Chapter 1

DEFINITE INTEGRALS

MULTIPLE CHOICE TYPE QUESTIONS

1. If $f(a+b+x) = f(x)$, then $\int_a^b x f(x)\,dx$ is equal to

   (a) $\frac{a+b}{2}\int_a^b f(b-x)\,dx$

   (b) $\frac{a+b}{2}\int_a^b f(b+x)\,dx$

   (c) $\frac{b-a}{2}\int_a^b f(x)\,dx$

   (d) $\frac{a+b}{2}\int_a^b f(x)\,dx$

   SOLUTION

   Let $I = \int_a^b x f(x)\,dx$ Let $a+b-x = z \Rightarrow -dx = dz$ When $x = a, z = b$ and when $x = b, z = a$ $\therefore I = -\int_b^a (a+b-z) f(z)\,dz = \int_a^b (a+b) f(z)\,dz - \int_a^b z f(z)\,dz = (a+b)\int_a^b f(x)\,dx - \int_a^b x f(x)\,dx = (a+b)\int_a^b f(x)\,dx - I \Rightarrow 2I = (a+b)\int_a^b f(x)\,dx$ Hence, $I = \left(\frac{a+b}{2}\right)\int_a^b f(x)dx$

2. $\int_0^1 e^{2logx} dx =$

   (a) 1/3

   (b) 2/3

   (c) 7/3

   (d) 1/2

   SOLUTION

   $\int_0^1 e^{2logx} dx = \int_0^1 e^{logx^2} dx$

$=\int_0^1 x^2dx$ [$\because e^{logx}=x$]

$=[\frac{x^3}{3}]_0^1=\frac{1}{3}$

3. $\int_0^{\pi/4} tan^2xdx =$

(a) $1+\frac{\pi}{4}$

(b) $1-\frac{\pi}{4}$

(c) $\frac{\pi}{4}$

(d) $2-\frac{\pi}{4}$

$\int_0^{\pi/4} tan^2xdx = \int_0^{\pi/4}(sec^2x-1)dx$

= $[tanx-x]_0^{\pi/4}$

$=1-\frac{\pi}{4}$

4. $\int_0^{\pi/2} e^x sinxdx =$

(a) $\frac{1}{3}(e^{\pi/2}+1)$

(b) $\frac{1}{2}(e^{\pi/2}+2)$

(c) $\frac{1}{2}(e^{\pi/2}-1)$

(d) $\frac{1}{2}(e^{\pi/2}+1)$

SOLUTION

I = $\int_0^{\pi/2} e^x sinxdx$

= $-[e^x cosx]_0^{\pi/2}+\int_0^{\pi/2} e^x cosxdx$

= $-[e^x cosx]_0^{\pi/2}+[e^x sinx]_0^{\pi/2}-\int_0^{\pi/2} e^x sinxdx$

$\Rightarrow 2I = [e^x(sinx-cosx)]_0^{\pi/2}$

= $(e^{\pi/2}+1)$

Hence I = $\frac{1}{2}(e^{\pi/2}+1)$

5. $\int_1^2 e^x(\frac{1}{x}-\frac{1}{x^2})dx =$

(a) $\frac{e^2}{2}-e$

(b) $\frac{e^2}{2}+e$

(c) $3\frac{e^2}{2}-e$

(d) $\frac{e^2}{2}-2e$

SOLUTION

Let $\frac{1}{x}=t \Rightarrow -\frac{1}{x^2}dx=dt$

$\int_1^2 e^x(\frac{1}{x}-\frac{1}{x^2})dx$

$=e^x\frac{1}{x}+c$ [Using : $\int e^x(f(x)+f'(x))dx=e^xf(x)+c$]

6. $\int_0^{\pi/2}\frac{dx}{2+cosx}=$

(a) $\frac{2}{\sqrt{3}}tan^{-1}(\frac{1}{\sqrt{3}})$

(b) $\frac{2}{\sqrt{3}}tan^{-1}(\frac{1}{\sqrt{3}})$

(c) $\frac{2}{\sqrt{3}}tan^{-1}(\frac{1}{\sqrt{3}})$

(d) $\frac{2}{\sqrt{3}}tan^{-1}(\frac{1}{\sqrt{3}})$

SOLUTION

$\int_0^{\pi/2}\frac{dx}{2+cosx}=$

$=\int_0^{\pi/2}\frac{dx}{2cos^2\frac{x}{2}+2sin^2\frac{x}{2}+cos^2\frac{x}{2}-sin^2\frac{x}{2}}$

$=\int_0^{\pi/2}\frac{dx}{sin^2\frac{x}{2}+3cos^2\frac{x}{2}}$

Dividing numerator and denominator by $cos^2\frac{x}{2}$

$\int_0^{\pi/2}\frac{sec^2\frac{x}{2}}{3+tan^2\frac{x}{2}}dx$

put $tan\frac{x}{2}=t \Rightarrow \frac{1}{2}sec^2\frac{x}{2}dx=dt$

$I=\int_0^1\frac{dt}{3+t^2}$

$I=\int_0^1\frac{dt}{(\sqrt{3})^2+t^2}$

$I=\frac{2}{\sqrt{3}}tan^{-1}(\frac{1}{\sqrt{3}})$

7. $\int_0^1\frac{tan^{-1}dx}{1+x^2}=$

(a) $\frac{\pi^2}{52}$

(b) $\frac{\pi^2}{16}$

(c) $\frac{\pi^2}{32}$

(d) $\frac{2\pi^2}{33}$

SOLUTION

let $tan^{-1}x = t \Rightarrow \frac{1}{1+x^2}dx = dt$

$\int_0^1 \frac{tan^{-1}dx}{1+x^2}$

$= \int_0^{\pi/4} tdt$

$[\frac{t^2}{2}]_0^{\pi/4}$

$\frac{\pi^2}{32}$

8. The value of the integral $\int_{1/x}^{2/\pi} \frac{sin(1/x)}{x^2}dx =$

(a) 1

(b) -1

(c) π

(d) 2

SOLUTION

Let $\frac{1}{x} = t \Rightarrow -\frac{1}{x^2}dx = dt$

$\int_{1/x}^{2/\pi} \frac{sin(1/x)}{x^2}dx$

$= -\int_{\pi/2}^{\pi} sintdt$

$= 1$

9. $\int_0^1 \frac{e^{-x}dx}{1+e^{-x}} =$

(a) $log(\frac{1-e}{2e}) + \frac{1}{e} + 1$

(b) $log(\frac{1-e}{2e}) + \frac{1}{e} + 1$

(c) $log(\frac{1+e}{2e}) - \frac{1}{e} + 1$

(d) $2log(\frac{1-e}{e}) + \frac{1}{e} + 1$

SOLUTION

Let $1 + e^{-x} = t \Rightarrow -e^{-x}dx = dt$

$\int_0^1 \frac{e^{-x}dx}{1+e^{-x}} =$

$\int_0^{1/e} -\frac{1}{t}dt$

$= log(\frac{e+1}{2e}) - \frac{1}{e} + 1$

10. $\int_{\pi/4}^{\pi/2} e^x(logsinx + cotx)dx =$

(a) $\frac{1}{3}e^{\pi/3}log2$

(b) $\frac{1}{5}e^{\pi/4}log2$

(c) $\frac{1}{2}e^{\pi/4}log3$

(d) $\frac{1}{2}e^{\pi/4}log2$

SOLUTION

Derivative of log sinx = cotx

therefore, $\int_{\pi/4}^{\pi/2} e^x(logsinx + cotx)dx$

$=[e^x logsinx]_{\pi/4}^{\pi/2}$

$= \frac{1}{2}e^{\pi/4}log2$

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VERY SHORT ANSWER QUESTIONS

11. . Evaluate : $\int_{-1}^{1} e^{|x|} dx$

SOLUTION

we have $I = \int_{-1}^{1} e^{|x|} dx$ Here, $f(x) = e^{|x|}$ and $f(-x) = e^{|-x|} = e^{|x|} = f(x) \therefore f(x)$ is an even function $\therefore I = \int_{-1}^{1} e^{|x| dx} = 2\int_{0}^{1} e^{x} dx = 2\left[e^{x}\right]_0^1 = 2(e-1)$

12. Evaluate : $\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} |\sin x|\, dx$

SOLUTION

we have $I = \int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} |\sin x|\, dx = 2\int_{0}^{\frac{\pi}{4}} \sin x dx = 2\left[-\cos x\right]_0^{\frac{\pi}{4}} = -2\left[\frac{1}{\sqrt{2}} - 1\right] = 2 - \sqrt{2}$

13. Evaluate : $\int_{1}^{2} \frac{dx}{x + (1+\log x)^2}$

SOLUTION

. Let $I = \int_{1}^{2} \frac{dx}{x(1+\log x)^2}$ Put $1 + \log x = t \Rightarrow \frac{dx}{x} = dt$ When $x = 1, t = 1$ and when $x = 2, t = 1 + \log 2 \therefore I = \int_{1}^{1+\log 2} \frac{dt}{t} = \left[\frac{-1}{t}\right]_1^{1+\log 2} = -\left[\frac{1}{1+\log 2} - 1\right] = -\left[\frac{1-1-\log 2}{1+\log 2}\right] = \frac{\log 2}{1+\log 2}$

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FILL IN THE BLANKS

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14. The value of $\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \sin^3 x\, dx$ is $\cdots$

SOLUTION

. Let $I = \int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \sin^3 x\, dx$ and $f(x) = \sin^3 x \Rightarrow f(-x) = (\sin(-x))^3 = -\sin^3 x = -f(x) \Rightarrow f(x)$ is an odd function. $\therefore \int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \sin^3 x dx = 0$

15. . The value of $\int_2^3 \frac{1}{x} dx$ is $\cdots$

SOLUTION

$\int_2^3 \frac{1}{x} dx = [\text{In}\, x]_2^3 = \text{In}\, 3 - \text{In}\, 2 = \text{In}\, \frac{3}{2}$

16. The value of $\int_0^{\pi/2} e^x (\sin x - \cos x)\, dx$ is $\cdots$

SOLUTION

$\int_0^{\pi/2} e^x (\sin x - \cos x)\, dx = \int_0^{\pi/2} e^x \{\cos x + (-\sin x)\}\, dx$ Put $\cos x = t \Rightarrow -\sin x dx = dt$ $I = [e^x \cos x\, dx]_0^{\pi/2} = \left(e^{\pi/2} \times 0\right) - \left(e^0 \times 1\right) =$ -1

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