x+C$

SOLUTION

(b) $\int e^x(\sec x+\sec x\tan x)dx=e^x\sec x+C$

3. . $\int \frac{x\,dx}{(x-1)(x-2)}$ equals

(a) $\log\left|\frac{(x-1)^2}{x-2}\right|+C$

(b)) $\log\left|\frac{(x-2)^2}{x-1}\right|+C$

(c) $\log\left|\left(\frac{x-1}{x-2}\right)^2\right|+C$

(d) $\log|(x-1)(x-2)|+C$

SOLUTION

$I=\int \frac{x}{(x-1)(x-2)}dx$ We write $\frac{x}{(x-1)(x-2)}=\frac{A}{x-1}+\frac{B}{x-2} \Rightarrow x=A(x-2)+B(x-1)$ Comparing like terms, we get $A+B=1$ and $-2A-B=0$ Solving, we get $A=-1, B=2 \therefore I=\int\left(\frac{-1}{x-1}+\frac{2}{x-2}\right)dx=-\log|x-1|+2\log|x-2|+C=\log\left|\frac{(x-2)^2}{(x-1)}\right|+C$

4. $\int \frac{dx}{x(x^2+1)}$ equals

(a) $\log|x| - \frac{1}{2}\log\left(x^2+1\right)+C$

(b) $\log|x| + \frac{1}{2}\log\left(x^2+1\right)+C$

(c) $-\log|x| + \frac{1}{2}\log\left(x^2+1\right)+C$

(d) $\frac{1}{2}\log|x| + \log\left(x^2+1\right)+C$

SOLUTION

(a) Let $\frac{1}{x\left(x^2+1\right)} = \frac{A}{x} + \frac{Bx+C}{x^2+1} \Rightarrow 1 = A\left(x^2+1\right)+(Bx+C)(x)$ ....(i) Putting $x=0$ in (i), we get $1 = A(0+1) \Rightarrow A=1$ Comparing coefficients of x^2 in (i) on both sides , we get $0=A+B \Rightarrow B=-1$ Comparing coefficients of x in (i) on both sides , we get$C=0 \therefore \int \frac{1}{x\left(x^2+1\right)}dx = \int \left[\frac{1}{x}+\frac{-x}{x^2+1}\right]dx = \log x - \frac{1}{2}\log\left(x^2+1\right)+C$

5. . $\int \frac{dx}{x^2+2x+2}$ equals

(a) $x\tan^{-1}(x+1)+C$
(b) $\tan^{-1}(x+1)+C$
(c) $(x+1)\tan^{-1}x+C$
(d) $\tan^{-1}x+C$

SOLUTION

(b) Let $I = \int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1} = \tan^{-1}(x+1)+C$

6. . $\int \frac{dx}{\sqrt{9x-4x^2}}$ equals

(a) $\frac{1}{9}\sin^{-1}\left(\frac{9x-8}{8}\right)+C$

(b) $\frac{1}{2}\sin^{-1}\left(\frac{8x-9}{9}\right)+C$

(c) $\frac{1}{3}\sin^{-1}\left(\frac{9x-8}{8}\right)+C$

(d) $\frac{1}{2}\sin^{-1}\left(\frac{9x-8}{9}\right)+C$

SOLUTION

(b) Let $I = \int \frac{dx}{\sqrt{9x-4x^2}} = \frac{1}{2}\int \frac{dx}{\sqrt{-x^2+\frac{9}{4}x}} = \frac{1}{2}\int \frac{dx}{\sqrt{-\left(x^2-\frac{9}{4}x\right)}} = \frac{1}{2}\int \frac{dx}{\sqrt{\left(\frac{9}{8}\right)^2-\left[x^2-\frac{9}{4}x+\left(\frac{9}{8}\right)^2\right]}} = \frac{1}{2}\int \frac{dx}{\sqrt{\left(\frac{9}{8}\right)^2-\left(x-\frac{9}{8}\right)^2}} =$

$C = \frac{1}{2}\sin^{-1}\left(\frac{8x-9}{9}\right)+C$

7. $\int \frac{\sin^2 x-\cos^2 x}{sin^2 x\cos^2 x}dx$ is equals to

(a) $\tan x+\cot x+C$

(b) $\tan x+\operatorname{cosec} x+C$

(c) $-\tan x+\cot x+C$

(d) $\tan x+\sec x+C$

SOLUTION

Let $I=\int \frac{\sin^2 x-\cos^2 x}{sin^2 x\cos^2 x}dx=\int\left(\sec^2 x-\cos ec^2 x\right)dx=\tan x+\cot x+C$

8. . $\int \frac{e^x(1+x)}{\cos^2(e^x x)}dx$ equals

(a) $-\cot(ex^x)+C$

(b) $\tan(xe^x)+C$

(c) $\tan(e^x)+C$

(d) $\cot(e^x)+C$

SOLUTION

Let $I=\int \frac{e^x(1+x)}{\cos^2(e^x x)}dx$ Put $xe^x=t \Rightarrow (e^x\cdot 1+e^x x)\,dx=dt \therefore I=\int \frac{dt}{\cos^2 t}=\int \sec^2 t\,dt=\tan t+C=tan(xe^x)+C$

9. $\int\left(\frac{10x^9+10^x\log_e 10}{x^{10}+10^x}\right)dx$equals

(a) $10^x-x^{10}+C$

(b) $10^x+x^{10}+C$

(c) $\left(10^x-x^{10}\right)^{-1}+C$

(d) $\log\left(10^x-x^{10}\right)+C$

SOLUTION

(d) Let $I=\int\left(\frac{10x^9+10^x\log_e 10}{x^{10}+10^x}\right)dx$ Put $x^{10}+10^x=t \Rightarrow \left(10x^9+\log_e 10\cdot 10^x\right)dx=dt \Rightarrow I=\int \frac{dt}{t}=\log|t|+C=\log\left(x^{10}+10^x\right)+C$

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VERY SHORT ANSWER QUESTIONS

10. Evaluate : $\int (x^2+5)^3 dx$

SOLUTION

.(i) Let$I=\int (x^2+5)^3 dx$ Expanding the in integrand by the binomial formula, $I=\int \left(x^6+15x^4+75x^2+125\right) dx=\frac{x^7}{7}+\frac{15x^5}{5}+\frac{75x^3}{3}+125x+C \Rightarrow I=\frac{x^7}{7}+3x^5+25x^3+125x+C$

11. Evaluate $\int \frac{x^3}{x+2}dx$

SOLUTION

.: Let $I=\int \frac{x^3}{x+2}dx$ Dividing x^2 by $x+2$,we get $=\int \left(x^2-2x+4-\frac{8}{x+2}\right) dx=\frac{x^3}{3}-x^2+4x-8\log|x+2|+C$

12. Evaluate $\int \left(e^{x\log a}+e^{a\log x}+e^{a\log a}\right) dx$

SOLUTION

$I=\int \left(e^{x\log a}+e^{a\log x}+e^{a\log a}\right) dx=\int \left(e^{\log a^x}+e^{\log x^a}+e^{\log a^a}\right) dx=\int (a^x+x^a+a^a) dx=\frac{a^x}{\log a}+\frac{x^{a+1}}{a+1}+a^a x+C$

13. $\int \frac{2^x+3^x}{5^x}dx$

SOLUTION

Let $I=\int \frac{2^x+3^x}{5^x}dx \Rightarrow I=\int \frac{2^x}{5^x}dx+\int \frac{3^x}{5^x}dx=\int \left(\frac{2}{5}\right)^x dx+\int \left(\frac{3}{5}\right)^x dx \Rightarrow I=\frac{\left(\frac{2}{5}\right)^x}{\log_e\left(\frac{2}{5}\right)}+\frac{\left(\frac{3}{5}\right)^x}{\log_e\left(\frac{3}{5}\right)}+C$

14. $\int \frac{\sin x}{1+\sin x}dx$

SOLUTION

Let $I=\int \frac{\sin x}{1+\sin x}dx \Rightarrow I=\int \frac{\sin x\,(1-\sin x)}{(1+\sin x)\,(1-\sin x)}dx \Rightarrow I=\int \frac{\sin x-\sin^2 x}{1-\sin^2 x}dx=\int \frac{\sin x-\sin^2 x}{\cos^2 x}dx \Rightarrow I=\int \frac{\sin x}{\cos^2 x}dx-\int \frac{\sin^2 x}{\cos^2 x}dx$
$=\int \sec x\tan x dx-\int \tan^2 x dx=\int \sec x \tan x dx-\int \left(\sec^2 x-1\right) dx=\sec x-\tan x+x+C$

15. If $f'(x)=2x^3-\frac{6}{x^4}$ and $f(2)=4$ find $f(x)$

SOLUTION

we have $f(x)=\int f'(x)\,dx \Rightarrow f(x)=\int \left(2x^3-\frac{6}{x^4}\right) dx \Rightarrow f(x)=2\left(\frac{x^4}{4}\right)-6\left(\frac{x^{-4+1}}{-4+1}\right)+C \Rightarrow f(x)=\frac{x^4}{2}+\frac{6\left(x^{-3}\right)}{3}+C$
$\Rightarrow f(x)=\frac{x^4}{2}+\frac{2}{x^3}+C$ When $x=2, f(2)=4$ (given) $\therefore f(2)=\frac{2^4}{2}+\frac{2}{2^3}+C \Rightarrow 4=8+\frac{1}{4}+C \Rightarrow C=4-8-\frac{1}{4}=\frac{-17}{4}$ $\therefore$
$f(x)=\frac{x^4}{2}+\frac{2}{x^3}-\frac{17}{4}$

16. Evaluate $\int \frac{\tan^4\sqrt{x}\sec^2\sqrt{x}}{\sqrt{x}}dx$

SOLUTION

Let $I=\int \frac{\tan^4\sqrt{x}\sec^2\sqrt{x}}{\sqrt{x}}dx$ Put $\tan\sqrt{x}=t \Rightarrow \frac{\sec^2\sqrt{x}}{2\sqrt{x}}dx=dt \therefore I=\int \frac{\tan^4\sqrt{x}\sec^2\sqrt{x}dx}{\sqrt{x}}=\int t^4(2dt)=2\int t^4dt \Rightarrow I=\frac{2t^5}{5}+C=\frac{2\tan^5\sqrt{x}}{5}+C$

17. $\int \frac{\sin(\tan^{-1}x)}{1+x^2}dx$

SOLUTION

Let $I=\int \frac{\sin(\tan^{-1}x)}{1+x^2}dx$ Put $\tan^{-1}x=t \Rightarrow \frac{1}{1+x^2}dx=dt \therefore I=\int \frac{\sin(\tan^{-1}x)}{1+x^2}dx=\int \sin t\,dt=-\cos t+C \Rightarrow I=-\cos(\tan^{-1}x)+C$

18. Evaluate $\int \frac{1}{\sqrt{2x+3}+\sqrt{2x-1}}dx$

SOLUTION

Let$I=\int \frac{1}{\sqrt{2x+3}+\sqrt{2x-1}}dx \Rightarrow I=\int \frac{\sqrt{2x+3}-\sqrt{2x-1}}{(\sqrt{2x+3}+\sqrt{2x-1})(\sqrt{2x+3}-\sqrt{2x-1})}dx \Rightarrow I=\int \frac{\sqrt{2x+3}-\sqrt{2x-1}}{(2x+3)-(2x-1)}dx \Rightarrow I=\frac{1}{4}\int\left[(2x+3)^{\frac{1}{2}}-(2x-1)^{\frac{1}{2}}\right]dx=\frac{1}{4}\left[\frac{(2x+3)^{\frac{3}{2}}}{2\times\frac{3}{2}}-\frac{(2x-1)^{\frac{3}{2}}}{2\times\frac{3}{2}}\right]+C=\frac{1}{12}\left[(2x+3)^{\frac{3}{2}}-(2x-1)^{\frac{3}{2}}\right]+C$

19. Evaluate $\int \frac{1}{\sqrt{2-3x}+\sqrt{1-3x}}dx$

Let $I=\int \frac{1}{\sqrt{2-3x}+\sqrt{1-3x}}dx \Rightarrow I=\int \frac{\sqrt{2-3x}-\sqrt{1-3x}}{(\sqrt{2-3x}+\sqrt{1-3x})(\sqrt{2-3x}-\sqrt{1-3x})}dx \Rightarrow I=\int \frac{\sqrt{2-3x}-\sqrt{1-3x}}{(2-3x)-(1-3x)}dx \Rightarrow I=\int \sqrt{2-3x}-\sqrt{1-3x}dx=\frac{(2-3x)^{\frac{3}{2}}}{-3\times\frac{3}{2}}-\frac{(1-3x)^{\frac{3}{2}}}{-3\times\frac{3}{2}}+C=-\frac{2}{9}\left[(2-3x)^{\frac{3}{2}}-(1-3x)^{\frac{3}{2}}\right]+C=\frac{2}{9}\left[(1-3x)^{\frac{3}{2}}-(2-3x)^{\frac{3}{2}}\right]+C$

20. Evaluate $\int \frac{\sin 4x}{\sin x}dx$

SOLUTION

Let $I=\int \frac{\sin 4x}{\sin x}dx \Rightarrow I=\int \frac{2\sin 2x\cos 2x}{\sin x}dx=\int \frac{4\sin x\cos x\cos 2x}{\sin x}dx \Rightarrow I=2\int 2\cos x cos2x\,dx=2\int(\cos 3x+\cos x)\,dx \Rightarrow I=2\left[\frac{\sin 3x}{3}+\sin x\right]+C$

21. Evaluate $\int \tan x\tan 2x\tan 3x\,dx$

SOLUTION

$\tan 3x=\tan(2x+x)=\frac{\tan 2x+\tan x}{1-\tan x\tan 2x} \Rightarrow \tan 3x(1-\tan x\tan 2x)=\tan 2x+\tan x \Rightarrow \tan 3x-\tan x\tan 2x\tan 3x=\tan x+\tan 2x \Rightarrow \tan x\tan 2x\tan 3x=\tan 3x-\tan 2x-\tan x \therefore I=\int \tan x\tan 2x\tan 3x\,dx=\int(\tan 3x-\tan 2x-\tan x)\,dx \Rightarrow I=\frac{1}{3}\log|\sec 3x|-\frac{1}{2}|\sec 2x|-\log|\sec x|+C$

22. Evaluate $\int [1+2\tan x(\tan x+\sec x)]^{\frac{1}{2}} dx$

SOLUTION

Let $I=\int [1+2\tan x(\tan x+\sec x)]^{\frac{1}{2}} dx \Rightarrow I=\int \left(1+2\tan^2 x+2\tan x\sec x\right)^{\frac{1}{2}} dx \Rightarrow I=\int \left(1+\tan^2 x+\tan^2 x+2\tan x\sec x\right)^{\frac{1}{2}} dx$

$\Rightarrow I=\int \left(\sec^2 x+\tan^2 x+2\tan x\sec x\right)^{\frac{1}{2}} dx \Rightarrow I=\int \left[(\sec x+tan x)^2\right]^{\frac{1}{2}} dx=\int (\sec x+\tan x)dx \Rightarrow I=\log|\sec x+\tan x|+\log|\sec x|+ C$

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FILL IN THE BLANKS

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23. Integration of $\sin 2x$ is $\cdots$

SOLUTION.

: Let $I=\int \sin 2x dx=-\frac{\cos 2x}{2}+C$

24. Integration of $\cos 3x$

SOLUTION.

: Let $I=\int \cos 3x dx=\frac{\sin 3x}{3}+C$

25. Integration of e^{2x} is $\cdots$

SOLUTION.: Let $I=\int e^{2x} dx=\frac{e^{2x}}{2}+C$

26. $(ax+b)^2$ SOLUTION.

: Let $I=\int (ax+b)^2 dx=\frac{(ax+b)^3}{3a}+C$

27. Integration of $\sin 2x-4e^{3x}$ is $\cdots$ SOLUTION.

: Let $I=\int \left(\sin 2x-4e^{3x}\right) dx=-\frac{\cos 2x}{2}-\frac{4e^{3x}}{3}+C$

28. . Integration of $\int \left(4e^{3x}+1\right) dx$ is $\cdots$

SOLUTION.

: Let $I=\int \left(4e^{3x}+1\right) dx=\frac{4e^{3x}}{3}+x+C$

29. Integration of $\int x^2\left(1-\frac{1}{x^2}\right) dx$ is $\cdots$

SOLUTION.

: Let$I=\int x^2\left(1-\frac{1}{x^2}\right) dx=\int \left(x^2-1\right) dx=\frac{x^3}{3}-x+C$

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