NCERT - Miscellaneous Exercise

1. Let A = $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, show that $(aI+bA)^n + a^nI + na^{n-1}bA$,where I is the identity matrix of order 2 and $n \in N$.

SOLUTION .:

We have, $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $(aI+bA)^n = a^nI + na^{n-1}bA$ ...(i)

For n = 1, $(aI+bA)^1 + a^1I + 1a^{1-1}bA \Rightarrow al + bA = al + bA$

So, it is true for n = 1 Let us assume that (i) is true for n = k, i.e., $(aI+bA)^k = a^kI + ka^{k-1}bA$

Then $(aI+bA)^{k+1} = (aI+bA)^k + (aI+bA) = (a^kI + ka^{k-1}bA)(aI+bA)$

$= a^{k+1}I \times I + ka^kbAI + a^kbAI + ka^{k-1}b^2A \cdot A = a^{k+1} + ka^kbA + a^kbA + ka^{k-1}b^2 \times O$

$= d^{k+1}I + (k+1)d^kbA = a^{k+1}I + (k+1)d^{k+1-1}bA \Rightarrow$ (i) is true for n = k + 1

Hence, by mathematical induction it is true for all n $\in$ N.

2. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, prove that $A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$, $n \in N$.

SOLUTION .:

We shall prove it by mathematical induction.

To prove that n = 1 is true.

$$A^1 = \begin{bmatrix} 3^{1-1} & 3^{1-1} & 3^{1-1} \\ 3^{1-1} & 3^{1-1} & 3^{1-1} \\ 3^{1-1} & 3^{1-1} & 3^{1-1} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = A$$

We have, A = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ and $A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$.(i)

Thus, it is true for n = 1. Let us assume that (i) is true for n = k, i.e., $A^k = \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix}$, $k \in N$

Then, $A^{k+1} = A^k \cdot A = \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} \\ 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} \\ 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} \end{bmatrix} = \begin{bmatrix} 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \end{bmatrix}$$

$\Rightarrow$ (i) is true for n = k + 1 So, by mathematical induction $A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$, $n \in N$is true.

3. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then prove that $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$,where n is any positive integer.

SOLUTION .:

We have, $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ and $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$.(i) For n = 1, $A^1 = \begin{bmatrix} 1+2\times 1 & -4\times 1 \\ 1 & 1-2\times 1 \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} = A$

So, (i) is true for n = 1. Assume that (i) is true for n = k i.e., $A^k = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}$

Also, $A^{k+1} = \begin{bmatrix} 1+2(k+1) & -4(k+1) \\ k+1 & 1-2(k+1) \end{bmatrix}$ for $n = k+1$. $\Rightarrow A^{k+1} = \begin{bmatrix} 2k+3 & -4k-4 \\ k+1 & -2k-1 \end{bmatrix}$

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Now, $A^{k+1} = A \cdot A^k = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix} = \begin{bmatrix} 3+6k-4k & -12k-4+8k \\ 1+k-k & -4k-1+2k \end{bmatrix}$

$= \begin{bmatrix} 2k+3 & -4k-4 \\ k+1 & -2k-1 \end{bmatrix} = A^{k+1}$

So, (i) is true for n = k + 1. Hence, by mathematical induction $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ is true.

4. If A and B are symmetric matrices, prove that AB−BA is a skew symmetric matrix.

SOLUTION .:

Given. : A and B are symmetric matrices, therefore $A' = A, B' = B$.

To prove :(AB−$BA)' = -$(AB−BA)

Proof : $(AB - BA)'$=$(AB)'$ -

(BA)=$B'A'$ - $A'B' = BA - AB =$ -

$(AB - BA)$

So, AB−BA is a skew-symmetric matrix.

5. . Show that the matrix BAB is symmetric or skew symmetric according as A is symmetric or skew symmetric.

SOLUTION .:

Case1: Given that A is symmetric. We will prove BAB is symmetric. As A is symmetric, so $A' = A$. Now, $(B'AB)' = B'A'(B')' = B'A'B = B'AB$ Thus, $B'AB$ is a symmetric matrix.

Case II: Given is skew symmetric, i.e., $A' = -$A. We will prove that $B'AB$ is skew symmetric.

Now, $(B'AB)' = B'A'(B')' = B'A'B$

=$B'(- A)B = - B'AB$

Hence, $B'AB$ is a skew-symmetric matrix.

6. Find the values of x, y, z if the matrix $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfy the equation $A'A = I$.

SOLUTION .:

Given that, matrix $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ and $A'A = I. \Rightarrow \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 0+x^2+x^2 & 0+xy-xy & 0-zx+zx \\ 0+xy-xy & 4y^2+y^2+y^2 & 2yz-yz-yz \\ 0-zx+xz & 2yz-zy-zy & z^2+z^2+z^2 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow 2x^2 = 1, 6y^2 = 1, 3z^2 = 1 \Rightarrow x^2 = \frac{1}{2}, y^2 = \frac{1}{6}, z^2 = \frac{1}{3}$

Hence, $x = \pm\frac{1}{\sqrt{2}}, y = \pm\frac{1}{\sqrt{6}}, z = \pm\frac{1}{\sqrt{3}}$

7. For what values of x : $\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$?

SOLUTION .:

$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$

$\Rightarrow \begin{bmatrix} 1+4+1 & 2+0+0 & 0+2+0 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$

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$\Rightarrow \begin{bmatrix} 6 & 2 & 4 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$

$0+4+4x = 0 \Rightarrow 4(x+1) = 0 \Rightarrow x+1 = 0 \Rightarrow x = -1.$

8. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then show that $A^2 - 5A + 7I = O$.

SOLUTION .:

Given that, $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

and $5A = 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$

Now, substituting the values, we have $A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$

Hence, proved.

9. Find x, if $\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O.$

SOLUTION .:

$\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$

$\Rightarrow \begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} x+2 \\ 8+1 \\ 2x+3 \end{bmatrix} = O \Rightarrow \begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} x+2 \\ 9 \\ 2x+3 \end{bmatrix} = O$

$\Rightarrow x(x+2) - 45 - 2x - 3 = 0 \Rightarrow x^2 - 48 = 0 \Rightarrow x = \pm 4\sqrt{3}.$

10. A manufacturer produces three products x, y, z which he sells in two markets. Annual sales are indicated as :

(a) If unit sale prices of x,y and z are Rs 2.50, Rs 1.50 and Rs 1.00, respectively, find the total revenue in each market with the help of matrix algebra.

(b) If the unit costs of the above three commodities are Rs 2.00, Rs 1.00 and 50 paise respectively. Find the gross profit

. SOLUTION .:

Let quantity matrix be $A = \begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix}$

(a) Selling Price $B = \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix}$

Now, Total Selling Price, $AB = \begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix} \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix}$

$= \begin{bmatrix} 10,000 \times 2.50 + 2,000 \times 1.50 + 18,000 \times 1 \\ 6,000 \times 2.50 + 20,000 \times 1.50 + 8,000 \times 1 \end{bmatrix}$

$= \begin{bmatrix} 25,000 + 3,000 + 18,000 \\ 15,000 + 30,000 + 8,000 \end{bmatrix}$

Total revenue in market I = Rs. 46,000.

Total revenue in market II = Rs. 53,000.

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(b) Now, cost price$=\begin{bmatrix} 2.00 \\ 1.00 \\ 0.50 \end{bmatrix}$

Total cost price$=\begin{bmatrix} 1000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 0.5 \end{bmatrix}$

$=\begin{bmatrix} 10,000\times 2+2,000\times 1+18,000\times 0.5 \\ 6,000\times 2+20,000\times 1+8,000\times 0.5 \end{bmatrix}$

Total cost price = 31000 + 36000 = Rs. 67,000.

Total selling price = 46000 + 53000 = Rs. 99,000

Profit = S.P.−C.P. = 99,000 −

67,000 = Rs. 32,000.

11. Find the matrix X so that $X\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}=\begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

SOLUTION .:

$X\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}=\begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

We can say that X is a 2 2 matrix.

Let $X=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\therefore \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}=\begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a+4b & 2a+5b & 3a+6b \\ c+4d & 2c+5d & 3c+6d \end{bmatrix}=\begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

$\Rightarrow a+4b=-7$ ....

(i) and $c+4d=2$ ...(ii)

$2a+5b=-8$ .... (iii)

and $2c+5d=4$ ...(iv)

Solving(i) and (iii), we get a = 1, b =−2 Solving (ii) and (iv), we get c = 2,d =0

Hence, $X=\begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$

12. If A and B are square matrices of the same order such that AB = BA, then prove by induction that ABn = BnA. Further, prove that $(AB)^n=A^nB^n$for all $n\in N$.

SOLUTION .: Given AB = BA,

To prove

(1) $AB^n=B^nA$ and (2) $(AB)^n=A^nB^n\forall n\in N$ ...(i) We will prove it by mathematical induction.

(1) Given that AB = BA ...(ii)

We have to prove $AB^n=B^nA$ For n = 1, $AB^1=B^1A\Rightarrow$ AB = BA, which is true [from (ii)]

Let it be true for n = ABm = BmA ...(iii)

Then, for n = m + 1,

$AB^{m+1}=A(B^mB)=(AB^m)B=(B^mA)B$ [using (iii)] $=B^m(AB)=B^m(BA)$ [using (ii)] $=(B^mB)A=B^{m+1}A$.So, it is true for n = m + 1

$\therefore AB^n=B^nA$.

(2) For $n=1,(AB)^1=A^1B^1\Rightarrow AB=BA$ which is true for n = 1 Let (i) be true for a positive integer n = m. i.e., $(AB)^m=A^mB^m$ ....(iv)

then for n = m + 1, $(AB)^m+1=(AB)^m(AB)=(A^mB^m)(AB)$ (from (iv))

$=A^m(B^mA)B=A^m(AB^m)B[AB^n=B^nA$

$\forall n$,whenever AB = BA]

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= $(A^m A)(B^m B) = A^m + 1B^m + 1$ So, it holds for n = m + 1 Hence. (AB)$^n - A^n B^n \forall n \in N$.

Choose the correct answer in the following questions : .

13. If$A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$is such that $A^2 = I$, then

    (a) $1 + \alpha^2 + \beta\gamma = 0$

    (b) $1 - \alpha^2 + \beta\gamma = 0$

    (c) $1 - \alpha^2 - \beta\gamma = 0$

    (d) $1 + \alpha^2 - \beta\gamma = 0$

SOLUTION .:

(C) Given $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$

Now, $A^2 = I \Rightarrow \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} \alpha^2 + \beta\gamma & \alpha\beta - \alpha\beta \\ \gamma\alpha - \alpha\gamma & \gamma\beta + \alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \gamma\beta + \alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \alpha^2 + \beta\gamma = 1 \Rightarrow 1 - \alpha^2 - \gamma\beta = 0$

14. If the matrix A is both symmetric and skew symmetric, then

    (a) A is a diagonal matrix

    (b) A is a zero matrix

    (c) A is a square matrix

    (d) None of these

SOLUTION .:

(B) Consider the matrix A. Clearly, $A' = A$ and A' =−A ∴ A =−A ⇒ 2A = 0 ⇒ A = 0 ∴ A is a zero matrix.

15. If A is a square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$is equal to

    (a) A

    (b) $I - A$

    (c) I

    (d) 3A

SOLUTION .:

(C) We are given that A2 = A Now, $(I + A)^3 - 7A = I^3 + A^3 + 3IA(I + A) - 7A$

$= I + A^2 + 3A(I + A) - 7A = I + A^2 + 3A + 3A^2 - 7A$

$= I + 4A^2 - 4A = I + 4A - 4A = I$

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