MULTIPLE CHOICE TYPE QUESTIONS

1. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3+\left(\frac{dy}{dx}\right)+\sin\left(\frac{dy}{dx}\right)+1=0$ is

   (a) 3
   (b) 2
   (c) 1
   (d) not defined

   SOLUTION

   (d) Since the given differential is not a polynomial in $\frac{dy}{dx}$, therefore, its degree is not defined.

2. The order of the differential equation $2x^2\frac{d^2y}{dx^2}-3\frac{dy}{dx}+y=0$ is

   (a) 2
   (b) 1
   (c) 0
   (d) not defined

   SOLUTION

   (a) Since the highest order derivative is $\frac{d^2y}{dx^2}$, therefore, its order is 2.

3. The number of arbitrary constants in the general solution of a differential equation of fourth order are:

   (a) 0
   (b) 2
   (c) 3
   (d) 4

   SOLUTION

   (d) The number of arbitary constants in the general solution of a differential equation of n^{th} order is n. So, differential equation of fourth order has 4 constants.

4. The order and degree, if defined, of the differential equation: $y'''+y^2+e^{y'}=0$

   (a) order = 2, degree = 1
   (b) order = 3, degree = 1
   (c) order = 3, degree = not defined
   (d) order = 2, degree = not defined

   SOLUTION

   The highest order derivative present in the differential equation is y so its order is three. The given differential equation is not a polynomial equation in its derivatives and so its degree is not defined.

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VERY SHORT ANSWER QUESTIONS

5. . Determine order and degree (if defined) of differential equation $y'' + 2y' + \sin y = 0$

SOLUTION

The highest order derivative is y'' and the degree of its highest order is one. Therefore, the given differential equation is of order 2 and degree 1.

6. Verify that the given functions (explicit or implicit) is a solution of the corresponding differential equation: $y = e^x + 1 : y'' - y' = 0$

SOLUTION

We have $y = e^x + 1 \ldots (1)$ Differentiating (1) w.r.t. x, we get $y' = \frac{d}{dx}(e^x + 1) = e^x$ and $y'' = \frac{d}{dx}(e^x) = e^x \Rightarrow y'' - y' = 0$ Hence, $y = e^x + 1$ is a solution of the given differential equation.

7. Verify that the given functions (explicit or implicit) is a solution of the corresponding differential equation: $y = x^2 + 2x + C : y' - 2x - 2 = 0$

SOLUTION

We have, $y = x^2 + 2x + C \ldots (1)$ Differentiating (1) w.r.t. x, we get $y' = 2x + 2 \Rightarrow y' - 2x - 2 = 0$ Hence, $y = x^2 + 2x + C$ is a solution of the given differential equation.

8. Verify that the given functions (explicit or implicit) is a solution of the corresponding differential equation: $y = \cos x + C : y' + \sin x = 0$

SOLUTION

: We have, $y = \cos x + C \ldots (1)$ Differentiating (1) w.r.t. x, we get $y' = -\sin x \Rightarrow y' + \sin x = 0$ Hence, $y = \cos x + C$ is a solution of the given differential equation.

9. Verify that the given functions (explicit or implicit) is a solution of the corresponding differential equation: $y = \sqrt{1+x^2}y' = \frac{W}{1+x^2}$

SOLUTION

We have, $y = \sqrt{1+x^2} \ldots (1)$ Differentiating (1) w.r.t. x, we get $y' = \frac{1 \times (2x)}{2\sqrt{1+x^2}} \Rightarrow y' = \frac{X}{\sqrt{1+x^2}} \ldots (2)$ Dividing (2) by (1), we get $\frac{y'}{y} = \frac{x}{1+x^2} \Rightarrow y' = \frac{\theta}{1+x^2}$ Hence, $y = \sqrt{1+x^2}$ is a solution of the given differential equation.

10. Verify that the given functions (explicit or implicit) is a solution of the corresponding differential equation: $y = Ax : xy' = y (x \neq 0)$

SOLUTION

: We have, $y = Ax \ldots (1)$ Differentiating (1) w.r.t. x, we get $y' = A \ldots (2)$ Dividing (2) by (1), we get $\frac{y'}{y} = \frac{1}{x} \Rightarrow xy' = y$ Hence, $y = Ax$ is a solution of the given differential equation.

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FILL IN THE BLANKS

11. The order and degree of the differential equation $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2+xy=0$ is $\cdots$

SOLUTION

Highest order derivative present is $\frac{d^2y}{dx^2}$. So, order is 2. It is polynomial in differential cofficents and highest power of $\frac{d^2y}{dx^2}$ is one. So, degree is 1.

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12. The order and degree of the differential equation $x+\frac{dy}{dx}=\sqrt{1+\left(\frac{dy}{dx}\right)^2}$ is $\cdots$.

SOLUTION

$x+\frac{dy}{dx}=\sqrt{1+\left(\frac{dy}{dx}\right)^2} \Rightarrow \left(x+\frac{dy}{dx}\right)^2=1+\left(\frac{dy}{dx}\right)^2 \Rightarrow x^2+\left(\frac{dy}{dx}\right)^2+2x\frac{dy}{dx}=1+\left(\frac{dy}{dx}\right)^2 \Rightarrow x^2+2x\frac{dy}{dx}=1$. Here, highest order derivative is $\frac{dy}{dx}$. So, order is 1 and its power is one. So, degree is 1.

13. . The order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2+\cos\left(\frac{dy}{dx}\right)=0$. is $\cdots$

SOLUTION

$\left(\frac{d^2y}{dx^2}\right)^2+\cos\left(\frac{dy}{dx}\right)=0$. The highest order derivative is $\frac{d^2y}{dx^2}$, so order is 2. Since given differential equation is not a polynomial in differential coefficients, so its degree is not defined.

14. . The order and degree of the differential equation $\frac{5d^2y}{dx^2}=\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}$. is $\cdots$ SOLUTION

$5\frac{d^2y}{dx^2}=\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}} \Rightarrow 25\left(\frac{d^2y}{dx^2}\right)^2=\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^3$ The highest order derivative is $\left(\frac{d^2y}{dx^2}\right)^2$, so order is 2 and its power is 2.

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