NCERT - Exercise 3.1

1. In the matrix $A = \begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & 5/2 & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$,write :

   (a) The order of the matrix,

   (b) The number of elements,

   (c) Write the elements

   $a_{13}, a_{21}, a_{33}, a_{24}, a_{23}$.

   SOLUTION

   (a) The matrix A has 3 rows and 4 columns. Thus, order of the matrix A is 3 4.

   (b) There are 3 4 = 12 elements in the matrix A.

   (c) $a_{13} = 19, a_{21} = 35, a_{33} = -5, a_{24} = 12, a_{23} = \frac{5}{2}$.

2. If a matrix has 24 elements, what are the possible orders it can have? What if, it has 13 elements?

   SOLUTION .:

   We know that a matrix of order m n, has mn elements. Thus, all possible ordered pairs are (1, 24), (24, 1), (2, 12), (12, 2), (3, 8), (8, 3), (4, 6), (6, 4). The matrix with 13 elements has possible order 1 13 and 13 1.

3. If a matrix has 18 elements, what are the possible orders it can have? What, if it has 5 elements?

   SOLUTION .:

   We know that a matrix of order m n has mn elements. Then the possible orders are 1 18, 18 1, 2 9, 9 2, 3 6, 6 3. If a matrix has 5 elements, then possible orders are 1 5 and 5 1.

4. Construct a 2 2 matrix, $A = [a_{ij}]$, whose elements are given by :

   (a) $a_{ij} = \frac{(i+j)^2}{2}$

   (b) $a_{ij} = \frac{i}{j}$

   (c) $a_{ij} = \frac{(i+2j)^2}{2}$

   SOLUTION .:

   (a) A 2 x 2 matrix has 2 rows and 2 columns. So, it is given by $A = [a_{ij}]_{2\times 2} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

   $a_{11} = \frac{(1+1)^2}{2} = \frac{4}{2} = 2, a_{12} = \frac{(1+2)^2}{2} = \frac{9}{2},$

   $a_{21} = \frac{(2+1)^2}{2} = \frac{9}{2}, a_{22} = \frac{(2+2)^2}{2} = \frac{16}{2} = 8$

   $\therefore A = \begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}$

   (b) For $a_{ij} = \frac{i}{j}$,we have $a_{11} = \frac{1}{1} = 1, a_{12} = \frac{1}{2}, a_{21} = \frac{2}{1} = 2, a_{22} = \frac{2}{2} = 1 \therefore A = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{bmatrix}$

(c) For $a_{ij}=\frac{(i+2j)^2}{2}$,we have $a_{11}=\frac{(1+2\times1)^2}{2}=\frac{9}{2}, a_{12}=\frac{(1+2\times2)^2}{2}=\frac{25}{2},$
$a_{21}=\frac{(2+2\times1)^2}{2}=8, a_{22}=\frac{(2+2\times2)^2}{2}=18$
$\therefore A=\begin{bmatrix}\frac{9}{2} & \frac{25}{2}\\ 8 & 18\end{bmatrix}$

5. Construct a 3 x 4 matrix, whose elements are given by :

(a) $a_{ij}=\frac{1}{2}|-3i+j|$
(b) $a_{ij}=2i-j$

SOLUTION .:

A 3 4 matrix has 3 rows and 4 columns. In general, 3 4 matrix is given by $A=[a_{ij}]_{3\times4}=\begin{bmatrix}a_{11} & a_{12} & a_{13} & a_{14}\\ a_{21} & a_{22} & a_{23} & a_{24}\\ a_{31} & a_{32} & a_{33} & a_{34}\end{bmatrix}$

(a) (i) For $a_{ij}=\frac{1}{2}|-3i+j|$,
we have $a_{11}=\frac{1}{2}|-3\times1+1|=\frac{1}{2}|-2|=1, a_{12}=\frac{1}{2}|-3\times1+2|=\frac{1}{2}|-1|=\frac{1}{2},$
$a_{13}=\frac{1}{2}|-3\times1+3|=0\ a_{14}=\frac{1}{2}|-3\times1+4|=\frac{1}{2}$
$a_{21}=\frac{1}{2}|-3\times2+1|=\frac{5}{2}, a_{22}=\frac{1}{2}|-3\times2+2|=2\ a_{23}=\frac{1}{2}|-3\times2+3|=\frac{3}{2}, a_{24}=\frac{1}{2}|-3\times2+4|=1$
$a_{31}=\frac{1}{2}|-3\times3+1|=4, a_{32}=\frac{1}{2}|-3\times3+2|=\frac{7}{2},$
$a_{33}=\frac{1}{2}|-3\times3+3|=3, a_{34}=\frac{1}{2}|-3\times3+4|=\frac{5}{2}.$
$\therefore A=\begin{bmatrix}1 & \frac{1}{2} & 0 & \frac{1}{2}\\ \frac{5}{2} & 2 & \frac{3}{2} & 1\\ 4 & \frac{7}{2} & 3 & \frac{5}{2}\end{bmatrix}$

(b) For $a_{ij}=2i-j$,we have
$a_{11}=2\times1-1=1, a_{12}=2\times1-2=0,$
$a_{13}=2\times1-3=-1, a_{14}=2\times1-4=-2,$
$a_{21}=2\times2-1=3,$
$a_{22}=2\times2-2=2, a_{23}=2\times2-3=1, a_{24}=2\times2-4=0, a_{31}=2\times3-1=5,$
$a_{32}=2\times3-2=4, a_{33}=2\times3-3=3, a_{34}=2\times3-4=2,$
Hence,$A=\begin{bmatrix}1 & 0 & -1 & -2\\ 3 & 2 & 1 & 0\\ 5 & 4 & 3 & 2\end{bmatrix}$

6. Find the values of x, y and z from the following equations:

(a) $\begin{bmatrix}4 & 3\\ x & 5\end{bmatrix}=\begin{bmatrix}y & z\\ 1 & 5\end{bmatrix}$

(b) $\begin{bmatrix}x+y & 2\\ 5+z & xy\end{bmatrix}=\begin{bmatrix}6 & 2\\ 5 & 8\end{bmatrix}$

(c) $\begin{bmatrix}x+y+z\\ x+z\\ y+z\end{bmatrix}=\begin{bmatrix}9\\ 5\\ 7\end{bmatrix}$

SOLUTION .:

(a) Since the corresponding elements of equal matrices are equal, we have x = 1, y = 4, z = 3.

(b) We are given that $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

$\Rightarrow$ 5 + z = 5 $\Rightarrow$ z = 0

Also, x + y = 6 $\Rightarrow$ y = 6 x .(i)

and xy = 8 (ii)

Solving (i) & (ii), we have x(6−x) = 8

$\Rightarrow 6x - x^2 = 8 \Rightarrow x^2 - 6x + 8 = 0$

$\Rightarrow (x-4)(x-2) = 0 \Rightarrow x = 2,4$

When x = 2, we get y = 4 and when x = 4, we get y = 6 − 4 = 2

Hence, x = 2, y = 4, z = 0 or x = 4, y = 2, z = 0.

(c) From the given matrix, we have

x + y + z = 9 .(i)

x + z = 5 .(ii)

y + z = 7 ..(iii) From (i) and (ii), we get y + 5 = 9 $\Rightarrow$ y = 4

From (i) and (iii), we get x + 7 = 9 $\Rightarrow$ x = 2

Now, from (ii), we get 2 + z = 5 $\Rightarrow$ z = 3

Hence, x = 2, y = 4, z = 3.

7. Find the values of a, b, c and d from the equation :

$$\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$

SOLUTION .:

From the given matrix, we have a− b =−1 (i) and 2a−b = 0 .... (ii)

Solving (i) & (ii), we get a = 1 and b = 2

Similarly 2a + c = 5 $\Rightarrow$ 2 1 + c = 5 $\Rightarrow$ c = 3

Also, 3c + d = 13 $\Rightarrow$ 3 3 + d = 13 $\Rightarrow$ d = 4

Hence, a = 1, b = 2, c = 3, d= 4.

8. $A = [a_{ij}]_{m \times n}$is a square matrix, if

(a) $m < n$

(b) $m > n$

(c) m = n

(d) None of these

SOLUTION .:

(c) For a square matrix, we have m = n.

9. Which of the given values of x and y make the following pair of matrices equal?

$$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix}, \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$$

(a) $x = \frac{-1}{3}, y = 7$

(b) Not possible to find

(c) $y = 7, x = \frac{-2}{3}$

(d) $x = \frac{-1}{3}, y = \frac{-2}{3}$

SOLUTION .:

(b) $\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix}, \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$

$\Rightarrow$ 3x + 7 = 0, y− 2 = 5, y + 1 = 8, 2−3x = 4.

Solving first two equations, we get x = $\frac{-7}{3}$ and y = 7

But x = does not satisfy other equation in x.

So, it is not possible to find the required values of x and y.

10. The number of all possible matrices of order 3 3 with each entry 0 or 1 is :

(a) 27
(b) 18
(c) 81
(d) 512

SOLUTION .:

(d) The matrix has 3 x 3 = 9 elements with entries 0 or 1. i.e., 2 entries.

Therefore, number of possible matrices = (2)9 = 512.

NCERT - Exercise 3.2

11. Let $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}, C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

Find each of the following :

(i) A+ B
(ii) A−B
(iii) 3A− C
(iv) AB
(v) BA

SOLUTION .:

Here, A is a 2 2 matrix, B is a 2 2 matrix and C is a 2 2 matrix. So, A, B, C are comparable.

(i)

$$A+B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 2+1 & 4+3 \\ 3+(-2) & 2+5 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 1 & 7 \end{bmatrix}$$

(ii)

$$A-B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 2-1 & 4-3 \\ 3+2 & 2-5 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 5 & -3 \end{bmatrix}$$

(iii)

$$3A-C = 3\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 12 \\ 9 & 6 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 6+2 & 12-5 \\ 9-3 & 6-4 \end{bmatrix} = \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$$

(iv)

$$AB = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 2\times1+4\times(-2) & 2\times3+4\times5 \\ 3\times1+2\times(-2) & 3\times3+2\times5 \end{bmatrix}$$

$$= \begin{bmatrix} 2-8 & 6+20 \\ 3-4 & 9+10 \end{bmatrix} = \begin{bmatrix} -6 & 26 \\ -1 & 19 \end{bmatrix}$$

(v)

$$BA = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 1\times2+3\times3 & 1\times4+3\times2 \\ -2\times2+5\times3 & -2\times4+5\times2 \end{bmatrix}$$

$$= \begin{bmatrix} 2+9 & 4+6 \\ -4+15 & -8+10 \end{bmatrix} = \begin{bmatrix} 11 & 10 \\ 11 & 2 \end{bmatrix}$$

12. Compute the following s

(i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix}$

(ii) $\begin{bmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix}$

(iii) $\begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix}$

(iv) $\begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix}$

SOLUTION .:

(i)

We have, $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix} = \begin{bmatrix} a+a & b+b \\ -b+b & a+a \end{bmatrix} = \begin{bmatrix} 2a & 2b \\ 0 & 2a \end{bmatrix}$

(ii)

We have, $\begin{bmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix} = \begin{bmatrix} a^2+b^2+2ab & b^2+c^2+2bc \\ a^2+c^2-2ac & a^2+b^2-2ab \end{bmatrix} = \begin{bmatrix} (a+b)^2 & (b+c)^2 \\ (a-c)^2 & (a-b)^2 \end{bmatrix}$

(iii)

We have, $\begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix} = \begin{bmatrix} -1+12 & 4+7 & -6+6 \\ 8+8 & 5+0 & 16+5 \\ 2+3 & 8+2 & 5+4 \end{bmatrix} = \begin{bmatrix} 11 & 11 & 0 \\ 16 & 5 & 21 \\ 5 & 10 & 9 \end{bmatrix}$

(iv)

We have, $\begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix}$

$= \begin{bmatrix} \cos^2 x+\sin^2 x & \sin^2 x+\cos^2 x \\ \sin^2 x+\cos^2 x & \cos^2 x+\sin^2 x \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

13. Compute the following products.

(i)

$\begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

(ii)

$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$

(iii)

$\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

(iv) $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$

(v) $\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$

(vi) $\begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$

SOLUTION .:

(i)

We have, $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

$$= \begin{bmatrix} a^2+b^2 & -ab+ab \\ -ab+ab & b^2+a^2 \end{bmatrix} = \begin{bmatrix} a^2+b^2 & 0 \\ 0 & a^2+b^2 \end{bmatrix}$$

(ii) We have, $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 2 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 1\times 2 & 1\times 3 & 1\times 4 \\ 2\times 2 & 2\times 3 & 2\times 4 \\ 3\times 2 & 3\times 3 & 3\times 4 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 6 & 8 \\ 6 & 9 & 12 \end{bmatrix}$

(iii)

We have, $\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1\times 1+(-2)\times 2 & 1\times 2+(-2)\times 3 & 1\times 3+(-2)\times 1 \\ 2\times 1+3\times 2 & 2\times 2+3\times 3 & 2\times 3+3\times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-4 & 2-6 & 3-2 \\ 2+6 & 4+9 & 6+3 \end{bmatrix} = \begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix}$$

(iv)

$$\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times 1+3\times 0+4\times 3 & 2\times(-3)+3\times 2+4\times 0 & 2\times 5+3\times 4+4\times 5 \\ 3\times 1+4\times 0+5\times 3 & 3\times(-3)+4\times 2+5\times 0 & 3\times 5+4\times 4+5\times 5 \\ 4\times 1+5\times 0+6\times 3 & 4\times(-3)+5\times 2+6\times 0 & 4\times 5+5\times 4+6\times 5 \end{bmatrix} \begin{bmatrix} 2+12 & -6+6 & 10+12+20 \\ 3+15 & -9+8 & 15+16+25 \\ 4+18 & -12+10 & 20+20+30 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 0 & 42 \\ 18 & -1 & 56 \\ 22 & -2 & 70 \end{bmatrix}$$

(v) $\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 2\times 1+1\times(-1) & 2\times 0+1\times 2 & 2\times 1+1\times 1 \\ 3\times 1+2\times(-1) & 3\times 0+2\times 2 & 3\times 1+2\times 1 \\ -1\times 1+1\times(-1) & -1\times 0+1\times 2 & -1\times 1+1\times 1 \end{bmatrix}$

$$= \begin{bmatrix} 2-1 & 2 & 2+1 \\ 3-2 & 4 & 3+2 \\ -1-1 & 2 & -1+1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ -2 & 2 & 0 \end{bmatrix}$$

(vi)

$$\begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3\times 2+(-1)\times 1+3\times 3 & 3\times(-3)+(-1)\times 0+3\times 1 \\ -1\times 2+0\times 1+2\times 3 & -1\times(-3)+0\times 0+2\times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6-1+9 & -9+0+3 \\ -2+0+6 & 3+0+2 \end{bmatrix} = \begin{bmatrix} 14 & -6 \\ 4 & 5 \end{bmatrix}$$

14. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}, B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$, then compute (A + B) and (B−C). Also, verify that A + (B−C) = (A + B)−C.

SOLUTION :

Here, A, B and C is a 3 3 matrix. So, A, B and C are comparable. So, (A + B), (B−C), A + (B−C) and (A + B)−C are defined and each one is 3 x 3 matrix.

$A+B = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix}$ $B-C = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix}$

$$A+(B-C) = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix}$$

$$(A+B)-C = \begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix}$$

Hence, A + (B−C) = (A + B)−C.

15. If $A+\begin{bmatrix} \frac{2}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ \frac{7}{3} & 2 & \frac{2}{3} \end{bmatrix}$ and $B=\begin{bmatrix} \frac{2}{5} & \frac{3}{5} & 1 \\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5} \\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5} \end{bmatrix}$, then compute $3A-5B$.

SOLUTION .:

$$3A-5B=3\begin{bmatrix} \frac{2}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ \frac{7}{3} & 2 & \frac{2}{3} \end{bmatrix}-5\begin{bmatrix} \frac{2}{5} & \frac{3}{5} & 1 \\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5} \\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5} \end{bmatrix}$$

$$=\begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix}-\begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix}=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

16. . Simplify,$\cos\theta\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}+\sin\theta\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$

SOLUTION .:

We have, $\cos\theta\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}+\sin\theta\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$

$$=\begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ -\sin\theta\cos\theta & \cos^2\theta \end{bmatrix}+\begin{bmatrix} \sin^2\theta & -\sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$$

$$=\begin{bmatrix} \cos^2\theta+\sin^2\theta & \sin\theta\cos\theta-\sin\theta\cos\theta \\ -\sin\theta\cos\theta+\cos\theta\sin\theta & \cos^2\theta+\sin^2\theta \end{bmatrix}$$

$$=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

17. Find X and Y, if

(i) $X+Y=\begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $X-Y=\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

(ii) $2X+3Y=\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ and $3X+2Y=\begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$

SOLUTION .:

(i)

We are given that $X+Y=\begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ ..(i)

and X − Y = $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ ..(ii)

Adding (i) and (ii), we get

$$(X+Y)+(X-Y)=\begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}+\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\Rightarrow 2X=\begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix}\Rightarrow X=\begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$$

Subtracting (ii) from (i), we get

$$\Rightarrow 2Y=\begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}-\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}=\begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix}\Rightarrow Y=\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

Hence, $X=\begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$ and $Y=\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

(ii) We have, $2X+3Y=\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$.(i)

and $3X+2Y=\begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$

Adding (i) & (ii), we get

$$5X+5Y=\begin{bmatrix}2&3\\4&0\end{bmatrix}+\begin{bmatrix}2&-2\\-1&5\end{bmatrix}$$

$$\Rightarrow 5X+5=\begin{bmatrix}4&1\\3&5\end{bmatrix}\Rightarrow X+Y=\frac{1}{5}\begin{bmatrix}4&1\\3&5\end{bmatrix}\text{ .(iii)}$$

Subtracting (i) from (ii),

we get (3X + 2y)−(2X + 3Y)= $\begin{bmatrix}2&-2\\-1&5\end{bmatrix}-\begin{bmatrix}2&3\\4&0\end{bmatrix}$

$$\Rightarrow X-Y=\begin{bmatrix}0&-5\\-5&5\end{bmatrix}\text{ .(iv)}$$

Finally, adding (iii) and (iv), we get

$$2X=\frac{1}{5}\begin{bmatrix}4&1\\3&5\end{bmatrix}+\begin{bmatrix}0&-5\\-5&5\end{bmatrix}$$

$$\Rightarrow 2X=\begin{bmatrix}\frac{4}{5}&\frac{1}{5}\\\frac{3}{5}&1\end{bmatrix}+\begin{bmatrix}0&-5\\-5&5\end{bmatrix}$$

$$\Rightarrow 2X=\begin{bmatrix}\frac{4}{5}&\frac{1}{5}-5\\\frac{3}{5}-5&1+5\end{bmatrix}=\begin{bmatrix}\frac{4}{5}&\frac{-24}{5}\\\frac{-22}{5}&6\end{bmatrix}$$

$$\Rightarrow X=\frac{1}{2}\begin{bmatrix}\frac{4}{5}&\frac{-24}{5}\\\frac{-22}{5}&6\end{bmatrix}\Rightarrow X=\begin{bmatrix}\frac{2}{5}&\frac{-12}{5}\\\frac{-11}{5}&3\end{bmatrix}$$

Subtracting (iii) and (iv), we get

$$2Y=\frac{1}{5}\begin{bmatrix}4&1\\3&5\end{bmatrix}-\begin{bmatrix}0&-5\\-5&5\end{bmatrix}$$

$$\Rightarrow 2Y=\begin{bmatrix}\frac{4}{5}&\frac{1}{5}\\\frac{3}{5}&1\end{bmatrix}+\begin{bmatrix}0&5\\5&-5\end{bmatrix}=\begin{bmatrix}\frac{4}{5}&\frac{26}{5}\\\frac{28}{5}&-4\end{bmatrix}$$

$$\Rightarrow Y=\frac{1}{2}\begin{bmatrix}\frac{4}{5}&\frac{26}{5}\\\frac{28}{5}&-4\end{bmatrix}=\begin{bmatrix}\frac{2}{5}&\frac{13}{5}\\\frac{14}{5}&-2\end{bmatrix}$$

Hence, $X-\begin{bmatrix}\frac{2}{5}&\frac{-12}{5}\\\frac{-11}{5}&3\end{bmatrix}, Y=\begin{bmatrix}\frac{2}{5}&\frac{13}{5}\\\frac{14}{5}&-2\end{bmatrix}$

18. Find X, if $Y=\begin{bmatrix}3&2\\1&4\end{bmatrix}$ and $2X+Y=\begin{bmatrix}1&0\\-3&2\end{bmatrix}$

SOLUTION .:

We are given that, $Y=\begin{bmatrix}3&2\\1&4\end{bmatrix}$ ..(i)

and $3X+Y=\begin{bmatrix}1&0\\-3&2\end{bmatrix}$.(ii)

Substuting the value of Y in (ii), we have

$$2X+\begin{bmatrix}3&2\\1&4\end{bmatrix}=\begin{bmatrix}1&0\\-3&2\end{bmatrix}\Rightarrow 2X=\begin{bmatrix}1&0\\-3&2\end{bmatrix}-\begin{bmatrix}3&2\\1&4\end{bmatrix}$$

$$\Rightarrow 2X=\begin{bmatrix}-2&-2\\-4&-2\end{bmatrix}\Rightarrow X=\frac{1}{2}\begin{bmatrix}-2&-2\\-4&-2\end{bmatrix}$$

Hence, $X=\begin{bmatrix}-1&-1\\-2&-1\end{bmatrix}$

19. . Find x and y, if $2\begin{bmatrix}1 & 3\\0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$

SOLUTION .:

We have, $2\begin{bmatrix}1 & 3\\0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$

$\Rightarrow \begin{bmatrix}2 & 6\\0 & 2x\end{bmatrix}+\begin{bmatrix}y & 0\\1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 8\end{bmatrix} \Rightarrow \begin{bmatrix}2+y & 6\\1 & 2x+2\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$

$\Rightarrow$ 2 + y = 5 and 2x + 2 = 8

$\Rightarrow$ y = 3 and x = 3

Hence, x = 3 and y = 3.

20. Solve the equation for x, y, z and t, if

$2\begin{bmatrix}x & z\\y & t\end{bmatrix}+3\begin{bmatrix}1 & -1\\0 & 2\end{bmatrix}=3\begin{bmatrix}3 & 5\\4 & 6\end{bmatrix}$

SOLUTION .:

$2\begin{bmatrix}x & z\\y & t\end{bmatrix}+3\begin{bmatrix}1 & -1\\0 & 2\end{bmatrix}=3\begin{bmatrix}3 & 5\\4 & 6\end{bmatrix}$

$\Rightarrow \begin{bmatrix}2x & 2z\\2y & 2t\end{bmatrix}+\begin{bmatrix}3 & -3\\0 & 6\end{bmatrix}=\begin{bmatrix}9 & 15\\12 & 18\end{bmatrix}$

$\Rightarrow \begin{bmatrix}2x+3 & 2z-3\\2y+0 & 2t+6\end{bmatrix}=\begin{bmatrix}9 & 15\\12 & 18\end{bmatrix}$

Now, 2x + 3 = 9 $\Rightarrow$ 2x = 6 $\Rightarrow$ x = 3

2z− 3 = 15 $\Rightarrow$ 2z = 18 $\Rightarrow$ z = 9

2y = 12 $\Rightarrow$ y = 6

2t + 6 = 18 $\Rightarrow$ 2t = 12 $\Rightarrow$ t = 6

Hence, x = 3, y = 6, z = 9 and t = 6.

21. If $x\begin{bmatrix}2\\3\end{bmatrix}+y\begin{bmatrix}-1\\1\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$,find the values of x and y.

SOLUTION .:

We have, $x\begin{bmatrix}2\\3\end{bmatrix}+y\begin{bmatrix}-1\\1\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$,

$\Rightarrow \begin{bmatrix}2x\\3x\end{bmatrix}+\begin{bmatrix}-y\\y\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix} \Rightarrow \begin{bmatrix}2x-y\\3x+y\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$

$\Rightarrow 2x-y=10$.(i) and $3x+y=5$ ..(ii)

Solving (i) & (ii), we get x = 3 and y = − 4

Hence, x = 3 and y =− 4.

22. Given $3\begin{bmatrix}x & y\\z & w\end{bmatrix}=\begin{bmatrix}x & 6\\-1 & 2w\end{bmatrix}+\begin{bmatrix}4 & x+y\\z+w & 3\end{bmatrix}$,

find the values of x, y, z and w.

SOLUTION . :

$3\begin{bmatrix}x & y\\z & w\end{bmatrix}=\begin{bmatrix}x & 6\\-1 & 2w\end{bmatrix}+\begin{bmatrix}4 & x+y\\z+w & 3\end{bmatrix}$

$\Rightarrow \begin{bmatrix}3x & 3y\\3z & 3w\end{bmatrix}=\begin{bmatrix}x+4 & 6+x+y\\-1+z+w & 2w+3\end{bmatrix}$

Now, 3x = x + 4 $\Rightarrow$ 2x = 4 $\Rightarrow$ x = 2

3y = 6 + x + y $\Rightarrow$ 2y = x + 6 $\Rightarrow$ 2y = 2 + 6

$\Rightarrow$ 2y = 8 $\Rightarrow$ y = 4 3w = 2w + 3 $\Rightarrow$ w = 3

3z =−1 + z + w $\Rightarrow$ 2z = -

1 + 3 $\Rightarrow 2z=2 \Rightarrow z=1$

Hence, x = 2, y = 4, z = 1 and w = 3

23. If $F(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$,then show that F(x)·F(y) = F(x + y).

SOLUTION .:

We are given that, $F(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$

Now, $F(y)=\begin{bmatrix}\cos y & -\sin y & 0\\ \sin y & \cos y & 0\\ 0 & 0 & 1\end{bmatrix}$

$\Rightarrow$ F(x)·F(y) = $\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos y & -\sin y & 0\\ \sin y & \cos y & 0\\ 0 & 0 & 1\end{bmatrix}$

$=\begin{bmatrix}\cos x\cos y-\sin x\sin y & -\sin y\cos x-\sin x\cos y & 0\\ \sin x\cos y+\cos x\sin y & -\sin x\sin y+\cos x\cos y & 0\\ 0 & 0 & 1\end{bmatrix}$

$=\begin{bmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{bmatrix}$ Also, $F(x+y)=\begin{bmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{bmatrix}$ $\therefore$ F(x) ·F(y) = F(x + y).

24. Show that

(i) $\begin{bmatrix}5 & -1\\ 6 & 7\end{bmatrix}\begin{bmatrix}2 & 1\\ 3 & 4\end{bmatrix}\neq\begin{bmatrix}2 & 1\\ 3 & 4\end{bmatrix}\begin{bmatrix}5 & -1\\ 6 & 7\end{bmatrix}$

(ii) $\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & 0\\ 1 & 1 & 0\end{bmatrix}\begin{bmatrix}-1 & 1 & 0\\ 0 & -1 & 1\\ 2 & 3 & 4\end{bmatrix}\neq\begin{bmatrix}-1 & 1 & 0\\ 0 & -1 & 1\\ 2 & 3 & 4\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & 0\\ 1 & 1 & 0\end{bmatrix}$

SOLUTION .:

(i)

Let A= $\begin{bmatrix}5 & -1\\ 6 & 7\end{bmatrix}$ and $B=\begin{bmatrix}2 & 1\\ 3 & 4\end{bmatrix}\Rightarrow AB=\begin{bmatrix}5 & -1\\ 6 & 7\end{bmatrix}\begin{bmatrix}2 & 1\\ 3 & 4\end{bmatrix}=\begin{bmatrix}10-3 & 5-4\\ 12+21 & 6+28\end{bmatrix}=\begin{bmatrix}7 & 1\\ 33 & 34\end{bmatrix}$

and

$BA=\begin{bmatrix}2 & 1\\ 3 & 4\end{bmatrix}\begin{bmatrix}5 & -1\\ 6 & 7\end{bmatrix}=\begin{bmatrix}10+6 & -2+7\\ 15+24 & -3+28\end{bmatrix}=\begin{bmatrix}16 & 5\\ 39 & 25\end{bmatrix}$

Hence, AB $\neq$ BA.

(ii)

Let A = $\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & 0\\ 1 & 1 & 0\end{bmatrix}$ and $B=\begin{bmatrix}-1 & 1 & 0\\ 0 & -1 & 1\\ 2 & 3 & 4\end{bmatrix}\Rightarrow AB=\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & 0\\ 1 & 1 & 0\end{bmatrix}\begin{bmatrix}-1 & 1 & 0\\ 0 & -1 & 1\\ 2 & 3 & 4\end{bmatrix}$

$=\begin{bmatrix}1\times(-1)+2\times0+3\times2 & 1\times1+2\times(-1)+3\times3 & 1\times0+2\times1+3\times4\\ 0\times(-1)+1\times0+0\times2 & 0\times1+1\times(-1)+0\times3 & 0\times0+1\times1+0\times4\\ 1\times(-1)+1\times0+0\times2 & 1\times1+1\times(-1)+0\times3 & 1\times0+1\times1+0\times4\end{bmatrix}$

$=\begin{bmatrix}-1+0+6 & 1-2+9 & 0+2+12\\ 0+0+0 & 0-1+0 & 0+1+0\\ -1+0+0 & 1-1+0 & 0+1+0\end{bmatrix}=\begin{bmatrix}5 & 8 & 14\\ 0 & -1 & 1\\ -1 & 0 & 1\end{bmatrix}$

and $BA=\begin{bmatrix}-1 & 1 & 0\\ 0 & -1 & 1\\ 2 & 3 & 4\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & 0\\ 1 & 1 & 0\end{bmatrix}=\begin{bmatrix}-1\times1+1\times0+0\times1 & -1\times2+1\times1+0\times1 & -1\times3+1\times0+0\times0\\ 0\times1-1\times0+1\times1 & 0\times2-1\times1+1\times1 & 0\times3-1\times0+1\times0\\ 2\times1+3\times0+4\times1 & 2\times2+3\times1+4\times1 & 2\times3+3\times0+4\times0\end{bmatrix}$

$=\begin{bmatrix}-1+0+0 & -2+1+0 & -3+0+0\\ 0-0+1 & 0-1+1 & 0-0+0\\ 2+0+4 & 4+3+4 & 6+0+0\end{bmatrix}=\begin{bmatrix}-1 & -1 & -3\\ 1 & 0 & 0\\ 6 & 11 & 6\end{bmatrix}$

Hence, L.H.S.$\neq$ R.H.S.

25. Find $A^2-5A+6I$,If $A=\begin{bmatrix}2&0&1\\2&1&3\\1&-1&0\end{bmatrix}$

SOLUTION .:

We are given that $A=\begin{bmatrix}2&0&1\\2&1&3\\1&-1&0\end{bmatrix}$

$\Rightarrow A^2=A\cdot A=\begin{bmatrix}2&0&1\\2&1&3\\1&-1&0\end{bmatrix}\begin{bmatrix}2&0&1\\2&1&3\\1&-1&0\end{bmatrix}$

$=\begin{bmatrix}2\times2+0\times2+1\times1&2\times0+0\times1+1\times(-1)&2\times1+0\times3+1\times0\\2\times2+1\times2+3\times1&2\times0+1\times1+3\times(-1)&2\times1+1\times3+3\times0\\1\times2+2\times(-1)+0\times1&1\times0+(-1)\times1+0\times(-1)&1\times1+(-1)\times3+0\times0\end{bmatrix}$

$=\begin{bmatrix}4+0+1&0+0-1&2+0+0\\4+2+3&0+1-3&2+3+0\\2-2+0&0-1+0&1-3+0\end{bmatrix}$

$=\begin{bmatrix}5&-1&2\\9&-2&5\\0&-1&-2\end{bmatrix}$ $5A=5\begin{bmatrix}2&0&1\\2&1&3\\1&-1&0\end{bmatrix}=\begin{bmatrix}10&0&5\\10&5&15\\5&-5&0\end{bmatrix}$

$6I=6\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=\begin{bmatrix}6&0&0\\0&6&0\\0&0&6\end{bmatrix}$

$\therefore A^2-5A+6I=\begin{bmatrix}5&-1&2\\9&-2&5\\0&-1&-2\end{bmatrix}-\begin{bmatrix}10&0&5\\10&5&15\\5&-5&0\end{bmatrix}+\begin{bmatrix}6&0&0\\0&6&0\\0&0&6\end{bmatrix}$

$=\begin{bmatrix}-5&-1&-3\\-1&-7&-10\\-5&4&-2\end{bmatrix}+\begin{bmatrix}6&0&0\\0&6&0\\0&0&6\end{bmatrix}=\begin{bmatrix}1&-1&-3\\-1&-1&-10\\-5&4&4\end{bmatrix}$

26. If $A=\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}$, prove that $A^3-6A^2+7A+2I=O$.

SOLUTION .:

We are given that, $A=\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}\Rightarrow A^2=A\cdot A=\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}$

$=\begin{bmatrix}1\times1+0\times0+2\times2&0\times1+0\times2+0\times2&1\times2+0\times1+2\times3\\0\times1+2\times0+1\times2&0\times0+2\times2+1\times0&0\times2+2\times1+3\times1\\2\times1+0\times0+3\times2&2\times0+0\times2+3\times0&2\times2+0\times1+3\times3\end{bmatrix}$

$=\begin{bmatrix}1+0+4&0+0+0&2+0+6\\0+0+2&0+4+0&0+2+3\\2+0+6&0+0+0&4+0+9\end{bmatrix}=\begin{bmatrix}5&0&8\\2&4&5\\8&0&13\end{bmatrix}$

Now, $A^3=A^2\cdot A=\begin{bmatrix}5&0&8\\2&4&5\\8&0&13\end{bmatrix}\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}=\begin{bmatrix}5\times1+0\times0+2\times8&5\times0+0\times2+8\times0&5\times2+0\times1+8\times3\\2\times1+4\times0+5\times2&2\times0+4\times2+5\times0&2\times2+4\times1+5\times3\\8\times1+0\times0+13\times2&8\times0+0\times2+13\times0&2\times8+1\times0+13\times3\end{bmatrix}$

$=\begin{bmatrix}5+0+16&0+0+0&10+0+24\\2+0+10&0+8+0&4+4+15\\8+0+26&0+0+0&16+0+39\end{bmatrix}=\begin{bmatrix}21&0&34\\12&8&23\\34&0&55\end{bmatrix}$

Now L.H.S. $=A^3-6A^2+7A+2I=\begin{bmatrix}21&0&34\\12&8&23\\34&0&55\end{bmatrix}-6\begin{bmatrix}5&0&8\\2&4&5\\8&0&13\end{bmatrix}+7\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}+2\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

$=\begin{bmatrix}21&0&34\\12&8&23\\34&0&55\end{bmatrix}-\begin{bmatrix}30&0&48\\12&24&30\\48&0&78\end{bmatrix}+\begin{bmatrix}7&0&14\\0&14&7\\14&0&21\end{bmatrix}+\begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix}$

$= \begin{bmatrix} 21-30 & 0-0 & 34-48 \\ 12-12 & 8-24 & 23-30 \\ 34-48 & 0-0 & 55-78 \end{bmatrix} + \begin{bmatrix} 7+2 & 0+0 & 14+0 \\ 0+0 & 14+2 & 7+0 \\ 14+0 & 0+0 & 21+2 \end{bmatrix} = \begin{bmatrix} -9 & 0 & -14 \\ 0 & -16 & -7 \\ -14 & 0 & -23 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 14 \\ 0 & 16 & 7 \\ 14 & 0 & 23 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} =$ 0 =R.H.S

Hence, proved.

27. If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find k so that $A^2 = kA = 2I$.

SOLUTION .:

We are given that, $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Also, A2 = kA − 2I

Substituting the values of A and I from above, we get

$\Rightarrow \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 3\times3+(-2)\times4 & 3\times(-2)+(-2)\times(-2) \\ 4\times3+(-2)\times4 & 4\times(-2)\times(-2)\times(-2) \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$

$\Rightarrow 4k = 4 \Rightarrow k = 1$ Hence, $k = 1$

28. If $A = \begin{bmatrix} 0 & -\tan\dfrac{\alpha}{2} \\ \tan\dfrac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2,

then show that I + A = (I− A)$\begin{bmatrix} \cos\alpha & -\sin\alpha \\ cin\alpha & \cos\alpha \end{bmatrix}$.

SOLUTION .:

We know that, $\cos\alpha = \dfrac{1-\tan^2\left(\dfrac{\alpha}{2}\right)}{1+\tan^2\left(\dfrac{\alpha}{2}\right)} = \dfrac{1-t^2}{1+t^2}$, where $\tan\dfrac{\alpha}{2} = t$

and $\sin\alpha = \dfrac{2\tan\left(\dfrac{\alpha}{2}\right)}{1+\tan^2\left(\dfrac{\alpha}{2}\right)} = \dfrac{2t}{1+t^2}$

Now, $I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -t \\ t & 0 \end{bmatrix} = \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix}$

and $I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -t \\ t & 0 \end{bmatrix} = \begin{bmatrix} 1 & t \\ -t & 1 \end{bmatrix} \therefore (I-A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & t \\ -t & 1 \end{bmatrix}\begin{bmatrix} \dfrac{(1-t^2)}{(1+t^2)} & \dfrac{-2t}{(1+t^2)} \\ \dfrac{2t}{(1+t^2)} & \dfrac{(1-t^2)}{(1+t^2)} \end{bmatrix}$

$= \begin{bmatrix} \dfrac{1-t^2}{1+t^2} + \dfrac{2t^2}{1+t^2} & \dfrac{-2t}{1+t^2} + \dfrac{t(1-t^2)}{1+t^2} \\ \dfrac{-t(1-t^2)}{1+t^2} + \dfrac{2t}{1+t^2} & \dfrac{2t^2}{(1+t^2)} + \dfrac{(1-t^2)}{(1+t^2)} \end{bmatrix} = \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix} = (I+A)$

Hence, $(I-A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} = (I+A)$.

29. A trust fund has Rs. 30, 000 that must be invested in two different types of bonds. The first bond pays 5% interest per year, and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide Rs. 30, 000 among the two types of bonds, if the trust fund obtains an annual total interest of :

(A) Rs. 1800

(B) Rs. 2000

SOLUTION .:

Let us take that the trust invests Rs. x at 5% p.a. and then the trust invests Rs. (30, 000−x) at 7% p.a.

(A) So, [x 30,000−x] $\begin{bmatrix} 5\% \\ 7\% \end{bmatrix}$ = 1800

$\Rightarrow \frac{5x}{100}$+ (30, 000− x) $\frac{7}{100}$= 1800 ⇒ 5x + 2,10,000−7x = 1,80,000

⇒ 2x = 2,10,000 − 1,80,000 ⇒ 2x = 30,000 ⇒ x = 15,000

Hence, the trust invests Rs. 15,000 at 5% p.a. and Rs. (30,000−x) = Rs. (30,000−15000)

= Rs. 15,000 at 7% p.a.

(B) [x 30,000− x] $\begin{bmatrix} 5\% \\ 7\% \end{bmatrix}$ = 2000 ⇒ $x\times = \frac{5}{100}$+ (30, 000−x) $\frac{7}{100}$= 2000

⇒ 5x + 2,10,000−7x = 200000

⇒ 2x = 2,10,000−2,00,000 ⇒ 2x = 10,000 ⇒ x = 5,000 Hence, the trust invests Rs. 5,000 at 5% p.a. and Rs.(30,000− 5,000) = Rs. 25,000 at 7% p.a.

30. The bookshop of a particular school has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are Rs. 80, Rs. 60 and Rs. 40 each respectively. Find the total amount the bookshop will receive from selling all the books using matrix algebra.

SOLUTION .:

Number of Chemistry books = 10 dozen books = 120 books

Number of Physics books = 8 dozen books = 96 books Number of Economics books = 10 dozen books = 120 books

Now, [120 96 120] $\begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix}$ = 120 80 + 96 60 + 120 40 = 9,600 + 5,760 + 4,800 = 20,160

Hence, total amount received = Rs. 20,160. Assume X, Y, Z, W and P are matrices of order 2 x n,3 x k, 2 x p, n x 3 and p x k, respectively.

Choose the correct answer in questions 21 and 22.

31. The restriction on n, k and p so that PY + WY will be defined are ⋯

(a) k = 3, p= n
(b) k is arbitrary, p = 2
(c) p is arbitrary, k = 3
(d) k = 2, p= 3

SOLUTION .:

(A) Given : $X_{2\times n}, Y_{3\times k}, Z_{2\times p}, W_{n\times 3}, P_{p\times k}$

Now, $PY + WY = P_{p\times k} \times Y_{3\times k} + Q_{n\times 3} \times Y_{3\times k}$

Clearly, k = 3 and p = n

32. If n= p, then the order of the matrix 7X−5Z is

(a) p x 2
(b) 2 x n
(c) n x 3
(d) p x n

SOLUTION .:

(B) $7X - 5Z = 7X_{2\times n} - 5Z_{2\times p}$

We can add two matrices if their order is same. $n = p$

∴ Order of 7X−5Z is 2 n.

NCERT - Exercise 3.3

1. Find the transpose of each of the following matrices :

(i) $\begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

(iii) $\begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$

SOLUTION .:

(i) Transpose of $\begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}$

$= \begin{bmatrix} 5 & \frac{1}{2} & -1 \end{bmatrix}$

(ii) Transpose of $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

(iii) Transpose of $\begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$

$= \begin{bmatrix} -1 & \sqrt{3} & 2 \\ 5 & 5 & 3 \\ 6 & 6 & -1 \end{bmatrix}$

2. If $A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$, then verify that

(i) $(A+B)' = A' + B'$,

(ii) $(A-B)' = A' - B'$

SOLUTION .:

(i) $A+B = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$

$= \begin{bmatrix} -1-4 & 2+1 & 3-5 \\ 5+1 & 7+2 & 9+0 \\ -2+1 & 1+3 & 1+1 \end{bmatrix} = \begin{bmatrix} -5 & 3 & -2 \\ 6 & 9 & 9 \\ -1 & 4 & 2 \end{bmatrix}$

Now, $A' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix}$ and $B' = \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix} \Rightarrow (A+B)' = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix}$

and $A' + B' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} -1-4 & 5+1 & -2+1 \\ 2+1 & 7+2 & 1+3 \\ 3-5 & 9+0 & 1+1 \end{bmatrix} = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix} \therefore (A+B)' = A' + B'$

(ii) $A-B=\begin{bmatrix}-1 & 2 & 3\\ 5 & 7 & 9\\ -2 & 1 & 1\end{bmatrix}=\begin{bmatrix}-4 & 1 & -5\\ 1 & 2 & 0\\ 1 & 3 & 1\end{bmatrix}$

$=\begin{bmatrix}-1+4 & 2-1 & 3+5\\ 5-1 & 7-2 & 9-0\\ -2-1 & 1-3 & 1-1\end{bmatrix}=\begin{bmatrix}3 & 1 & 8\\ 4 & 5 & 9\\ -3 & -2 & 0\end{bmatrix}$

$\Rightarrow (A-B)'$

$=\begin{bmatrix}3 & 4 & -3\\ 1 & 5 & -2\\ 8 & 9 & 0\end{bmatrix}$

and $A'-B'=\begin{bmatrix}-1 & 5 & -2\\ 2 & 7 & 1\\ 3 & 9 & 1\end{bmatrix}-\begin{bmatrix}-4 & 1 & 1\\ 1 & 2 & 3\\ -5 & 0 & 1\end{bmatrix}$

$=\begin{bmatrix}-1+4 & 5-1 & -2-1\\ 2-1 & 7-2 & 1-3\\ 3+5 & 9-0 & 1-1\end{bmatrix}=\begin{bmatrix}3 & 4 & -3\\ 1 & 5 & -2\\ 8 & 9 & 0\end{bmatrix}\therefore (A-B)'=A'-B'.$

3. If $A'=\begin{bmatrix}3 & 4\\ -1 & 2\\ 0 & 1\end{bmatrix}$ and $B=\begin{bmatrix}-1 & 2 & 1\\ 1 & 2 & 3\end{bmatrix}$, then verify that

(i) $(A+B)'=A'+B'$

(ii) $(A-B)'=A'-B'$

SOLUTION .:

$A'=\begin{bmatrix}3 & 4\\ -1 & 2\\ 0 & 1\end{bmatrix}\Rightarrow A=\begin{bmatrix}3 & -1 & 0\\ 4 & 2 & 1\end{bmatrix}$

and $B=\begin{bmatrix}-1 & 2 & 1\\ 1 & 2 & 3\end{bmatrix}\Rightarrow B'=\begin{bmatrix}-1 & 1\\ 2 & 2\\ 1 & 3\end{bmatrix}$

(i) Now, $A+B=\begin{bmatrix}3 & -1 & 0\\ 4 & 2 & 1\end{bmatrix}+\begin{bmatrix}-1 & 2 & 1\\ 1 & 2 & 3\end{bmatrix}=\begin{bmatrix}3-1 & -1+2 & 0+1\\ 4+1 & 2+2 & 1+3\end{bmatrix}=\begin{bmatrix}2 & 1 & 1\\ 5 & 4 & 4\end{bmatrix}$

$\Rightarrow (A+B)'=\begin{bmatrix}2 & 5\\ 1 & 4\\ 1 & 4\end{bmatrix}A'+B'=\begin{bmatrix}3 & 4\\ -1 & 2\\ 0 & 1\end{bmatrix}+\begin{bmatrix}-1 & 1\\ 2 & 2\\ 1 & 3\end{bmatrix}=\begin{bmatrix}3-1 & 4+1\\ -1+2 & 2+2\\ 0+1 & 1+3\end{bmatrix}=\begin{bmatrix}2 & 5\\ 1 & 4\\ 1 & 4\end{bmatrix}$

$\therefore (A+B)'=A'+B'.$

(ii) $A-B=\begin{bmatrix}3 & -1 & 0\\ 4 & 2 & 1\end{bmatrix}-\begin{bmatrix}-1 & 2 & 1\\ 1 & 2 & 3\end{bmatrix}=\begin{bmatrix}3-(-1) & -1-2 & 0-1\\ 4-1 & 2-2 & 1-3\end{bmatrix}$

$=\begin{bmatrix}4 & -3 & -1\\ 3 & 0 & -2\end{bmatrix}\Rightarrow (A-B)'=\begin{bmatrix}4 & 3\\ -3 & 0\\ -1 & -2\end{bmatrix},A'-B'$

$=\begin{bmatrix}3 & 4\\ -1 & 2\\ 0 & 1\end{bmatrix}-\begin{bmatrix}-1 & 1\\ 2 & 2\\ 1 & 3\end{bmatrix}=\begin{bmatrix}3+1 & 4-1\\ -1-1 & 2-2\\ 0-1 & 1-3\end{bmatrix}$

$=\begin{bmatrix}4 & 3\\ -3 & 0\\ -1 & -2\end{bmatrix}\therefore (A-B)'=A'=B'.$

4. If $A'=\begin{bmatrix}-2 & 3\\ 1 & 2\end{bmatrix}$ and $B=\begin{bmatrix}-1 & 0\\ 1 & 2\end{bmatrix}$, then find $(A+2B)'$.

SOLUTION .:

$A'=\begin{bmatrix}-2 & 3\\ 1 & 2\end{bmatrix}\Rightarrow A=\begin{bmatrix}-2 & 1\\ 3 & 2\end{bmatrix},B$

$= \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix} \therefore A+2B = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} + 2\begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 0 \\ 2 & 4 \end{bmatrix}$

$= \begin{bmatrix} -2-2 & 1+0 \\ 3+2 & 2+4 \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ 5 & 6 \end{bmatrix}$

Hence, $(A+2B)' = \begin{bmatrix} -4 & 5 \\ 1 & 6 \end{bmatrix}$

5. For the matrices A and B, verify that (AB) =BA, where

(i) $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}, B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$

(ii) $A = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 5 & 7 \end{bmatrix}$

SOLUTION .:

(i) $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 1 & -4 & 3 \end{bmatrix}$

and $B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix} \Rightarrow B' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$

$AB = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix} \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1\times(-1) & 1\times 2 & 1\times 1 \\ -4\times(-1) & (-4)\times 2 & (-4)\times 1 \\ 3\times(-1) & 3\times 2 & 3\times 1 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}$

L.H.S. = $(AB) = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$

R.H.S. = $BA = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & -4 & 3 \end{bmatrix}$

$= \begin{bmatrix} -1\times 1 & -1\times(-4) & -1\times 3 \\ 2\times 1 & 2\times(-4) & 2\times 3 \\ 1\times 1 & 1\times(-4) & 1\times 3 \end{bmatrix} = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$

Hence, $(AB) = BA$.

(ii) $A = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 5 & 7 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 0 & 1 & 2 \end{bmatrix}$

and $B' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}$

Now, $AB = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 5 & 7 \end{bmatrix}$

$= \begin{bmatrix} 0\times 1 & 0\times 5 & 0\times 7 \\ 1\times 1 & 1\times 5 & 1\times 7 \\ 2\times 1 & 2\times 5 & 2\times 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 5 & 7 \\ 2 & 10 & 14 \end{bmatrix}$

$\Rightarrow (AB)'$

$= \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$

Now, $B'A' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 1\times 0 & 1\times 1 & 1\times 2 \\ 5\times 0 & 5\times 1 & 5\times 2 \\ 7\times 0 & 7\times 1 & 7\times 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$

$\therefore (AB)' = B'A'$.

6. If

$(i) A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$,then verify that $A'A = I$

(ii) $A = \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$, then verify that $A'A = I$

SOLUTION .:

(i) $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \Rightarrow A' = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

So, $A'A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$= \begin{bmatrix} \cos^2\alpha + \sin^2\alpha & \cos\alpha\sin\alpha - \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha - \cos\alpha\sin\alpha & \sin^2\alpha + \cos^2\alpha \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

Hence, $A'A = I$.

(ii) Given that, $A = \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix} \Rightarrow A' = \begin{bmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{bmatrix}$

So, $A'A = \begin{bmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{bmatrix} \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$

$= \begin{bmatrix} \sin^2\alpha + \cos^2\alpha & \sin\alpha\cos\alpha - \cos\alpha\sin\alpha \\ \cos\alpha\sin\alpha - \sin\alpha\cos\alpha & \cos^2\alpha + \sin^2\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

Hence, $A'A = I$.

7. (i) Show that the matrix $A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$ is a symmetric matrix.

(ii) Show that the matrix $A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ is a skew symmetric matrix.

SOLUTION .:

(i) A square matrix A is said to be symmetric, if A = A. As, $A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix} \Rightarrow A' = A$

So, A is a symmetric matrix.

(ii) A square matrix A is said to be skew symmetric matrix if A = −A.

As, $A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} = -A \Rightarrow A' = -A.$

So, A is a skew symmetric matrix.

8. For the matrix $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$, verify that

(i) (A + A) is a symmetric matrix.

(ii) (A−A) is a skew-symmetric matrix.

SOLUTION .:

$A=\begin{bmatrix}1&5\\6&7\end{bmatrix}\Rightarrow A'=\begin{bmatrix}1&6\\5&7\end{bmatrix}$

(i) $A+A'=\begin{bmatrix}1&5\\6&7\end{bmatrix}+\begin{bmatrix}1&6\\5&7\end{bmatrix}=\begin{bmatrix}1+1&5+6\\6+5&7+7\end{bmatrix}=\begin{bmatrix}2&11\\11&14\end{bmatrix}$

$\Rightarrow (A+A')'=\begin{bmatrix}2&11\\11&14\end{bmatrix}=(A+A')$

So, A + A is a symmetric matrix.

(ii) $A-A'=\begin{bmatrix}1&5\\6&7\end{bmatrix}-\begin{bmatrix}1&6\\5&7\end{bmatrix}=\begin{bmatrix}1-1&5-6\\6-5&7-7\end{bmatrix}=\begin{bmatrix}0&-1\\1&0\end{bmatrix}\Rightarrow (A-A')$

$=\begin{bmatrix}0&1\\-1&0\end{bmatrix}=-\begin{bmatrix}0&-1\\1&0\end{bmatrix}=-(A-A')$

So, A−A is a skew−symmetric matrix.

9. Find (A+ A) and $\frac{1}{2}(A-A')$, when , $A=\begin{bmatrix}0&a&b\\-a&0&c\\-b&-c&0\end{bmatrix}$

SOLUTION .:

$A=\begin{bmatrix}0&a&b\\-a&0&c\\-b&-c&0\end{bmatrix}\Rightarrow A'=\begin{bmatrix}0&-a&-b\\a&0&-c\\b&c&0\end{bmatrix}$

Now, A + A= $\begin{bmatrix}0&a&b\\-a&0&c\\-b&-c&0\end{bmatrix}+\begin{bmatrix}0&-a&-b\\a&0&-c\\b&c&0\end{bmatrix}$

$=\begin{bmatrix}0+0&a-a&b-b\\-a+a&0+0&c-c\\-b+b&-c+c&0+0\end{bmatrix}=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}\therefore \frac{1}{2}(A+A')=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$

Now, $A-A'=\begin{bmatrix}0&a&b\\-a&0&c\\-b&-c&0\end{bmatrix}-\begin{bmatrix}0&-a&-b\\a&0&-c\\b&c&0\end{bmatrix}$

$=\begin{bmatrix}0-0&a+a&b+b\\-a-a&0-0&c+c\\-b-b&-c-c&0-0\end{bmatrix}-\begin{bmatrix}0&2a&2b\\-2a&0&2c\\-2b&-2c&0\end{bmatrix}$

$\therefore \frac{1}{2}(A-A')=\frac{1}{2}\begin{bmatrix}0&2a&2b\\-2a&0&2c\\-2b&-2c&0\end{bmatrix}=\begin{bmatrix}0&a&b\\-a&0&c\\-b&-c&0\end{bmatrix}$

10. Express the following matrices as the sum of a symmetric and a skew symmetric matrix :

(i) $\begin{bmatrix}3&5\\1&-1\end{bmatrix}$

(ii) $\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}$

(iii) $\begin{bmatrix}3&3&-1\\-2&-2&1\\-4&-5&2\end{bmatrix}$

(iv) $\begin{bmatrix}1&5\\-1&2\end{bmatrix}$

SOLUTION .

:(i) Here, A = $\begin{bmatrix}3&5\\1&-1\end{bmatrix}\Rightarrow A'=\begin{bmatrix}3&1\\5&-1\end{bmatrix}$

Let $P=\frac{1}{2}(A+A')$

Now, $A+A'=\begin{bmatrix}3&5\\1&-1\end{bmatrix}+\begin{bmatrix}3&1\\5&-1\end{bmatrix}=\begin{bmatrix}3+3&5+1\\1+5&-1-1\end{bmatrix}=\begin{bmatrix}6&6\\6&-2\end{bmatrix}\Rightarrow P=\frac{1}{2}(A+A')$

$=\frac{1}{2}\begin{bmatrix}6&6\\6&-2\end{bmatrix}=\begin{bmatrix}3&3\\3&-1\end{bmatrix}$

$\Rightarrow P'=\begin{bmatrix}3&3\\3&-1\end{bmatrix}=P$

Thus, P = $\frac{1}{2}$ (A+ A) is a symmetric matrix. Also, let Q= $\frac{1}{2}$(A−A)

Now, A−A= $\begin{bmatrix}3&5\\1&-1\end{bmatrix}-\begin{bmatrix}3&1\\5&-1\end{bmatrix}=\begin{bmatrix}3-3&5-1\\1-5&-1+1\end{bmatrix}=\begin{bmatrix}0&4\\-4&0\end{bmatrix}$

$\Rightarrow Q=\frac{1}{2}(A-A')=\frac{1}{2}\begin{bmatrix}0&4\\-4&0\end{bmatrix}=\begin{bmatrix}0&2\\-2&0\end{bmatrix}$

$\Rightarrow Q'=\begin{bmatrix}0&-2\\2&0\end{bmatrix}=-\begin{bmatrix}0&2\\-2&0\end{bmatrix}=-Q$ Thus, $Q=\frac{1}{2}$= (A− A) is a skew−symmetric matrix. $\therefore P+Q=A=\begin{bmatrix}3&3\\3&-1\end{bmatrix}+\begin{bmatrix}0&2\\-2&0\end{bmatrix}$

(ii) Here, $A=\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}\Rightarrow A'=\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}$

Let $P=\frac{1}{2}(A+A')\therefore A+A'=\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}+\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}$

$=\begin{bmatrix}6+6&-2-2&2+2\\-2+(-2)&3+3&-1+(-1)\\2+2&-1+(-1)&3+3\end{bmatrix}=\begin{bmatrix}12&-4&4\\-4&6&-2\\4&-2&6\end{bmatrix}$

$\Rightarrow P=\frac{1}{2}(A+A')=\frac{1}{2}\begin{bmatrix}12&-4&4\\-4&6&-2\\4&-2&6\end{bmatrix}=\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}\Rightarrow P'=\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}=\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}=P$

Thus, $P=\frac{1}{2}(A+A')$is a symmetric matrix.

Also, let $Q=\frac{1}{2}(A-A')$ Now, A−A= $\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}-\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}$

$=\begin{bmatrix}6-6&-2+2&2-2\\-2+2&3-3&-1+1\\2-2&-1+1&3-3\end{bmatrix}=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}\Rightarrow Q=\frac{1}{2}(A-A')=\frac{1}{2}\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$ and $Q'=-Q$

So, Q = $\frac{1}{2}$(A−A) is a skew symmetric matrix.

Hence, $P+Q=A=\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}+\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$

(iii) Here, $A=\begin{bmatrix}3&3&-1\\-2&-2&1\\-4&-5&2\end{bmatrix}\Rightarrow A'=\begin{bmatrix}3&-2&-4\\3&-2&-5\\-1&1&2\end{bmatrix}$

Let $P=\frac{1}{2}(A+A')$

Now, $A+A'=\begin{bmatrix}3&3&-1\\-2&-2&1\\-4&-5&2\end{bmatrix}+\begin{bmatrix}3&-2&-4\\3&-2&-5\\-1&1&2\end{bmatrix}$

$=\begin{bmatrix}3+3&3-2&-1-4\\-2+3&-2-2&1-5\\-4-1&-5+1&2+2\end{bmatrix}=\begin{bmatrix}6&1&-5\\1&-4&-4\\-5&-4&4\end{bmatrix}$

So, $P=\frac{1}{2}(A+A')=\frac{1}{2}\begin{bmatrix}6&1&-5\\1&-4&-4\\-5&-4&4\end{bmatrix}$

$=\begin{bmatrix}3&1/2&-5/2\\1/2&-2&-2\\-5/2&-2&2\end{bmatrix}$ and $P'=\begin{bmatrix}3&1/2&-5/2\\1/2&-2&-2\\-5/2&-2&2\end{bmatrix}=P$

Thus, P = $\frac{1}{2}(A+A')$is a symmetric matrix. Also, let Q = $\frac{1}{2}(A-A')$

Again, $A-A'=\begin{bmatrix}3&3&-1\\-2&-2&1\\-4&-5&2\end{bmatrix}-\begin{bmatrix}3&-2&-4\\3&-2&-5\\-1&1&2\end{bmatrix}=\begin{bmatrix}3-3&3+2&-1+4\\-2-3&-2+2&1+5\\-4+1&-5-1&2-2\end{bmatrix}=\begin{bmatrix}0&5&3\\-5&0&6\\-3&-6&0\end{bmatrix}$

$\therefore\frac{1}{2}(A-A')=\frac{1}{2}\begin{bmatrix}0&5&3\\-5&0&6\\-3&-6&0\end{bmatrix}=\begin{bmatrix}0&5/2&3/2\\-5/2&0&3\\-3/2&-3&0\end{bmatrix}$

$\Rightarrow Q'=\begin{bmatrix}0&-5/2&-3/2\\5/2&0&-3\\3/2&-3&0\end{bmatrix}=-\begin{bmatrix}0&5/2&3/2\\-5/2&0&3\\-3/2&-3&0\end{bmatrix}=-Q.$

Thus, Q =$\frac{1}{2}$(A−A) is a skew symmetric matrix.

Hence, $P+Q=A=\begin{bmatrix}3&\frac{1}{2}&\frac{-5}{2}\\\frac{1}{2}&-2&-2\\\frac{-5}{2}&-2&2\end{bmatrix}+\begin{bmatrix}0&\frac{5}{2}&\frac{3}{2}\\\frac{-5}{2}&0&3\\\frac{-3}{2}&-3&0\end{bmatrix}$

(iv) Here, $A=\begin{bmatrix}1&5\\-1&2\end{bmatrix}\Rightarrow A'=\begin{bmatrix}1&-1\\5&2\end{bmatrix}$

Let $P=\frac{1}{2}(A+A')$ Now, $A+A'=\begin{bmatrix}1&5\\-1&5\end{bmatrix}+\begin{bmatrix}1&-1\\5&2\end{bmatrix}=\begin{bmatrix}1+1&5-1\\-1+5&2+2\end{bmatrix}=\begin{bmatrix}2&4\\4&4\end{bmatrix}$

$\Rightarrow P=\frac{1}{2}(A+A')$

$=\frac{1}{2}\begin{bmatrix}2&4\\4&4\end{bmatrix}=\begin{bmatrix}1&2\\2&2\end{bmatrix}\Rightarrow P'=\begin{bmatrix}1&2\\2&2\end{bmatrix}=P$

Thus, P = $\frac{1}{2}$(A+ A) is a symmetric matrix.

Also, let Q = $\frac{1}{2}$(A−A)

Now, A−A=$\begin{bmatrix}1&5\\-1&2\end{bmatrix}-\begin{bmatrix}1&-1\\5&2\end{bmatrix}=\begin{bmatrix}1-1&5+1\\-1-5&2-2\end{bmatrix}=\begin{bmatrix}0&6\\-6&0\end{bmatrix}$

$\Rightarrow Q=\frac{1}{2}(A-A')=\frac{1}{2}\begin{bmatrix}0&6\\-6&0\end{bmatrix}=\begin{bmatrix}0&3\\3&0\end{bmatrix}$

$\Rightarrow Q'=\begin{bmatrix}0&-3\\3&0\end{bmatrix}=-\begin{bmatrix}0&3\\-3&0\end{bmatrix}=-Q.$

Thus, Q =$\frac{1}{2}$(A−A) is a skew symmetric matrix.

Hence, P + Q = A=$\begin{bmatrix}1&2\\2&2\end{bmatrix}+\begin{bmatrix}0&3\\-3&0\end{bmatrix}$.

Choose the correct answer in the questions 11 and 12.

11. If A, B are symmetric matrices of same order, then AB−BA is a

(a) Skew symmetric matrix

(b) Symmetric matrix
(c) Zero matrix
(d) Identity matrix

SOLUTION .:

(A) We know that, A = A, B = B So, (AB−BA) = (AB)−(BA) = BA−AB = BA−AB=−(AB−BA) $\therefore$ AB−BA is a skew−symmetric matrix.

12. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$,then $A + A' = I$,If the value of α is

(a) $\dfrac{\pi}{6}$
(b) $\dfrac{\pi}{3}$
(c) π
(d) $\dfrac{3\pi}{2}$

SOLUTION .:

(B) Given that, $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \Rightarrow A' = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

We know that, A + A = I $\Rightarrow \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow 2\cos\alpha = 1 \Rightarrow \cos\alpha = \dfrac{1}{2}$

$\Rightarrow \cos\alpha = \cos\dfrac{\pi}{3} \Rightarrow \alpha = \dfrac{\pi}{3}$

NCERT - Exercise 3.

Using elementary transformations, find the inverse of each of the matrices, if it exists in questions 1 to 17..

1. $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

SOLUTION .:

Let us take $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ We know that, A = IA $\Rightarrow \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$ Applying $R_2 \to R_2 - 2R_1$ $\begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$

Applying $R_2 \to \frac{1}{5}R_2$ $\begin{bmatrix} 0 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{-2}{5} & \frac{1}{5} \end{bmatrix} A$

Applying $R_1 \to R_1 + R_2$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ \frac{-2}{5} & \frac{1}{5} \end{bmatrix} A$

Hence, $A^{-1} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ \frac{-2}{5} & \frac{1}{5} \end{bmatrix}$

2. $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

SOLUTION .:

Let us take $A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

We know that, A = IA $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Applying $R_1 \to R_1 - R_2$ $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$

Applying $R_2 \to R_2 - R_1$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} A$

Hence, $A^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$

3. $\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$

SOLUTION .:

Let us take $A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$

We know that, A = IA

$\Rightarrow \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Applying $R_2 \to R_2 - 2R_1$ $\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$

Applying $R_1 \to R_1 - 3R_2$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix} A$ Hence, $A^{-1} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$

4. $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$

SOLUTION .:

Let us take $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$

We know that, A = IA $\Rightarrow \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Applying $R_2 \to R_2 - 2R_1$ $\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$

Applying $R_1 \to R_1 - R_2$ $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix} A$

Applying $R_2 \to R_2 - R_1$ $\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix} A$

Applying $R_2 \to -R_2$ $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 5 & -2 \end{bmatrix} A$

Applying $R_1 \to R_1 - 2R_2$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix} A$

Hence, $A^{-1} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix}$

5. $\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$

SOLUTION .:

Let us take A = $\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$

We know that, A = IA $\Rightarrow \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$ Applying $R_2 \to R_2 - 3R_1$ $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} A$

Applying $R_1 \to R_1 - R_2$ $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix} A$

Applying$R_2 \to R_2 - R_1$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix} A$ Hence, $A^{-1} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$

6. $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

SOLUTION .:

Let us take $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$ We know that, A = IA $\Rightarrow \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Applying $R_1 \to R_1 - R_2$ $\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$

Applying $R_2 \to R_2 - R_1$ $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} A$

Applying $R_1 \to R_1 - 2R_2$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} A$

Hence, $A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$

7. $\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$

SOLUTION .:

Let us take A = $\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$

We know that, A = IA

$\Rightarrow \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Applying $R_2 \to R_2 - R_1$ $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} A$

Applying $R_1 \to R_1 - R_2$ $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} A$

Applying $R_2 \to R_2 - 2R_1$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix} A$ Hence, $A^{-1} = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$

8. $\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$

SOLUTION .:

Let us take A= $\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$

We know that, A = IA $\Rightarrow \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Applying $R_1 \to R_1 - R_2$ $\begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$

Applying $R_2 \to R_2 - 3R_1$ $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix} A$

Applying $R_1 \to R_1 - R_2$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix} A$

Hence, $A^{-1} = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}$

9. $\begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$

SOLUTION .:

Let us take A = $\begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$ We know that, A = IA $\Rightarrow \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Applying $R_1 \to R_1 - R_2 \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$

Applying $R_2 \to R_2 - 2R_1 \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} A$

Applying $R_1 \to R_1 - 3R_2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix} A$

Hence, $A^{-1} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix}$

10. $\begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$

SOLUTION .:

Let us take A = $\begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$ We know that, A = IA $\Rightarrow \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Applying $R_1 \to R_1 + R_2 \begin{bmatrix} -1 & 1 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} A$

Applying $R_1 \to -R_1 \begin{bmatrix} 1 & -1 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 0 & 1 \end{bmatrix} A$

Applying $R_2 \to R_2 + 4R_1 \begin{bmatrix} 1 & -1 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ -4 & -3 \end{bmatrix} = A$

Applying $R_2 \to -R_2 \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix} = A$

Applying $R_2 \to \frac{1}{2} R_2 \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 2 & \frac{3}{2} \end{bmatrix} A$

Applying $R_1 \to R_1 + R_2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{bmatrix} A$

Hence $A^{-1} = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{bmatrix}$

11. $\begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$

SOLUTION .:

Let us take A = $\begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$

We know that, A = IA $\Rightarrow \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Applying $R_1 \to R_1 - R_2 \begin{bmatrix} 1 & -4 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$

Applying $R_2 \to R_2 - R_1 \begin{bmatrix} 1 & -4 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} A$

Applying $R_1 \to R_1 + 2R_2 \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ -1 & 2 \end{bmatrix} A$

Applying $R_2 \rightarrow \frac{1}{2}R_2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ \frac{-1}{2} & 1 \end{bmatrix} A$

Hence, $A^{-1} = \begin{bmatrix} -1 & 3 \\ \frac{-1}{2} & 1 \end{bmatrix}$

12. $\begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$

SOLUTION .:

Let us take A = $\begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$

We know that, A = IA $\begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Applying $R_1 \rightarrow R_1 + 3R_2 \begin{bmatrix} 0 & 0 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} A$

We have all zeroes in the first row of the left hand side matrix of the above equation. $\therefore A^{-1}$ does not exist.

13. $\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

SOLUTION .:

Let us take A = $\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

We know that, A = IA $\Rightarrow \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Applying $R_1 \rightarrow R_1 + R_2 \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} A$

Applying $R_2 \rightarrow R_2 + R_1 \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} A$

Applying $R_1 \rightarrow R_1 + R_2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} A$

Hence, $A^{-1} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

14. $\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$

SOLUTION .:

Let us take A = $\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$ We know that, A = IA $\Rightarrow \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Applying $R_2 \rightarrow R_2 - 2R_1 \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$

We have all zeroes in the second row of the left hand side matrix of the above equation. Therefore, A^{-1}does not exist.

15. $\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$

SOLUTION .:

Let us take A = $\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$

We know that, A = IA $\Rightarrow \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

Applying $R_1 \to R_1 - R_2$, we get $\begin{bmatrix} 0 & -5 & 0 \\ 2 & 2 & 3 \\ 2 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

Applying $R_3 \to R_3 - R_2$, we get $\begin{bmatrix} 0 & -5 & 0 \\ 2 & 2 & 3 \\ 1 & -4 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} A$

Applying $R_2 \to R_2 - 2R_3$, we get $\begin{bmatrix} 0 & -5 & 0 \\ 0 & 10 & 5 \\ 1 & -4 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 3 & -2 \\ 0 & -1 & 1 \end{bmatrix} A$

Applying $R_1 \leftrightarrow R_3$,we get $\begin{bmatrix} 1 & -4 & -1 \\ 0 & 10 & 5 \\ 0 & -5 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ 0 & 3 & -2 \\ 1 & -1 & 0 \end{bmatrix} A$

Applying $R_3 \leftrightarrow R_2$, we get $\begin{bmatrix} 1 & -4 & -1 \\ 0 & -5 & 0 \\ 0 & 10 & 5 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ 1 & -1 & 0 \\ 0 & 3 & -2 \end{bmatrix} A$

Applying $R_3 \to R_3 + 2R_2$,we get $\begin{bmatrix} 1 & -4 & -1 \\ 0 & -5 & 0 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ 1 & -1 & 0 \\ 2 & 1 & -2 \end{bmatrix} A$

Applying $R_2 \to -R_2$,we get $\begin{bmatrix} 1 & -4 & -1 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ -1 & 1 & 0 \\ 2 & 1 & -2 \end{bmatrix} A$

Applying $R_2 \to \frac{1}{5}R_2$and $R_3 \to \frac{1}{5}R_3$, we get $\begin{bmatrix} 1 & -4 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ \frac{-1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & \frac{-2}{5} \end{bmatrix} A$

Applying $R_1 \to R_1 + 4R_2$,we get $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{4}{5} & -\frac{1}{5} & 1 \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & \frac{-2}{5} \end{bmatrix} A$

Applying $R_1 \to R_1 + R_3$,we get $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{-2}{5} & 0 & \frac{3}{5} \\ \frac{-1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & \frac{-2}{5} \end{bmatrix}$

Hence, $A^{-1} = \begin{bmatrix} \frac{-2}{5} & 0 & \frac{3}{5} \\ \frac{-1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & \frac{-2}{5} \end{bmatrix}$

16. $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$

SOLUTION .:

Let us take A = $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$

We know that, A = IA ⇒ $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

Applying $R_2 \to R_2 + 3R_1$,we get $\begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -11 \\ 2 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

Applying $R_3 \to R_3 - 2R_1$,we get $\begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -11 \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A$

Applying $R_1 \to R_1 + 3R_3$,wet get $\begin{bmatrix} 1 & 0 & 10 \\ 0 & 9 & -11 \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A$

Interchanging $R_2$and R_3we get $\begin{bmatrix} 1 & 0 & 10 \\ 0 & -1 & 4 \\ 0 & 9 & -11 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ -2 & 0 & 1 \\ 3 & 1 & 0 \end{bmatrix} A$

Applying $R_3 \to R_3 + 9R_2$, we get $\begin{bmatrix} 1 & 0 & 10 \\ 0 & -1 & 4 \\ 0 & 0 & 25 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ -2 & 0 & 1 \\ -15 & 1 & 9 \end{bmatrix} A$

Applying $R_2 \to -R_2$, we get $\begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & -4 \\ 0 & 0 & 25 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ 2 & 0 & -1 \\ -15 & 1 & 9 \end{bmatrix} A$

Applying $R_3 \to \frac{1}{25}R_3$,we get $\begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ 2 & 0 & -1 \\ \frac{-15}{25} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$

Applying $R_1 \to R_1 - 10R_3$, we get $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{-2}{5} & \frac{-3}{5} \\ 2 & 0 & -1 \\ \frac{-3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$

Applying $R_2 \to R_2 + 4R_3$,we get $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{-2}{5} & \frac{-3}{5} \\ \frac{-2}{5} & \frac{4}{25} & \frac{11}{25} \\ \frac{-3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$

Hence, $A^{-1} = \begin{bmatrix} 1 & \frac{-2}{5} & \frac{-3}{5} \\ \frac{-2}{5} & \frac{4}{25} & \frac{11}{25} \\ \frac{-3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix}$

17. $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

SOLUTION .:

Let us take A = $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ We know that, A = IA ⇒ $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

Interchanging $R_1$and R_2, we get

$$\begin{bmatrix} 5 & 1 & 0 \\ 2 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to R_1 - 2R_2$,we get $\begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

Applying $R_2 \to R_2 - 2R_1$,we get $\begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & -5 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 5 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

Applying $R_2 \to -R_2$, we get $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 5 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ -5 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

Applying $R_2 \to R_2 - R_3$,we get $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ -5 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix} A$

Applying $R_1 \to R_1 - R_2$,we get $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -5 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix} A$

Applying $R_3 \to R_3 - R_2$,we get $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -5 & 2 & -1 \\ 5 & -2 & 2 \end{bmatrix} A$

Applying $R_2 \to R_2 - 2R_3$,we get $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$

Hence, $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$

18. Matrices A and B will be inverse of each other only if

    (a) $AB = BA$

    (b) $AB = BA = 0$

    (c) $AB = 0, BA = I$

    (d) $AB = BA = I$

    SOLUTION .:

    (D) Matrices A and B will be inverse of each other only if, AB = BA -1.

NCERT - Miscellaneous Exercise

1. Let A = $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, show that $(aI + bA)^n + a^n I + na^{n-1}bA$,where I is the identity matrix of order 2 and $n \in N$.

    SOLUTION .:

    We have, $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $(aI + bA)^n = a^n I + na^{n-1}bA$ ...(i)

    For n = 1, $(aI + bA)^1 + a^1 I + 1a^{1-1}bA \Rightarrow al + bA = al + bA$

    So, it is true for n = 1 Let us assume that (i) is true for n = k, i.e., $(aI + bA)^k = a^k I + ka^{k-1}bA$

    Then $(aI + bA)^{k+1} = (aI + bA)^k + (aI + bA) = (a^k I + ka^{k-1}bA)(aI + bA)$

    $= a^{k+1} I \times I + ka^k bAI + a^k bAI + ka^{k-1}b^2 A \cdot A = a^{k+1} + ka^k bA + a^k bA + ka^{k-1}b^2 \times O$

    $= d^{k+1} I + (k+1)d^k bA = a^{k+1} I + (k+1)d^{k+1-1} bA \Rightarrow$ (i) is true for n = k + 1

    Hence, by mathematical induction it is true for all n $\in$ N.

2. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, prove that $A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$, $n \in N$.

    SOLUTION .:

    We shall prove it by mathematical induction.

To prove that n = 1 is true.

$$A^1 = \begin{bmatrix} 3^{1-1} & 3^{1-1} & 3^{1-1} \\ 3^{1-1} & 3^{1-1} & 3^{1-1} \\ 3^{1-1} & 3^{1-1} & 3^{1-1} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = A$$

We have, A = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ and $A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$.(i)

Thus, it is true for n = 1. Let us assume that (i) is true for n = k, i.e., $A^k = \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix}, k \in N$

Then, $A^{k+1} = A^k \cdot A = \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} \\ 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} \\ 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} & 3^{k-1}+3^{k-1}+3^{k-1} \end{bmatrix} = \begin{bmatrix} 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \end{bmatrix}$$

$\Rightarrow$ (i) is true for n = k + 1 So, by mathematical induction $A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}, n \in N$is true.

3. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then prove that $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$,where n is any positive integer.

SOLUTION .:

We have, $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$and $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$.(i) For n = 1, $A^1 = \begin{bmatrix} 1+2\times 1 & -4\times 1 \\ 1 & 1-2\times 1 \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} = A$

So, (i) is true for n = 1. Assume that (i) is true for n = k i.e., $A^k = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}$

Also, $A^{k+1} = \begin{bmatrix} 1+2(k+1) & -4(k+1) \\ k+1 & 1-2(k+1) \end{bmatrix}$for $n = k+1$. $\Rightarrow A^{k+1} = \begin{bmatrix} 2k+3 & -4k-4 \\ k+1 & -2k-1 \end{bmatrix}$

Now, $A^{k+1} = A \cdot A^k = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix} = \begin{bmatrix} 3+6k-4k & -12k-4+8k \\ 1+k-k & -4k-1+2k \end{bmatrix}$

$= \begin{bmatrix} 2k+3 & -4k-4 \\ k+1 & -2k-1 \end{bmatrix} = A^{k+1}$

So, (i) is true for n = k + 1. Hence, by mathematical induction $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$is true.

4. If A and B are symmetric matrices, prove that AB−BA is a skew symmetric matrix.

SOLUTION .:

Given. : A and B are symmetric matrices, therefore $A' = A, B' = B$.

To prove :(AB$-BA)' = -$(AB−BA)

Proof : $(AB$ - $BA)'$=$(AB)'$ -

(BA)=$B'A'$ - $A'B' = BA$ - $AB =$ -

$(AB$ - $BA)$

So, AB−BA is a skew-symmetric matrix.

5. . Show that the matrix BAB is symmetric or skew symmetric according as A is symmetric or skew symmetric.

SOLUTION .:

Case1: Given that A is symmetric. We will prove BAB is symmetric. As A is symmetric, so $A' = A$. Now, $(B'AB)' = B'A'(B')' = B'A'B = B'AB$ Thus, $B'AB$ is a symmetric matrix.

Case II: Given is skew symmetric, i.e., $A' = -$A. We will prove that $B'AB$ is skew symmetric.

Now, $(B'AB)' = B'A'(B')' = B'A'B$

$=B'(-A)B=-B'AB$

Hence, $B'AB$ is a skew-symmetric matrix.

6. Find the values of x, y, z if the matrix $A=\begin{bmatrix}0 & 2y & z\\ x & y & -z\\ x & -y & z\end{bmatrix}$ satisfy the equation $A'A=I$.

SOLUTION .:

Given that, matrix $A=\begin{bmatrix}0 & 2y & z\\ x & y & -z\\ x & -y & z\end{bmatrix}$ and $A'A=I. \Rightarrow \begin{bmatrix}0 & x & x\\ 2y & y & -y\\ z & -z & z\end{bmatrix}\begin{bmatrix}0 & 2y & z\\ x & y & -z\\ x & -y & z\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$

$\Rightarrow \begin{bmatrix}0+x^2+x^2 & 0+xy-xy & 0-zx+zx\\ 0+xy-xy & 4y^2+y^2+y^2 & 2yz-yz-yz\\ 0-zx+xz & 2yz-zy-zy & z^2+z^2+z^2\end{bmatrix}$

$=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} \Rightarrow \begin{bmatrix}2x^2 & 0 & 0\\ 0 & 6y^2 & 0\\ 0 & 0 & 3z^2\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} \Rightarrow 2x^2=1, 6y^2=1, 3z^2=1 \Rightarrow x^2=\frac{1}{2}, y^2=\frac{1}{6}, z^2=\frac{1}{3}$

Hence, $x=\pm\frac{1}{\sqrt{2}}, y=\pm\frac{1}{\sqrt{6}}, z=\pm\frac{1}{\sqrt{3}}$

7. For what values of x : $\begin{bmatrix}1 & 2 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 0\\ 2 & 0 & 1\\ 1 & 0 & 2\end{bmatrix}\begin{bmatrix}0\\ 2\\ x\end{bmatrix}=O$?

SOLUTION .:

$\begin{bmatrix}1 & 2 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 0\\ 2 & 0 & 1\\ 1 & 0 & 2\end{bmatrix}\begin{bmatrix}0\\ 2\\ x\end{bmatrix}=O$

$\Rightarrow \begin{bmatrix}1+4+1 & 2+0+0 & 0+2+0\end{bmatrix}\begin{bmatrix}0\\ 2\\ x\end{bmatrix}=O$

$\Rightarrow \begin{bmatrix}6 & 2 & 4\end{bmatrix}\begin{bmatrix}0\\ 2\\ x\end{bmatrix}=O$

$0+4+4x=0 \Rightarrow 4(x+1)=0 \Rightarrow x+1=0 \Rightarrow x=-1.$

8. If $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$, then show that $A^2-5A+7I=O$.

SOLUTION .:

Given that, $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix} \Rightarrow A^2=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$

$=\begin{bmatrix}9-1 & 3+2\\ -3-2 & -1+4\end{bmatrix}=\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}$

and $5A=5\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}=\begin{bmatrix}15 & 5\\ -5 & 10\end{bmatrix}$

Now, substituting the values, we have $A^2-5A+7I=\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}-\begin{bmatrix}15 & 5\\ -5 & 10\end{bmatrix}+\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}$

$=\begin{bmatrix}-7 & 0\\ 0 & -7\end{bmatrix}+\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}=O$

Hence, proved.

9. Find x, if $\begin{bmatrix}x & -5 & -1\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}\begin{bmatrix}x\\ 4\\ 1\end{bmatrix}=O.$

SOLUTION .:

$$\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$$

$$\Rightarrow \begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} x+2 \\ 8+1 \\ 2x+3 \end{bmatrix} = O \Rightarrow \begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} x+2 \\ 9 \\ 2x+3 \end{bmatrix} = O$$

$$\Rightarrow x(x+2) - 45 - 2x - 3 = 0 \Rightarrow x^2 - 48 = 0 \Rightarrow x = \pm 4\sqrt{3}.$$

10. A manufacturer produces three products x, y, z which he sells in two markets. Annual sales are indicated as :

(a) If unit sale prices of x,y and z are Rs 2.50, Rs 1.50 and Rs 1.00, respectively, find the total revenue in each market with the help of matrix algebra.

(b) If the unit costs of the above three commodities are Rs 2.00, Rs 1.00 and 50 paise respectively. Find the gross profit

. SOLUTION .:

Let quantity matrix be $A = \begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix}$

(a) Selling Price $B = \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix}$

Now, Total Selling Price, $AB = \begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix} \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix}$

$= \begin{bmatrix} 10,000 \times 2.50 + 2,000 \times 1.50 + 18,000 \times 1 \\ 6,000 \times 2.50 + 20,000 \times 1.50 + 8,000 \times 1 \end{bmatrix}$

$= \begin{bmatrix} 25,000 + 3,000 + 18,000 \\ 15,000 + 30,000 + 8,000 \end{bmatrix}$

Total revenue in market I = Rs. 46,000.

Total revenue in market II = Rs. 53,000.

(b) Now, cost price$= \begin{bmatrix} 2.00 \\ 1.00 \\ 0.50 \end{bmatrix}$

Total cost price$= \begin{bmatrix} 1000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 0.5 \end{bmatrix}$

$= \begin{bmatrix} 10,000 \times 2 + 2,000 \times 1 + 18,000 \times 0.5 \\ 6,000 \times 2 + 20,000 \times 1 + 8,000 \times 0.5 \end{bmatrix}$

Total cost price = 31000 + 36000 = Rs. 67,000.

Total selling price = 46000 + 53000 = Rs. 99,000

Profit = S.P.−C.P. = 99,000 −

67,000 = Rs. 32,000.

11. Find the matrix X so that $X \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

SOLUTION .:

$X \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

We can say that X is a 2 2 matrix.

Let $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\therefore \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a+4b & 2a+5b & 3a+6b \\ c+4d & 2c+5d & 3c+6d \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

$\Rightarrow a+4b=-7$ ....

(i) and $c+4d=2$ ...(ii)

$2a+5b=-8$ .... (iii)

and $2c+5d=4$ ...(iv)

Solving(i) and (iii), we get a = 1, b =−2 Solving (ii) and (iv), we get c = 2,d =0

Hence, $X=\begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$

12. If A and B are square matrices of the same order such that AB = BA, then prove by induction that ABn = BnA. Further, prove that $(AB)^n = A^nB^n$for all $n \in N$.

SOLUTION .: Given AB = BA,

To prove

(1) $AB^n = B^nA$ and (2) $(AB)^n = A^nB^n \forall n \in N$ ...(i) We will prove it by mathematical induction.

(1) Given that AB = BA ...(ii)

We have to prove $AB^n = B^nA$ For n = 1, $AB^1 = B^1A \Rightarrow$ AB = BA, which is true [from (ii)]

Let it be true for n = ABm = BmA ...(iii)

Then, for n = m + 1,

$AB^{m+1} = A(B^mB) = (AB^m)B = (B^mA)B$ [using (iii)] $= B^m(AB) = B^m(BA)$ [using (ii)] $= (B^mB)A = B^{m+1}A$.So, it is true for n = m + 1

$\therefore AB^n = B^nA$.

(2) For $n=1, (AB)^1 = A^1B^1 \Rightarrow AB = BA$ which is true for n = 1 Let (i) be true for a positive integer n = m. i.e., $(AB)^m = A^mB^m$ ....(iv)

then for n = m + 1, $(AB)^m + 1 = (AB)^m(AB) = (A^mB^m)(AB)$ (from (iv))

$= A^m(B^mA)B = A^m(AB^m)B[AB^n = B^nA$

$\forall n$,whenever AB = BA]

= $(A^mA)(B^mB) = A^m + 1B^m + 1$ So, it holds for n = m + 1 Hence. (AB)$^n - A^nB^n \forall n \in N$.

Choose the correct answer in the following questions : .

13. If$A=\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$is such that $A^2 = I$, then

(a) $1+\alpha^2+\beta\gamma=0$

(b) $1-\alpha^2+\beta\gamma=0$

(c) $1-\alpha^2-\beta\gamma=0$

(d) $1+\alpha^2-\beta\gamma=0$

SOLUTION .:

(C) Given $A=\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$

Now, $A^2 = I \Rightarrow \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} \alpha^2+\beta\gamma & \alpha\beta-\alpha\beta \\ \gamma\alpha-\alpha\gamma & \gamma\beta+\alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} \alpha^2+\beta\gamma & 0 \\ 0 & \gamma\beta+\alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \alpha^2+\beta\gamma=1 \Rightarrow 1-\alpha^2-\gamma\beta=0$

14. If the matrix A is both symmetric and skew symmetric, then

(a) A is a diagonal matrix

(b) A is a zero matrix

(c) A is a square matrix

(d) None of these

SOLUTION .:

(B) Consider the matrix A. Clearly, $A' = A$ and A' =−A ∴ A =−A ⇒ 2A = 0 ⇒ A = 0 ∴ A is a zero matrix.

15. If A is a square matrix such that $A^2 = A$, then $(I+A)^3 - 7A$is equal to

(a) A

(b) $I - A$

(c) I

(d) 3A

SOLUTION .:

(C) We are given that A2 = A Now, $(I+A)^3 - 7A = I^3 + A^3 + 3IA(I+A) - 7A$

$= I + A^2 + 3A(I+A) - 7A = I + A^2 + 3A + 3A^2 - 7A$

$= I + 4A^2 - 4A = I + 4A - 4A = I$