NCERT - Exercise 3.3

1. Find the transpose of each of the following matrices :

(i) $\begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

(iii) $\begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$

SOLUTION .:

(i) Transpose of $\begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}$

$= \begin{bmatrix} 5 & \frac{1}{2} & -1 \end{bmatrix}$

(ii) Transpose of $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

(iii) Transpose of $\begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$

$= \begin{bmatrix} -1 & \sqrt{3} & 2 \\ 5 & 5 & 3 \\ 6 & 6 & -1 \end{bmatrix}$

2. If $A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$, then verify that

(i) $(A+B)' = A' + B'$,

(ii) $(A-B)' = A' - B'$

SOLUTION .:

(i) $A+B = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$

$= \begin{bmatrix} -1-4 & 2+1 & 3-5 \\ 5+1 & 7+2 & 9+0 \\ -2+1 & 1+3 & 1+1 \end{bmatrix} = \begin{bmatrix} -5 & 3 & -2 \\ 6 & 9 & 9 \\ -1 & 4 & 2 \end{bmatrix}$

Now, $A' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix}$ and $B' = \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix} \Rightarrow (A+B)' = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix}$

and $A' + B' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} -1-4 & 5+1 & -2+1 \\ 2+1 & 7+2 & 1+3 \\ 3-5 & 9+0 & 1+1 \end{bmatrix} = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix} \therefore (A+B)' = A' + B'$

(ii) $A-B=\begin{bmatrix}-1&2&3\\5&7&9\\-2&1&1\end{bmatrix}=\begin{bmatrix}-4&1&-5\\1&2&0\\1&3&1\end{bmatrix}$

$=\begin{bmatrix}-1+4&2-1&3+5\\5-1&7-2&9-0\\-2-1&1-3&1-1\end{bmatrix}=\begin{bmatrix}3&1&8\\4&5&9\\-3&-2&0\end{bmatrix}$

$\Rightarrow (A-B)'$

$=\begin{bmatrix}3&4&-3\\1&5&-2\\8&9&0\end{bmatrix}$

and $A'-B'=\begin{bmatrix}-1&5&-2\\2&7&1\\3&9&1\end{bmatrix}-\begin{bmatrix}-4&1&1\\1&2&3\\-5&0&1\end{bmatrix}$

$=\begin{bmatrix}-1+4&5-1&-2-1\\2-1&7-2&1-3\\3+5&9-0&1-1\end{bmatrix}=\begin{bmatrix}3&4&-3\\1&5&-2\\8&9&0\end{bmatrix}$ $\therefore (A-B)'=A'-B'$.

3. If $A'=\begin{bmatrix}3&4\\-1&2\\0&1\end{bmatrix}$ and $B=\begin{bmatrix}-1&2&1\\1&2&3\end{bmatrix}$, then verify that

(i) $(A+B)'=A'+B'$

(ii) $(A-B)'=A'-B'$

SOLUTION .:

$A'=\begin{bmatrix}3&4\\-1&2\\0&1\end{bmatrix}\Rightarrow A=\begin{bmatrix}3&-1&0\\4&2&1\end{bmatrix}$

and $B=\begin{bmatrix}-1&2&1\\1&2&3\end{bmatrix}\Rightarrow B'=\begin{bmatrix}-1&1\\2&2\\1&3\end{bmatrix}$

(i) Now, $A+B=\begin{bmatrix}3&-1&0\\4&2&1\end{bmatrix}+\begin{bmatrix}-1&2&1\\1&2&3\end{bmatrix}=\begin{bmatrix}3-1&-1+2&0+1\\4+1&2+2&1+3\end{bmatrix}=\begin{bmatrix}2&1&1\\5&4&4\end{bmatrix}$

$\Rightarrow (A+B)'=\begin{bmatrix}2&5\\1&4\\1&4\end{bmatrix}$ $A'+B'=\begin{bmatrix}3&4\\-1&2\\0&1\end{bmatrix}+\begin{bmatrix}-1&1\\2&2\\1&3\end{bmatrix}=\begin{bmatrix}3-1&4+1\\-1+2&2+2\\0+1&1+3\end{bmatrix}=\begin{bmatrix}2&5\\1&4\\1&4\end{bmatrix}$

$\therefore (A+B)'=A'+B'$.

(ii) $A-B=\begin{bmatrix}3&-1&0\\4&2&1\end{bmatrix}-\begin{bmatrix}-1&2&1\\1&2&3\end{bmatrix}=\begin{bmatrix}3-(-1)&-1-2&0-1\\4-1&2-2&1-3\end{bmatrix}$

$=\begin{bmatrix}4&-3&-1\\3&0&-2\end{bmatrix}\Rightarrow (A-B)'=\begin{bmatrix}4&3\\-3&0\\-1&-2\end{bmatrix}$, $A'-B'$

$=\begin{bmatrix}3&4\\-1&2\\0&1\end{bmatrix}-\begin{bmatrix}-1&1\\2&2\\1&3\end{bmatrix}=\begin{bmatrix}3+1&4-1\\-1-1&2-2\\0-1&1-3\end{bmatrix}$

$=\begin{bmatrix}4&3\\-3&0\\-1&-2\end{bmatrix}$ $\therefore (A-B)'=A'=B'$.

4. If $A'=\begin{bmatrix}-2&3\\1&2\end{bmatrix}$ and $B=\begin{bmatrix}-1&0\\1&2\end{bmatrix}$, then find $(A+2B)'$.

SOLUTION .:

$A'=\begin{bmatrix}-2&3\\1&2\end{bmatrix}\Rightarrow A=\begin{bmatrix}-2&1\\3&2\end{bmatrix}$, B

$= \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix} \therefore A+2B = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} + 2\begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 0 \\ 2 & 4 \end{bmatrix}$

$= \begin{bmatrix} -2-2 & 1+0 \\ 3+2 & 2+4 \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ 5 & 6 \end{bmatrix}$

Hence, $(A+2B)' = \begin{bmatrix} -4 & 5 \\ 1 & 6 \end{bmatrix}$

5. For the matrices A and B, verify that (AB) =BA, where

(i) $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}, B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$

(ii) $A = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 5 & 7 \end{bmatrix}$

SOLUTION .:

(i) $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 1 & -4 & 3 \end{bmatrix}$

and $B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix} \Rightarrow B' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$

$AB = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix} \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1\times(-1) & 1\times 2 & 1\times 1 \\ -4\times(-1) & (-4)\times 2 & (-4)\times 1 \\ 3\times(-1) & 3\times 2 & 3\times 1 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}$

L.H.S. = $(AB) = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$

R.H.S. = $BA = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & -4 & 3 \end{bmatrix}$

$= \begin{bmatrix} -1\times 1 & -1\times(-4) & -1\times 3 \\ 2\times 1 & 2\times(-4) & 2\times 3 \\ 1\times 1 & 1\times(-4) & 1\times 3 \end{bmatrix} = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$

Hence, $(AB) = BA$.

(ii) $A = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 5 & 7 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 0 & 1 & 2 \end{bmatrix}$

and $B' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}$

Now, $AB = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 5 & 7 \end{bmatrix}$

$= \begin{bmatrix} 0\times 1 & 0\times 5 & 0\times 7 \\ 1\times 1 & 1\times 5 & 1\times 7 \\ 2\times 1 & 2\times 5 & 2\times 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 5 & 7 \\ 2 & 10 & 14 \end{bmatrix}$

$\Rightarrow (AB)'$

$= \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$

Now, $B'A' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 1\times 0 & 1\times 1 & 1\times 2 \\ 5\times 0 & 5\times 1 & 5\times 2 \\ 7\times 0 & 7\times 1 & 7\times 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$

$\therefore (AB)' = B'A'$.

6. If

$(i) A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$,then verify that $A'A = I$

(ii) $A = \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$, then verify that $A'A = I$

SOLUTION .:

(i) $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \Rightarrow A' = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

So, $A'A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$= \begin{bmatrix} \cos^2\alpha + \sin^2\alpha & \cos\alpha\sin\alpha - \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha - \cos\alpha\sin\alpha & \sin^2\alpha + \cos^2\alpha \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

Hence, $A'A = I$.

(ii) Given that, $A = \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix} \Rightarrow A' = \begin{bmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{bmatrix}$

So, $A'A = \begin{bmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{bmatrix} \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$

$= \begin{bmatrix} \sin^2\alpha + \cos^2\alpha & \sin\alpha\cos\alpha - \cos\alpha\sin\alpha \\ \cos\alpha\sin\alpha - \sin\alpha\cos\alpha & \cos^2\alpha + \sin^2\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

Hence, $A'A = I$.

7. (i) Show that the matrix $A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$ is a symmetric matrix.

(ii) Show that the matrix $A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ is a skew symmetric matrix.

SOLUTION .:

(i) A square matrix A is said to be symmetric, if A = A. As, $A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix} \Rightarrow A' = A$

So, A is a symmetric matrix.

(ii) A square matrix A is said to be skew symmetric matrix if A = −A.

As, $A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} = -A \Rightarrow A' = -A$.

So, A is a skew symmetric matrix.

8. For the matrix $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$, verify that

(i) (A + A) is a symmetric matrix.

(ii) (A−A) is a skew-symmetric matrix.

SOLUTION .:

$A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$

(i) $A + A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 1+1 & 5+6 \\ 6+5 & 7+7 \end{bmatrix} = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}$

$\Rightarrow (A + A')' = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix} = (A + A')$

So, A + A is a symmetric matrix.

(ii) $A - A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 1-1 & 5-6 \\ 6-5 & 7-7 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \Rightarrow (A - A')$

$= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = -(A - A')$

So, A−A is a skew−symmetric matrix.

9. Find (A+ A) and $\frac{1}{2}(A - A')$, when , $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$

SOLUTION .:

$A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$

Now, A + A= $\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} + \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$

$= \begin{bmatrix} 0+0 & a-a & b-b \\ -a+a & 0+0 & c-c \\ -b+b & -c+c & 0+0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \therefore \frac{1}{2}(A + A') = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Now, $A - A' = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} - \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$

$= \begin{bmatrix} 0-0 & a+a & b+b \\ -a-a & 0-0 & c+c \\ -b-b & -c-c & 0-0 \end{bmatrix} - \begin{bmatrix} 0 & 2a & 2b \\ -2a & 0 & 2c \\ -2b & -2c & 0 \end{bmatrix}$

$\therefore \frac{1}{2}(A - A') = \frac{1}{2}\begin{bmatrix} 0 & 2a & 2b \\ -2a & 0 & 2c \\ -2b & -2c & 0 \end{bmatrix} = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$

10. Express the following matrices as the sum of a symmetric and a skew symmetric matrix :

(i) $\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$

(ii) $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

(iii) $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$

(iv) $\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$

SOLUTION .

:(i) Here, A = $\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$

Let $P = \frac{1}{2}(A + A')$

Now, $A+A'=\begin{bmatrix}3&5\\1&-1\end{bmatrix}+\begin{bmatrix}3&1\\5&-1\end{bmatrix}=\begin{bmatrix}3+3&5+1\\1+5&-1-1\end{bmatrix}=\begin{bmatrix}6&6\\6&-2\end{bmatrix}\Rightarrow P=\frac{1}{2}(A+A')$

$=\frac{1}{2}\begin{bmatrix}6&6\\6&-2\end{bmatrix}=\begin{bmatrix}3&3\\3&-1\end{bmatrix}$

$\Rightarrow P'=\begin{bmatrix}3&3\\3&-1\end{bmatrix}=P$

Thus, P = $\frac{1}{2}$ (A+ A) is a symmetric matrix. Also, let Q= $\frac{1}{2}$(A−A)

Now, A−A= $\begin{bmatrix}3&5\\1&-1\end{bmatrix}-\begin{bmatrix}3&1\\5&-1\end{bmatrix}=\begin{bmatrix}3-3&5-1\\1-5&-1+1\end{bmatrix}=\begin{bmatrix}0&4\\-4&0\end{bmatrix}$

$\Rightarrow Q=\frac{1}{2}(A-A')=\frac{1}{2}\begin{bmatrix}0&4\\-4&0\end{bmatrix}=\begin{bmatrix}0&2\\-2&0\end{bmatrix}$

$\Rightarrow Q'=\begin{bmatrix}0&-2\\2&0\end{bmatrix}=-\begin{bmatrix}0&2\\-2&0\end{bmatrix}=-Q$ Thus, $Q=\frac{1}{2}$= (A− A) is a skew−symmetric matrix. $\therefore P+Q=A=\begin{bmatrix}3&3\\3&-1\end{bmatrix}+$
$\begin{bmatrix}0&2\\-2&0\end{bmatrix}$

(ii) Here, $A=\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}\Rightarrow A'=\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}$

Let $P=\frac{1}{2}(A+A')\ \therefore A+A'=\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}+\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}$

$=\begin{bmatrix}6+6&-2-2&2+2\\-2+(-2)&3+3&-1+(-1)\\2+2&-1+(-1)&3+3\end{bmatrix}=\begin{bmatrix}12&-4&4\\-4&6&-2\\4&-2&6\end{bmatrix}$

$\Rightarrow P=\frac{1}{2}(A+A')=\frac{1}{2}\begin{bmatrix}12&-4&4\\-4&6&-2\\4&-2&6\end{bmatrix}=\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}\Rightarrow P'=\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}=\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}=P$

Thus, $P=\frac{1}{2}(A+A')$is a symmetric matrix.

Also, let $Q=\frac{1}{2}(A-A')$ Now, A−A= $\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}-\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}$

$=\begin{bmatrix}6-6&-2+2&2-2\\-2+2&3-3&-1+1\\2-2&-1+1&3-3\end{bmatrix}=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}\Rightarrow Q=\frac{1}{2}(A-A')=\frac{1}{2}\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$ and $Q'=-Q$

So, Q = $\frac{1}{2}$(A−A) is a skew symmetric matrix.

Hence, $P+Q=A=\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}+\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$

(iii) Here, $A=\begin{bmatrix}3&3&-1\\-2&-2&1\\-4&-5&2\end{bmatrix}\Rightarrow A'=\begin{bmatrix}3&-2&-4\\3&-2&-5\\-1&1&2\end{bmatrix}$

Let $P=\frac{1}{2}(A+A')$

Now, $A+A'=\begin{bmatrix}3&3&-1\\-2&-2&1\\-4&-5&2\end{bmatrix}+\begin{bmatrix}3&-2&-4\\3&-2&-5\\-1&1&2\end{bmatrix}$

$=\begin{bmatrix}3+3&3-2&-1-4\\-2+3&-2-2&1-5\\-4-1&-5+1&2+2\end{bmatrix}=\begin{bmatrix}6&1&-5\\1&-4&-4\\-5&-4&4\end{bmatrix}$

So, $P=\frac{1}{2}(A+A')=\frac{1}{2}\begin{bmatrix}6&1&-5\\1&-4&-4\\-5&-4&4\end{bmatrix}$

$=\begin{bmatrix}3&1/2&-5/2\\1/2&-2&-2\\-5/2&-2&2\end{bmatrix}$ and $P'=\begin{bmatrix}3&1/2&-5/2\\1/2&-2&-2\\-5/2&-2&2\end{bmatrix}=P$

Thus, P = $\frac{1}{2}(A+A')$is a symmetric matrix. Also, let Q = $\frac{1}{2}(A-A')$

Again, $A-A'=\begin{bmatrix}3&3&-1\\-2&-2&1\\-4&-5&2\end{bmatrix}-\begin{bmatrix}3&-2&-4\\3&-2&-5\\-1&1&2\end{bmatrix}=\begin{bmatrix}3-3&3+2&-1+4\\-2-3&-2+2&1+5\\-4+1&-5-1&2-2\end{bmatrix}=\begin{bmatrix}0&5&3\\-5&0&6\\-3&-6&0\end{bmatrix}$

$\therefore\frac{1}{2}(A-A')=\frac{1}{2}\begin{bmatrix}0&5&3\\-5&0&6\\-3&-6&0\end{bmatrix}=\begin{bmatrix}0&5/2&3/2\\-5/2&0&3\\-3/2&-3&0\end{bmatrix}$

$\Rightarrow Q'=\begin{bmatrix}0&-5/2&-3/2\\5/2&0&-3\\3/2&-3&0\end{bmatrix}=-\begin{bmatrix}0&5/2&3/2\\-5/2&0&3\\-3/2&-3&0\end{bmatrix}=-Q.$

Thus, Q =$\frac{1}{2}$(A−A) is a skew symmetric matrix.

Hence, $P+Q=A=\begin{bmatrix}3&\frac{1}{2}&\frac{-5}{2}\\\frac{1}{2}&-2&-2\\\frac{-5}{2}&-2&2\end{bmatrix}+\begin{bmatrix}0&\frac{5}{2}&\frac{3}{2}\\\frac{-5}{2}&0&3\\\frac{-3}{2}&-3&0\end{bmatrix}$

(iv) Here, $A=\begin{bmatrix}1&5\\-1&2\end{bmatrix}\Rightarrow A'=\begin{bmatrix}1&-1\\5&2\end{bmatrix}$

Let $P=\frac{1}{2}(A+A')$ Now, $A+A'=\begin{bmatrix}1&5\\-1&5\end{bmatrix}+\begin{bmatrix}1&-1\\5&2\end{bmatrix}=\begin{bmatrix}1+1&5-1\\-1+5&2+2\end{bmatrix}=\begin{bmatrix}2&4\\4&4\end{bmatrix}$

$\Rightarrow P=\frac{1}{2}(A+A')$

$=\frac{1}{2}\begin{bmatrix}2&4\\4&4\end{bmatrix}=\begin{bmatrix}1&2\\2&2\end{bmatrix}\Rightarrow P'=\begin{bmatrix}1&2\\2&2\end{bmatrix}=P$

Thus, P = $\frac{1}{2}$(A+ A) is a symmetric matrix.

Also, let Q = $\frac{1}{2}$(A−A)

Now, A−A= $\begin{bmatrix}1&5\\-1&2\end{bmatrix}-\begin{bmatrix}1&-1\\5&2\end{bmatrix}=\begin{bmatrix}1-1&5+1\\-1-5&2-2\end{bmatrix}=\begin{bmatrix}0&6\\-6&0\end{bmatrix}$

$\Rightarrow Q=\frac{1}{2}(A-A')=\frac{1}{2}\begin{bmatrix}0&6\\-6&0\end{bmatrix}=\begin{bmatrix}0&3\\3&0\end{bmatrix}$

$\Rightarrow Q'=\begin{bmatrix}0&-3\\3&0\end{bmatrix}=-\begin{bmatrix}0&3\\-3&0\end{bmatrix}=-Q.$

Thus, Q =$\frac{1}{2}$(A−A) is a skew symmetric matrix.

Hence, P + Q = A= $\begin{bmatrix}1&2\\2&2\end{bmatrix}+\begin{bmatrix}0&3\\-3&0\end{bmatrix}$.

Choose the correct answer in the questions 11 and 12.

11. If A, B are symmetric matrices of same order, then AB−BA is a

(a) Skew symmetric matrix

(b) Symmetric matrix

(c) Zero matrix

(d) Identity matrix

SOLUTION .:

(A) We know that, A = A, B = B So, (AB−BA) = (AB)−(BA) = BA−AB = BA−AB=−(AB−BA) ∴ AB−BA is a skew−symmetric matrix.

12. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$,then $A + A' = I$,If the value of α is

(a) $\frac{\pi}{6}$

(b) $\frac{\pi}{3}$

(c) π

(d) $\frac{3\pi}{2}$

SOLUTION .:

(B) Given that, $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \Rightarrow A' = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

We know that, A + A = I $\Rightarrow \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow 2\cos\alpha = 1 \Rightarrow \cos\alpha = \frac{1}{2}$

$\Rightarrow \cos\alpha = \cos\frac{\pi}{3} \Rightarrow \alpha = \frac{\pi}{3}$