



## NCERT - Exercise 3.2

1. Let  $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$ ,  $C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

Find each of the following :

(i)  $A + B$

(ii)  $A - B$

(iii)  $3A - C$

(iv)  $AB$

(v)  $BA$

**SOLUTION** ∴

Here, A is a 2 × 2 matrix, B is a 2 × 2 matrix and C is a 2 × 2 matrix. So, A, B, C are comparable.

(i)

$$A + B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 2+1 & 4+3 \\ 3+(-2) & 2+5 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 1 & 7 \end{bmatrix}$$

(ii)

$$A - B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 2-1 & 4-3 \\ 3-(-2) & 2-5 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 5 & -3 \end{bmatrix}$$

(iii)

$$3A - C = 3 \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 12 \\ 9 & 6 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} \\ = \begin{bmatrix} 6+2 & 12-5 \\ 9-3 & 6-4 \end{bmatrix} = \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$$

(iv)

$$AB = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 2 \times 1 + 4 \times (-2) & 2 \times 3 + 4 \times 5 \\ 3 \times 1 + 2 \times (-2) & 3 \times 3 + 2 \times 5 \end{bmatrix} \\ = \begin{bmatrix} 2-8 & 6+20 \\ 3-4 & 9+10 \end{bmatrix} = \begin{bmatrix} -6 & 26 \\ -1 & 19 \end{bmatrix}$$

(v)

$$BA = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 1 \times 2 + 3 \times 3 & 1 \times 4 + 3 \times 2 \\ -2 \times 2 + 5 \times 3 & -2 \times 4 + 5 \times 2 \end{bmatrix} \\ = \begin{bmatrix} 2+9 & 4+6 \\ -4+15 & -8+10 \end{bmatrix} = \begin{bmatrix} 11 & 10 \\ 11 & 2 \end{bmatrix}$$

2. Compute the following s

(i)  $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix}$

(ii)  $\begin{bmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix}$

(iii)  $\begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix}$

(iv)  $\begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix}$

**SOLUTION** ∴

(i)

We have,  $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix} = \begin{bmatrix} a+a & b+b \\ -b+b & a+a \end{bmatrix} = \begin{bmatrix} 2a & 2b \\ 0 & 2a \end{bmatrix}$

(ii)

$$\text{We have, } \begin{bmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix} = \begin{bmatrix} a^2+b^2+2ab & b^2+c^2+2bc \\ a^2+c^2-2ac & a^2+b^2-2ab \end{bmatrix} = \begin{bmatrix} (a+b)^2 & (b+c)^2 \\ (a-c)^2 & (a-b)^2 \end{bmatrix}$$

(iii)

$$\text{We have, } \begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix} = \begin{bmatrix} -1+12 & 4+7 & -6+6 \\ 8+8 & 5+0 & 16+5 \\ 2+3 & 8+2 & 5+4 \end{bmatrix} = \begin{bmatrix} 11 & 11 & 0 \\ 16 & 5 & 21 \\ 5 & 10 & 9 \end{bmatrix}$$

(iv)

$$\begin{aligned} \text{We have, } & \begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix} \\ &= \begin{bmatrix} \cos^2 x + \sin^2 x & \sin^2 x + \cos^2 x \\ \sin^2 x + \cos^2 x & \cos^2 x + \sin^2 x \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \end{aligned}$$

3. Compute the following products.

(i)

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$$

$$\text{(iv)} \quad \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$$

$$\text{(v)} \quad \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$$

$$\text{(vi)} \quad \begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$$

**SOLUTION** ∴

(i)

$$\begin{aligned} \text{We have, } & \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} a & -b \\ b & a \end{bmatrix} \\ &= \begin{bmatrix} a^2+b^2 & -ab+ab \\ -ab+ab & b^2+a^2 \end{bmatrix} = \begin{bmatrix} a^2+b^2 & 0 \\ 0 & a^2+b^2 \end{bmatrix} \end{aligned}$$

$$\text{(ii) We have, } \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 2 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 \times 2 & 1 \times 3 & 1 \times 4 \\ 2 \times 2 & 2 \times 3 & 2 \times 4 \\ 3 \times 2 & 3 \times 3 & 3 \times 4 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 6 & 8 \\ 6 & 9 & 12 \end{bmatrix}$$

(iii)

$$\begin{aligned} \text{We have, } & \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 \times 1 + (-2) \times 2 & 1 \times 2 + (-2) \times 3 & 1 \times 3 + (-2) \times 1 \\ 2 \times 1 + 3 \times 2 & 2 \times 2 + 3 \times 3 & 2 \times 3 + 3 \times 1 \end{bmatrix} \\ &= \begin{bmatrix} 1-4 & 2-6 & 3-2 \\ 2+6 & 4+9 & 6+3 \end{bmatrix} = \begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix} \end{aligned}$$

(iv)

$$\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \times 1 + 3 \times 0 + 4 \times 3 & 2 \times (-3) + 3 \times 2 + 4 \times 0 & 2 \times 5 + 3 \times 4 + 4 \times 5 \\ 3 \times 1 + 4 \times 0 + 5 \times 3 & 3 \times (-3) + 4 \times 2 + 5 \times 0 & 3 \times 5 + 4 \times 4 + 5 \times 5 \\ 4 \times 1 + 5 \times 0 + 6 \times 3 & 4 \times (-3) + 5 \times 2 + 6 \times 0 & 4 \times 5 + 5 \times 4 + 6 \times 5 \end{bmatrix} \begin{bmatrix} 2+12 & -6+6 & 10+12+20 \\ 3+15 & -9+8 & 15+16+25 \\ 4+18 & -12+10 & 20+20+30 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 0 & 42 \\ 18 & -1 & 56 \\ 22 & -2 & 70 \end{bmatrix}$$

$$(v) \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 2 \times 1 + 1 \times (-1) & 2 \times 0 + 1 \times 2 & 2 \times 1 + 1 \times 1 \\ 3 \times 1 + 2 \times (-1) & 3 \times 0 + 2 \times 2 & 3 \times 1 + 2 \times 1 \\ -1 \times 1 + 1 \times (-1) & -1 \times 0 + 1 \times 2 & -1 \times 1 + 1 \times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & 2 & 2+1 \\ 3-2 & 4 & 3+2 \\ -1-1 & 2 & -1+1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ -2 & 2 & 0 \end{bmatrix}$$

(vi)

$$\begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \times 2 + (-1) \times 1 + 3 \times 3 & 3 \times (-3) + (-1) \times 0 + 3 \times 1 \\ -1 \times 2 + 0 \times 1 + 2 \times 3 & -1 \times (-3) + 0 \times 0 + 2 \times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6-1+9 & -9+0+3 \\ -2+0+6 & 3+0+2 \end{bmatrix} = \begin{bmatrix} 14 & -6 \\ 4 & 5 \end{bmatrix}$$

4. If  $A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$  and  $C = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$ , then compute  $(A+B)$  and  $(B-C)$ . Also, verify that  $A + (B-C) = (A+B)-C$ .

**SOLUTION** ∴

Here, A, B and C is a 3 × 3 matrix. So, A, B and C are comparable. So,  $(A+B)$ ,  $(B-C)$ ,  $A + (B-C)$  and  $(A+B)-C$  are defined and each one is 3 × 3 matrix.

$$A+B = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix} \quad B-C = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix}$$

$$A + (B-C) = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix}$$

$$(A+B)-C = \begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix}$$

Hence,  $A + (B-C) = (A+B)-C$ .

5. If  $A = \begin{bmatrix} \frac{2}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & 2 & 4 \\ \frac{3}{7} & \frac{3}{3} & \frac{3}{2} \\ \frac{3}{3} & 2 & \frac{3}{3} \end{bmatrix}$  and  $B = \begin{bmatrix} \frac{2}{5} & \frac{3}{5} & 1 \\ \frac{1}{5} & 2 & 4 \\ \frac{5}{5} & \frac{5}{5} & \frac{5}{5} \\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5} \end{bmatrix}$ , then compute  $3A - 5B$ .

**SOLUTION** ∴

$$3A - 5B = 3 \begin{bmatrix} \frac{2}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & 2 & 4 \\ \frac{3}{7} & \frac{3}{3} & \frac{3}{2} \\ \frac{3}{3} & 2 & \frac{3}{3} \end{bmatrix} - 5 \begin{bmatrix} \frac{2}{5} & \frac{3}{5} & 1 \\ \frac{1}{5} & 2 & 4 \\ \frac{5}{5} & \frac{5}{5} & \frac{5}{5} \\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5} \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

6. . Simplify,  $\cos \theta \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} + \sin \theta \begin{bmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{bmatrix}$

**SOLUTION** .:

$$\begin{aligned} \text{We have, } & \cos \theta \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} + \sin \theta \begin{bmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{bmatrix} \\ = & \begin{bmatrix} \cos^2 \theta & \sin \theta \cos \theta \\ -\sin \theta \cos \theta & \cos^2 \theta \end{bmatrix} + \begin{bmatrix} \sin^2 \theta & -\sin \theta \cos \theta \\ \sin \theta \cos \theta & \sin^2 \theta \end{bmatrix} \\ = & \begin{bmatrix} \cos^2 \theta + \sin^2 \theta & \sin \theta \cos \theta - \sin \theta \cos \theta \\ -\sin \theta \cos \theta + \cos \theta \sin \theta & \cos^2 \theta + \sin^2 \theta \end{bmatrix} \\ = & \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

7. Find X and Y, if

(i)  $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$  and  $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$   
 (ii)  $2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$  and  $3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$

**SOLUTION** .:

(i)

We are given that  $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$  ..(i)

and  $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$  ..(ii)

Adding (i) and (ii), we get

$$(X + Y) + (X - Y) = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \Rightarrow 2X = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix} \Rightarrow X = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$$

Subtracting (ii) from (i), we get

$$\Rightarrow 2Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix} \Rightarrow Y = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

Hence,  $X = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$  and  $Y = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

(ii) We have,  $2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$  ..(i)

and  $3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$

Adding (i) & (ii), we get

$$5X + 5Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$$

$$\Rightarrow 5X + 5 = \begin{bmatrix} 4 & 1 \\ 3 & 5 \end{bmatrix} \Rightarrow X + Y = \frac{1}{5} \begin{bmatrix} 4 & 1 \\ 3 & 5 \end{bmatrix} \text{ ..(iii)}$$

Subtracting (i) from (ii),

$$\text{we get } (3X + 2Y) - (2X + 3Y) = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$\Rightarrow X - Y = \begin{bmatrix} 0 & -5 \\ -5 & 5 \end{bmatrix} \text{ ..(iv)}$$

Finally, adding (iii) and (iv), we get

$$2X = \frac{1}{5} \begin{bmatrix} 4 & 1 \\ 3 & 5 \end{bmatrix} + \begin{bmatrix} 0 & -5 \\ -5 & 5 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} \frac{4}{5} & \frac{1}{5} \\ \frac{3}{5} & 1 \end{bmatrix} + \begin{bmatrix} 0 & -5 \\ -5 & 5 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} \frac{4}{5} & \frac{1}{5}-5 \\ \frac{3}{5}-5 & 1+5 \end{bmatrix} = \begin{bmatrix} \frac{4}{5} & \frac{-24}{5} \\ \frac{-22}{5} & 6 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{2} \begin{bmatrix} \frac{4}{5} & \frac{-24}{5} \\ \frac{-22}{5} & 6 \end{bmatrix} \Rightarrow X = \begin{bmatrix} \frac{2}{5} & \frac{-12}{5} \\ \frac{-11}{5} & 3 \end{bmatrix}$$

Subtracting (iii) and (iv), we get

$$2Y = \frac{1}{5} \begin{bmatrix} 4 & 1 \\ 3 & 5 \end{bmatrix} - \begin{bmatrix} 0 & -5 \\ -5 & 5 \end{bmatrix}$$

$$\Rightarrow 2Y = \begin{bmatrix} \frac{4}{5} & \frac{1}{5} \\ \frac{3}{5} & 1 \end{bmatrix} + \begin{bmatrix} 0 & 5 \\ 5 & -5 \end{bmatrix} = \begin{bmatrix} \frac{4}{5} & \frac{26}{5} \\ \frac{28}{5} & -4 \end{bmatrix}$$

$$\Rightarrow Y = \frac{1}{2} \begin{bmatrix} \frac{4}{5} & \frac{26}{5} \\ \frac{28}{5} & -4 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$$

Hence,  $X = \begin{bmatrix} \frac{2}{5} & \frac{-12}{5} \\ \frac{-11}{5} & 3 \end{bmatrix}, Y = \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$

8. Find X, if  $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$  and  $2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$

**SOLUTION** ∴

We are given that,  $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$  ..(i)

and  $2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$  ..(ii)

Substituting the value of Y in (ii), we have

$$2X + \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} \Rightarrow 2X = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix} \Rightarrow X = \frac{1}{2} \begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}$$

Hence,  $X = \begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$

9. Find x and y, if  $2 \begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

**SOLUTION** ∴

We have,  $2 \begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$$\Rightarrow 2 + y = 5 \text{ and } 2x + 2 = 8$$

$$\Rightarrow y = 3 \text{ and } x = 3$$

Hence,  $x = 3$  and  $y = 3$ .

10. Solve the equation for x, y, z and t, if

$$2 \begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3 \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3 \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$$

**SOLUTION** ∴

$$2 \begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3 \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3 \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x & 2z \\ 2y & 2t \end{bmatrix} + \begin{bmatrix} 3 & -3 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x+3 & 2z-3 \\ 2y+0 & 2t+6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$$

$$\text{Now, } 2x + 3 = 9 \Rightarrow 2x = 6 \Rightarrow x = 3$$

$$2z - 3 = 15 \Rightarrow 2z = 18 \Rightarrow z = 9$$

$$2y = 12 \Rightarrow y = 6$$

$$2t + 6 = 18 \Rightarrow 2t = 12 \Rightarrow t = 6$$

Hence,  $x = 3$ ,  $y = 6$ ,  $z = 9$  and  $t = 6$ .

11. If  $x \begin{bmatrix} 2 \\ 3 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$ , find the values of  $x$  and  $y$ .

**SOLUTION** ∴

$$\text{We have, } x \begin{bmatrix} 2 \\ 3 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix},$$

$$\Rightarrow \begin{bmatrix} 2x \\ 3x \end{bmatrix} + \begin{bmatrix} -y \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 2x-y \\ 3x+y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

$$\Rightarrow 2x - y = 10 \text{ (i) and } 3x + y = 5 \text{ (ii)}$$

Solving (i) & (ii), we get  $x = 3$  and  $y = -4$

Hence,  $x = 3$  and  $y = -4$ .

12. Given  $3 \begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$ ,

find the values of  $x$ ,  $y$ ,  $z$  and  $w$ .

**SOLUTION** ∴

$$3 \begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3x & 3y \\ 3z & 3w \end{bmatrix} = \begin{bmatrix} x+4 & 6+x+y \\ -1+z+w & 2w+3 \end{bmatrix}$$

$$\text{Now, } 3x = x + 4 \Rightarrow 2x = 4 \Rightarrow x = 2$$

$$3y = 6 + x + y \Rightarrow 2y = x + 6 \Rightarrow 2y = 2 + 6$$

$$\Rightarrow 2y = 8 \Rightarrow y = 4 \quad 3w = 2w + 3 \Rightarrow w = 3$$

$$3z = -1 + z + w \Rightarrow 2z = -$$

$$1 + 3 \Rightarrow 2z = 2 \Rightarrow z = 1$$

Hence,  $x = 2$ ,  $y = 4$ ,  $z = 1$  and  $w = 3$

13. If  $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ , then show that  $F(x) \cdot F(y) = F(x+y)$ .

**SOLUTION** ∴

$$\text{We are given that, } F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Now, } F(y) = \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow F(x) \cdot F(y) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned}
&= \begin{bmatrix} \cos x \cos y - \sin x \sin y & -\sin y \cos x - \sin x \cos y & 0 \\ \sin x \cos y + \cos x \sin y & -\sin x \sin y + \cos x \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ Also, } F(x+y) = \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix} \therefore F(x) \cdot F(y) = F(x+y).
\end{aligned}$$

14. Show that

$$\begin{aligned}
\text{(i)} & \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \neq \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \\
\text{(ii)} & \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \neq \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}
\end{aligned}$$

**SOLUTION** ∴

(i)

$$\text{Let } A = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \Rightarrow AB = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 10-3 & 5-4 \\ 12+21 & 6+28 \end{bmatrix} = \begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix}$$

and

$$BA = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} = \begin{bmatrix} 10+6 & -2+7 \\ 15+24 & -3+28 \end{bmatrix} = \begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix}$$

Hence,  $AB \neq BA$ .

(ii)

$$\begin{aligned}
\text{Let } A &= \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \Rightarrow AB = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \\
&= \begin{bmatrix} 1 \times (-1) + 2 \times 0 + 3 \times 2 & 1 \times 1 + 2 \times (-1) + 3 \times 3 & 1 \times 0 + 2 \times 1 + 3 \times 4 \\ 0 \times (-1) + 1 \times 0 + 0 \times 2 & 0 \times 1 + 1 \times (-1) + 0 \times 3 & 0 \times 0 + 1 \times 1 + 0 \times 4 \\ 1 \times (-1) + 1 \times 0 + 0 \times 2 & 1 \times 1 + 1 \times (-1) + 0 \times 3 & 1 \times 0 + 1 \times 1 + 0 \times 4 \end{bmatrix} \\
&= \begin{bmatrix} -1+0+6 & 1-2+9 & 0+2+12 \\ 0+0+0 & 0-1+0 & 0+1+0 \\ -1+0+0 & 1-1+0 & 0+1+0 \end{bmatrix} = \begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix} \\
\text{and } BA &= \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} -1 \times 1 + 1 \times 0 + 0 \times 1 & -1 \times 2 + 1 \times 1 + 0 \times 1 & -1 \times 3 + 1 \times 0 + 0 \times 0 \\ 0 \times 1 - 1 \times 0 + 1 \times 1 & 0 \times 2 - 1 \times 1 + 1 \times 1 & 0 \times 3 - 1 \times 0 + 1 \times 0 \\ 2 \times 1 + 3 \times 0 + 4 \times 1 & 2 \times 2 + 3 \times 1 + 4 \times 1 & 2 \times 3 + 3 \times 0 + 4 \times 0 \end{bmatrix} \\
&= \begin{bmatrix} -1+0+0 & -2+1+0 & -3+0+0 \\ 0-0+1 & 0-1+1 & 0-0+0 \\ 2+0+4 & 4+3+4 & 6+0+0 \end{bmatrix} = \begin{bmatrix} -1 & -1 & -3 \\ 1 & 0 & 0 \\ 6 & 11 & 6 \end{bmatrix}
\end{aligned}$$

Hence, L.H.S.  $\neq$  R.H.S.

$$15. \text{ Find } A^2 - 5A + 6I, \text{ If } A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

**SOLUTION** ∴

$$\text{We are given that } A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$\Rightarrow A^2 = A \cdot A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \times 2 + 0 \times 2 + 1 \times 1 & 2 \times 0 + 0 \times 1 + 1 \times (-1) & 2 \times 1 + 0 \times 3 + 1 \times 0 \\ 2 \times 2 + 1 \times 2 + 3 \times 1 & 2 \times 0 + 1 \times 1 + 3 \times (-1) & 2 \times 1 + 1 \times 3 + 3 \times 0 \\ 1 \times 2 + 2 \times (-1) + 0 \times 1 & 1 \times 0 + (-1) \times 1 + 0 \times (-1) & 1 \times 1 + (-1) \times 3 + 0 \times 0 \end{bmatrix}$$

$$\begin{aligned}
&= \begin{bmatrix} 4+0+1 & 0+0-1 & 2+0+0 \\ 4+2+3 & 0+1-3 & 2+3+0 \\ 2-2+0 & 0-1+0 & 1-3+0 \end{bmatrix} \\
&= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} \quad 5A = 5 \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix} \\
6I &= 6 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} \\
\therefore A^2 - 5A + 6I &= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} \\
&= \begin{bmatrix} -5 & -1 & -3 \\ -1 & -7 & -10 \\ -5 & 4 & -2 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}
\end{aligned}$$

16. If  $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ , prove that  $A^3 - 6A^2 + 7A + 2I = O$ .

**SOLUTION** ∴

We are given that,  $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \Rightarrow A^2 = A \cdot A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$

$$\begin{aligned}
&= \begin{bmatrix} 1 \times 1 + 0 \times 0 + 2 \times 2 & 0 \times 1 + 0 \times 2 + 0 \times 2 & 1 \times 2 + 0 \times 1 + 2 \times 3 \\ 0 \times 1 + 2 \times 0 + 1 \times 2 & 0 \times 0 + 2 \times 2 + 1 \times 0 & 0 \times 2 + 2 \times 1 + 3 \times 1 \\ 2 \times 1 + 0 \times 0 + 3 \times 2 & 2 \times 0 + 0 \times 2 + 3 \times 0 & 2 \times 2 + 0 \times 1 + 3 \times 3 \end{bmatrix} \\
&= \begin{bmatrix} 1+0+4 & 0+0+0 & 2+0+6 \\ 0+0+2 & 0+4+0 & 0+2+3 \\ 2+0+6 & 0+0+0 & 4+0+9 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} \\
\text{Now, } A^3 &= A^2 \cdot A = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 5 \times 1 + 0 \times 0 + 8 \times 2 & 5 \times 0 + 0 \times 2 + 8 \times 0 & 5 \times 2 + 0 \times 1 + 8 \times 3 \\ 2 \times 1 + 4 \times 0 + 5 \times 2 & 2 \times 0 + 4 \times 2 + 5 \times 0 & 2 \times 2 + 4 \times 1 + 5 \times 3 \\ 8 \times 1 + 0 \times 0 + 13 \times 2 & 8 \times 0 + 0 \times 2 + 13 \times 0 & 8 \times 2 + 1 \times 0 + 13 \times 3 \end{bmatrix} \\
&= \begin{bmatrix} 5+0+16 & 0+0+0 & 10+0+24 \\ 2+0+10 & 0+8+0 & 4+4+15 \\ 8+0+26 & 0+0+0 & 16+0+39 \end{bmatrix} = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} \\
\text{Now L.H.S.} &= A^3 - 6A^2 + 7A + 2I = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - 6 \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} + 2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \\
&= \begin{bmatrix} 21-30 & 0-0 & 34-48 \\ 12-12 & 8-24 & 23-30 \\ 34-48 & 0-0 & 55-78 \end{bmatrix} + \begin{bmatrix} 7+2 & 0+0 & 14+0 \\ 0+0 & 14+2 & 7+0 \\ 14+0 & 0+0 & 21+2 \end{bmatrix} = \begin{bmatrix} -9 & 0 & -14 \\ 0 & -16 & -7 \\ -14 & 0 & -23 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 14 \\ 0 & 16 & 7 \\ 14 & 0 & 23 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O \\
&= \text{R.H.S.}
\end{aligned}$$

Hence, proved.

17. If  $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$  and  $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ , find k so that  $A^2 = kA = 2I$ .

**SOLUTION** ∴

We are given that,  $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$  and  $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Also,  $A^2 = kA - 2I$

Substituting the values of A and I from above, we get



$$\Rightarrow \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = k \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 \times 3 + (-2) \times 4 & 3 \times (-2) + (-2) \times (-2) \\ 4 \times 3 + (-2) \times 4 & 4 \times (-2) + (-2) \times (-2) \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$$

$$\Rightarrow 4k = 4 \Rightarrow k = 1 \text{ Hence, } k = 1$$

18. If  $A = \begin{bmatrix} 0 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 0 \end{bmatrix}$  and I is the identity matrix of order 2,

then show that  $I + A = (I - A) \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$ .

**SOLUTION** ∴

We know that,  $\cos \alpha = \frac{1 - \tan^2 \left( \frac{\alpha}{2} \right)}{1 + \tan^2 \left( \frac{\alpha}{2} \right)} = \frac{1 - t^2}{1 + t^2}$ , where  $\tan \frac{\alpha}{2} = t$

and  $\sin \alpha = \frac{2 \tan \left( \frac{\alpha}{2} \right)}{1 + \tan^2 \left( \frac{\alpha}{2} \right)} = \frac{2t}{1 + t^2}$

Now,  $I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -t \\ t & 0 \end{bmatrix} = \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix}$

and  $I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -t \\ t & 0 \end{bmatrix} = \begin{bmatrix} 1 & t \\ -t & 1 \end{bmatrix} \therefore (I - A) \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} = \begin{bmatrix} 1 & t \\ -t & 1 \end{bmatrix} \begin{bmatrix} \frac{(1-t^2)}{(1+t^2)} & \frac{-2t}{(1+t^2)} \\ \frac{2t}{(1+t^2)} & \frac{(1-t^2)}{(1+t^2)} \end{bmatrix}$

$= \begin{bmatrix} \frac{1-t^2}{1+t^2} + \frac{2t^2}{1+t^2} & \frac{-2t}{1+t^2} + \frac{t(1-t^2)}{1+t^2} \\ \frac{-t(1-t^2)}{1+t^2} + \frac{2t}{1+t^2} & \frac{1-t^2}{1+t^2} + \frac{2t^2}{1+t^2} \end{bmatrix} = \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix} = (I + A)$

Hence,  $(I - A) \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} = (I + A)$ .

19. A trust fund has Rs. 30,000 that must be invested in two different types of bonds. The first bond pays 5% interest per year, and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide Rs. 30,000 among the two types of bonds, if the trust fund obtains an annual total interest of :

(A) Rs. 1800

(B) Rs. 2000

**SOLUTION** ∴

Let us take that the trust invests Rs. x at 5% p.a. and then the trust invests Rs. (30,000-x) at 7% p.a.

(A) So,  $[x \ 30,000-x] \begin{bmatrix} 5\% \\ 7\% \end{bmatrix} = 1800$

$\Rightarrow \frac{5x}{100} + (30,000-x) \frac{7}{100} = 1800 \Rightarrow 5x + 2,10,000 - 7x = 1,80,000$

$\Rightarrow 2x = 2,10,000 - 1,80,000 \Rightarrow 2x = 30,000 \Rightarrow x = 15,000$

Hence, the trust invests Rs. 15,000 at 5% p.a. and Rs. (30,000-x) = Rs. (30,000-15,000)

= Rs. 15,000 at 7% p.a.

(B)  $[x \ 30,000-x] \begin{bmatrix} 5\% \\ 7\% \end{bmatrix} = 2000 \Rightarrow x \times \frac{5}{100} + (30,000-x) \frac{7}{100} = 2000$

$$\Rightarrow 5x + 2,10,000 - 7x = 200000$$

$$\Rightarrow 2x = 2,10,000 - 2,00,000 \Rightarrow 2x = 10,000 \Rightarrow x = 5,000 \text{ Hence, the trust invests Rs. 5,000 at 5\% p.a. and Rs. (30,000 - 5,000) = Rs. 25,000 at 7\% p.a.}$$

20. The bookshop of a particular school has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are Rs. 80, Rs. 60 and Rs. 40 each respectively. Find the total amount the bookshop will receive from selling all the books using matrix algebra.

**SOLUTION** ∴

Number of Chemistry books = 10 dozen books = 120 books

Number of Physics books = 8 dozen books = 96 books Number of Economics books = 10 dozen books = 120 books

$$\text{Now, } [120 \ 96 \ 120] \begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix} = 120 \cdot 80 + 96 \cdot 60 + 120 \cdot 40 = 9,600 + 5,760 + 4,800 = 20,160$$

Hence, total amount received = Rs. 20,160. Assume X, Y, Z, W and P are matrices of order  $2 \times n$ ,  $3 \times k$ ,  $2 \times p$ ,  $n \times 3$  and  $p \times k$ , respectively.

**Choose the correct answer in questions 21 and 22.**

21. The restriction on n, k and p so that  $PY + WY$  will be defined are ...

- (a)  $k = 3, p = n$
- (b) k is arbitrary,  $p = 2$
- (c) p is arbitrary,  $k = 3$
- (d)  $k = 2, p = 3$

**SOLUTION** ∴

(A) Given :  $X_{2 \times n}, Y_{3 \times k}, Z_{2 \times p}, W_{n \times 3}, P_{p \times k}$

$$\text{Now, } PY + WY = P_{p \times k} \times Y_{3 \times k} + Q_{n \times 3} \times Y_{3 \times k}$$

Clearly,  $k = 3$  and  $p = n$

22. If  $n = p$ , then the order of the matrix  $7X - 5Z$  is

- (a)  $p \times 2$
- (b)  $2 \times n$
- (c)  $n \times 3$
- (d)  $p \times n$

**SOLUTION** ∴

$$(B) 7X - 5Z = 7X_{2 \times n} - 5Z_{2 \times p}$$

We can add two matrices if their order is same.  $n = p$

∴ Order of  $7X - 5Z$  is  $2 \times n$ .