1. Given matrices $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ and $B = \begin{pmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{pmatrix}$, calculate $A+B$.

2. If $C = \begin{pmatrix} 2 & 0 & 1 \\ 3 & 3 & 3 \\ 1 & 1 & 2 \end{pmatrix}$, find $2C$.

3. Determine the product of $A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 2 & 1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}$.

4. What is the transpose of $M = \begin{pmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{pmatrix}$?

5. Is the matrix $S = \begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & 1 \\ -1 & -1 & 0 \end{pmatrix}$ symmetric, skew-symmetric, or neither? Justify your answer.

6. Given $Z = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, explain the significance of matrices Z and I in matrix operations.

7. Prove that matrix multiplication is not commutative using $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ and $B = \begin{pmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{pmatrix}$.

8. Show that there exist non-zero 2×2 matrices X and Y such that $XY = Z$, where Z is the zero matrix. Extend this concept to 3×3 matrices.

9. Find the inverse of $P = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$, if it exists.

10. Explain the concept of an identity matrix using 3×3 matrices as examples.

11. Calculate $AB - BA$ for $A = \begin{pmatrix} 3 & 2 & 1 \\ 1 & 0 & 2 \\ 2 & 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 0 & -1 \end{pmatrix}$.

12. Given $A = \begin{pmatrix} 1 & 3 & 2 \\ -3 & -1 & -2 \\ 2 & 2 & 1 \end{pmatrix}$, determine if A is invertible.

13. For $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{pmatrix}$, compute $A^T A$.

14. If $A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$ and $A^T = A$, find the conditions on a,b,c,d,e,f,g,h,i.

15. Show that the matrix $M = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ does not have an inverse.

16. Calculate the determinant of $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$. Use this to explain why A is not invertible.

17. Given $A = \begin{pmatrix} 1 & 0 & 2 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$, verify that AB is not equal to BA.

18. If $X = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$, is X symmetric? Why?

19. For the matrix $M = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 4 & 1 \\ 1 & 2 & 2 \end{pmatrix}$, determine if M is symmetric or skew-symmetric.

20. Calculate the product of $A = \begin{pmatrix} 2 & 4 & 6 \\ 0 & 1 & 2 \\ 3 & 5 & 7 \end{pmatrix}$ and its transpose.

21. Explain why the zero matrix does not have an inverse.

22. Given $A = \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}$, find A^{-1} if possible.

Answers

1. $A+B=\begin{pmatrix}10&10&10\\10&10&10\\10&10&10\end{pmatrix}$

2. $2C=\begin{pmatrix}4&0&2\\6&6&6\\2&2&4\end{pmatrix}$

3. $AB=\begin{pmatrix}5&6&8\\5&5&3\\4&5&7\end{pmatrix}$

4. $M^T=\begin{pmatrix}0&3&6\\1&4&7\\2&5&8\end{pmatrix}$

5. The matrix S is skew-symmetric because $S^T=-S$.

6. The zero matrix Z serves as the additive identity in matrix operations, while the identity matrix I serves as the multiplicative identity.

7. To prove non-commutativity, we calculate AB and BA and show they are different: $AB \neq BA$.

8. For 2×2 matrices, example matrices $X=\begin{pmatrix}1&2\\2&-1\end{pmatrix}$ and $Y=\begin{pmatrix}-1&2\\2&1\end{pmatrix}$ have $XY=Z$. This concept extends to 3×3 matrices with similar properties.

9. $P^{-1}=\frac{1}{4}\begin{pmatrix}3&-1&-1\\-1&3&-1\\-1&-1&3\end{pmatrix}$

10. An identity matrix is a square matrix in which all the elements of the principal diagonal are ones and all other elements are zeros. For 3×3, $I=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$.

11. $AB-BA=\begin{pmatrix}-4&6&-6\\-10&0&10\\4&-6&6\end{pmatrix}$

12. To determine if A is invertible, we calculate its determinant. Since $\det(A)\neq 0$, A is invertible.

13. $A^TA=\begin{pmatrix}26&32&10\\32&41&16\\10&16&25\end{pmatrix}$

14. For A to be symmetric, a,e,i are arbitrary real numbers, and $b=d$, $c=g$, $f=h$.

15. M does not have an inverse because its determinant is 0, indicating it is singular.

16. The determinant of A is 0, which means A is not invertible.

17. Calculating AB and BA shows that they are indeed different, verifying that matrix multiplication is not commutative.

18. Yes, X is symmetric because $X=X^T$.

19. M is neither symmetric nor skew-symmetric because $M\neq M^T$ and $M\neq -M^T$.

20. The product of A and its transpose is a symmetric matrix.

21. The zero matrix does not have an inverse because there is no matrix that, when multiplied by the zero matrix, yields the identity matrix.

22. The inverse of A does not exist because its determinant is 0, indicating it is singular.