MULTIPLE CHOICE TYPE QUESTIONS

1. If the points $(2,-3),(k,-1)$ and $(0,4)$ are collinear then the value of k. is

   (a) $\frac{5}{7}$
   (b) $\frac{10}{7}$
   (c) $\frac{2}{5}$
   (d) $\frac{11}{3}$

   SOLUTION:

   The given points are collinear $\therefore \begin{vmatrix} 2 & -3 & 1 \\ k & -1 & 1 \\ 0 & 4 & 1 \end{vmatrix} = 0$ Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get $\Rightarrow \begin{vmatrix} 2 & -3 & 1 \\ k-2 & 2 & 0 \\ -2 & 7 & 0 \end{vmatrix} = 0$

   $\Rightarrow \begin{vmatrix} k-2 & 2 \\ -2 & 7 \end{vmatrix} = 0 \Rightarrow 7k - 14 + 4 = 0 \Rightarrow k = \frac{10}{7}$.

2. Matrices A and B will be inverse of each other only if

   (a) $AB = BA$
   (b) $AB = BA = 0$
   (c) $AB = 0, BA = I$
   (d) $AB = BA = I$

   SOLUTION

   (d) Matrices A and B will be inverse of each other only if, AB = BA -1.

3. If $P(A) = \frac{2}{5}, P(B) = \frac{3}{10}$ and $P(A'/B').P(B'/A')$ is equal to

   (a) $\frac{25}{27}$
   (b) $\frac{25}{42}$
   (c) $\frac{5}{17}$
   (d) $\frac{2}{13}$

   SOLUTION

   $P(A'/B') = \frac{P(A' \cap B')}{P(B')} = \frac{1-P(A \cup B)}{1-P(B)}$

   $\Rightarrow P(A'/B') = \frac{1-(2/5+3/10-1/5)}{1-3/10}$

   $\Rightarrow P(A'/B') = \frac{5}{7}$

   Similarly $P(B'/A') = \frac{P(B' \cap A')}{P(A')} = \frac{1-P(A \cup B)}{1-P(A)}$

   $P(B'/A') = \frac{1-1/2}{1-2/5} = \frac{5}{6}$

   $P(A'/B').P(B'/A') = \frac{25}{42}$

4. Let S be the set of all triangles in a plane and let R be the congruence relation on S. Then , R is
   (a) reflexive, symmetric and transitive
   (b) reflexive and symmetric but not transitive
   (c) reflexive and transitive but not symmetric
   (d) symmetric and transitive but not reflexive

SOLUTION

Every triangle is congruent to itself. therefore the given relation is reflexive.

Now, if $\Delta_1 \cong \Delta_2 \Rightarrow \Delta_2 \cong \Delta_1$ hence the relation is reflexive.

Now consider, $\Delta_1 \cong \Delta_2, \Delta_2\Delta_3 \Rightarrow \Delta_1 \cong \Delta_3$ hence the relation is transitive as well. Hence the given relation is equivalence relation. Ans. (a)

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5. $\int \frac{\cos 2x}{(\sin x+\cos x)^2}dx$ is equal to
   (a) $\frac{-1}{\sin x+\cos x}+C$
   (b) $\log|\sin x+\cos x|+C$
   (c) $\log|\sin x-\cos x|+C$
   (d) $\frac{1}{(\sin x+\cos x)^2}+C$

SOLUTION

. (b) : Let $I=\int \frac{\cos 2x}{(\sin x+\cos x)^2}dx=\int \frac{\cos^2 x-\sin^2 x}{(\cos x-\sin x)^2}dx=\int \frac{\cos x-\sin x}{\cos x+\sin x}dx=\log(\cos x+\sin x)+C$

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6. The value of determinant of a skew symmetric matrix of odd order is
   (a) 1
   (b) -1
   (c) 0
   (d) none of these

SOLUTION:

Let A be an n × n skew symmetric matrix, where n is an odd positive integer, then $A^t=-A \Rightarrow \det(A^t)=\det(-A) \Rightarrow \det A=(-1)^n \det A \Rightarrow \det A=-\det A$ n is odd) $\Rightarrow 2\det A=0 \Rightarrow \det A=0$.

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7. Cartesian equation of the line through $A(3,4,-7)$ and $B(1,-1,6)$ is
   (a) $\frac{x-3}{-2}=\frac{y-4}{-5}=\frac{z-7}{17}$
   (b) $\frac{x+1}{2}=\frac{y-4}{5}=\frac{z+7}{13}$
   (c) $\frac{x+3}{-2}=\frac{y-2}{-5}=\frac{z+7}{13}$
   (d) $\frac{x-3}{-2}=\frac{y-4}{-5}=\frac{z+7}{13}$

SOLUTION

we have $\frac{x-3}{-2}=\frac{y-4}{-5}=\frac{z+7}{13}$ Hence, the cartesian form of the equation (i) is $\frac{x-3}{-2}=\frac{y-4}{-5}=\frac{z+7}{13}$

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8. $cos^{-1}(-\frac{1}{2}) - 2sin^{-1}(\frac{1}{2}) + 3cos^{-1}(-\frac{1}{\sqrt{2}}) - 4tan^{-1}(-1)$ equals

   (a) $\frac{13\pi}{12}$

   (b) $\frac{43\pi}{12}$

   (c) $\frac{23\pi}{12}$

   (d) $\frac{43\pi}{15}$

   SOLUTION

   $cos^{-1}(-\frac{1}{2}) - 2sin^{-1}(\frac{1}{2}) + 3cos^{-1}(-\frac{1}{\sqrt{2}}) - 4tan^{-1}(-1)$

   $= \pi - cos^{-1}(\frac{1}{2}) - 2(\frac{\pi}{6}) + 3(\pi - cos^{-1}(\frac{1}{\sqrt{2}})) + 4tan^{-1}(1)$

   $= \pi - \frac{\pi}{3} - \frac{\pi}{3} + 3(\pi - \frac{\pi}{4}) + 4\frac{\pi}{4}$

   $= \frac{43\pi}{12}$

9. The value of $cot(cosec^{-1}\frac{5}{3} + tan^{-1})(\frac{2}{3}))$ is

   (a) $\frac{6}{17}$

   (b) $\frac{6}{19}$

   (c) $\frac{1}{2}$

   (d) $\frac{3}{19}$

   SOLUTION

   $cot(cosec^{-1}\frac{5}{3} + tan^{-1})(\frac{2}{3}))$

   $=cot(tan^{-1}\frac{3}{4} + tan^{-1}\frac{2}{3})$

   $= cot\left[tan^{-1}\left(\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{1}{2}}\right)\right]$

   $= cot\left[tan^{-1}\left(\frac{17}{6}\right)\right]$

   $= \frac{6}{17}$

10. If $3tan^{-1}x + cot^{-1}x = \pi$ then x equals

    (a) 0

    (b) 1

    (c) 2

    (d) - 1

    SOLUTION

    $3tan^{-1}x + cot^{-1}x = \pi = 2tan^{-1}x = \pi - \frac{\pi}{2}$ [Using $tan^{-1}x + cot^{-1}x = \frac{\pi}{2}$]

    $\Rightarrow 2tan^{-1}x = \frac{\pi}{2}$

$tan^{-1}\dfrac{2x}{1-x^2} = \dfrac{\pi}{2}$

$\Rightarrow \dfrac{2x}{1-x^2} = tan\dfrac{\pi}{2} \Rightarrow \dfrac{2x}{1-x^2} = \dfrac{1}{0}$

$\Rightarrow x^2 = 1 \Rightarrow x = \pm 1 \Rightarrow x = 1$

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11. The value of $sin^{-1}[cos(\dfrac{33\pi}{5})]$ is

   (a) $\dfrac{-\pi}{6}$

   (b) $\dfrac{-\pi}{4}$

   (c) $\dfrac{-\pi}{10}$

   (d) $\dfrac{-\pi}{8}$

SOLUTION

$\dfrac{33\pi}{5} = 6\pi + \dfrac{3\pi}{5}$

Using the above,

$sin^{-1}[cos(\dfrac{33\pi}{5})]$ = $\sin^{-1}[cos(6\pi + \dfrac{3\pi}{5})]$

$= sin^{-1}[cos(\dfrac{3\pi}{5})]$

$= sin^{-1}[cos(\dfrac{\pi}{2} + \dfrac{\pi}{10})]$

$sin^{-1}[-sin\dfrac{\pi}{10}]$ = - $\dfrac{\pi}{10}$

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12. If $x = y\sqrt{1-y^2}$ then $\dfrac{dy}{dx}$ is equal to

   (a) $\dfrac{\sqrt{1-y^2}}{1+2y^2}$

   (b) $\dfrac{\sqrt{1-y^2}}{1-2y^2}$

   (c) $\dfrac{\sqrt{3-y^2}}{1+2y^2}$

   (d) $\dfrac{\sqrt{1+y^2}}{3-2y^2}$

SOLUTION

$x = y\sqrt{1-y^2}$ then $\dfrac{dy}{dx}$ On differentiating both sides w.r.t. x , we get :

$1 = \dfrac{dy}{dx}\sqrt{1-y^2} + y\dfrac{1}{2\sqrt{1-y^2}}(-2y)\dfrac{dy}{dx}$

$\Rightarrow 1 = \dfrac{dy}{dx}\left(\dfrac{1-y^2-y^2}{\sqrt{1-y^2}}\right)$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{\sqrt{1-y^2}}{1-2y^2}$

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13. The degree of the differential equation $\left(\dfrac{d^2y}{dx^2}\right)^3 + \left(\dfrac{dy}{dx}\right) + \sin\left(\dfrac{dy}{dx}\right) + 1 = 0$ is

(a) 3
(b) 2
(c) 1
(d) not defined

SOLUTION

(d) Since the given differential is not a polynomial in $\frac{dy}{dx}$, therefore, its degree is not defined.

VERY SHORT ANSWER QUESTIONS

14. . Differentiate : $\cos(\sqrt{x})$

SOLUTION

. : Let $y = \cos\left(\sqrt{x}\right) \Rightarrow \frac{dy}{dx} = \frac{d}{dx}\cos\left(\sqrt{x}\right) = -\sin\sqrt{x} \cdot \frac{d}{dx}(\sqrt{x}) = -\sin\sqrt{x} \cdot \frac{1}{2}(x)^{\frac{-1}{2}} = \frac{-\sin\sqrt{x}}{2\sqrt{x}}$

15. .Evaluate : $\int_0^2 e^{3-4x}dx$

SOLUTION

we have, $I = \int_0^2 e^{3-4x}dx = \left[\frac{e^{3-4x}}{-4}\right]_0^2 = \frac{1}{4}\left[e^{3-8} - e^{3-0}\right] = \frac{-1}{4}\left[e^{-5} - e^{3}\right]$

16. .Evaluate : $\int_{-\pi}^{\pi} x^{10}\sin^7 xdx$

SOLUTION

we have, $I = \int_{-\pi}^{\pi} x^{10}\sin^7 xdx$ Let $f(x) = x^{10}\sin^7 x$ And $f(-x) = (-x)^{10}[\sin(-x)]^7 = -x^{10}\sin^7 x = -f(x) \therefore f(x)$ is an odd function.

$\therefore I = \int_{-\pi}^{\pi} x^{10}\sin^7 xdx = 0$

17. .Evaluate : $\int_0^1 \frac{\tan^{-1}x}{1+x^2}dx$

SOLUTION

we have $I = \int_0^1 \frac{\tan^{-1}xdx}{1+x^2}$ Put $\tan^{-1}x = t \Rightarrow \frac{1}{1+x^2}dx = dt$ When $x = 0, t = 0$ and when $x = 1, t = \frac{\pi}{4} \therefore I = \int_0^1 \frac{\tan^{-1}x}{1+x^2}dx = \int_0^{\frac{\pi}{4}} t\,dt =$

$\left[\frac{t}{2}\right]_0^{\frac{\pi}{4}} = \frac{\pi^2}{32}$

18. Evaluate : $\int (x^2+5)^3 dx$

SOLUTION

.(i) Let$I = \int (x^2+5)^3 dx$ Expanding the in integrand by the binomial formula, $I = \int \left(x^6+15x^4+75x^2+125\right) dx = \frac{x^7}{7}+\frac{15x^5}{5}+\frac{75x^3}{3}+125x+C \Rightarrow I = \frac{x^7}{7}+3x^5+25x^3+125x+C$

19. Consider two points P and Q with position vectors $\overrightarrow{OP} = 3\vec{a}-2\vec{b}$ and $\overrightarrow{OQ} = \vec{a}+\vec{b}$. Find the position vector of a point R which divides the line joining P and Q in the ratio2 : 1 internally,

SOLUTION

The position vector of the point R dividing the line joining P and Q internally in the ratio 2 : 1 is $\overrightarrow{OR} = \frac{2\left(\vec{a}+\vec{b}\right)+\left(3\vec{a}-2\vec{b}\right)}{2+1} = \frac{5\vec{a}}{3}$

20. Find the principal value of $sin^{-1}(\frac{\sqrt{3}}{2})$

SOLUTION

Range of principal value branch of sos^{-1} function is $[-\frac{\pi}{2},\frac{\pi}{2}]$

$\Rightarrow sin\theta = \frac{\sqrt{3}}{2} \Rightarrow sin\theta = sin\frac{\pi}{3}$

$\Rightarrow \theta = \frac{\pi}{3}$

21. Prove that $3sin^{-1}x = sin^{-1}(3x-4x^3), x \in [-\frac{1}{2},\frac{1}{2}]$

SOLUTION

Let $x = sin\theta \Rightarrow \theta = sin^{-1}x$

$\Rightarrow sin^{-1}(3x-4x^3) = sin^{-1}(3sin\theta-4sin^3\theta) = sin^{-1}(sin3\theta)$ $=3\theta = 3sin^{-1}x$

22. Evaluate $\int \frac{x^3}{x+2}dx$

SOLUTION

.: Let $I = \int \frac{x^3}{x+2}dx$ Dividing x^2 by $x+2$,we get $= \int \left(x^2-2x+4-\frac{8}{x+2}\right) dx = \frac{x^3}{3}-x^2+4x-8\log|x+2|+C$

23. The value of the integral $\int_{1/x}^{2/\pi} \frac{sin(1/x)}{x^2}dx =$

(a) 1
(b) -1
(c) π
(d) 2

SOLUTION

Let $\frac{1}{x} = t \Rightarrow -\frac{1}{x^2}dx = dt$

$\int_{1/x}^{2/\pi} \frac{sin(1/x)}{x^2}dx$

=- $\int_{\pi/2}^{\pi} sint dt$

$= 1$

24. $\int_0^1 \frac{e^{-x}dx}{1+e^{-x}} =$

(a) $log(\frac{1-e}{2e}) + \frac{1}{e} + 1$

(b) $log(\frac{1-e}{2e}) + \frac{1}{e} + 1$

(c) $log(\frac{1+e}{2e}) - \frac{1}{e} + 1$

(d) $2log(\frac{1-e}{e}) + \frac{1}{e} + 1$

SOLUTION

Let $1+e^{-x} = t \Rightarrow -e^{-x}dx = dt$

$\int_0^1 \frac{e^{-x}dx}{1+e^{-x}} =$

$\int_0^{1/e} -\frac{1}{t}dt$

$=log(\frac{e+1}{2e}) - \frac{1}{e} + 1$

FILL IN THE BLANKS

25. If $cos[tan^{-1}x + cot^{-1}\sqrt{3}] = 0$ then the value of x is $\cdots$

SOLUTION

$tan^{-1}x + cot^{-1}\sqrt{3} = cos^{-1}0$

$\Rightarrow tan^{-1}x + cot^{-1}\sqrt{3} = \frac{\pi}{2}$

$\Rightarrow tan^{-1}x = \frac{\pi}{2} - cot^{-1}\sqrt{3}$

$\Rightarrow tan^{-1}x = tan^{-1}\sqrt{3}$

$\Rightarrow x = \sqrt{3}$

26. The value of $\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \sin^3 x\,dx$ is $\cdots$

SOLUTION

. Let $I = \int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \sin^3 x\,dx$ and $f(x) = \sin^3 x \Rightarrow f(-x) = (\sin(-x))^3 = -\sin^3 x = -f(x) \Rightarrow f(x)$ is an odd function. $\therefore \int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \sin^3 x dx = 0$

27. . The value of $\int_2^3 \frac{1}{x}dx$ is $\cdots$

SOLUTION

$\int_2^3 \frac{1}{x}dx = [\text{In}\, x]_2^3 = \text{In}\,3 - \text{In}\,2 = \text{In}\,\frac{3}{2}$

28. Integration of $\sin 2x - 4e^{3x}$ is $\cdots$ SOLUTION.

: Let $I = \int \left(\sin 2x - 4e^{3x}\right) dx = -\frac{\cos 2x}{2} - \frac{4e^{3x}}{3} + C$

29. . Integration of $\int \left(4e^{3x} + 1\right) dx$ is $\cdots$

SOLUTION.

: Let $I = \int \left(4e^{3x} + 1\right) dx = \frac{4e^{3x}}{3} + x + C$

30. Integration of $\int \frac{x^3 + 3x + 4}{\sqrt{x}} dx$ is $\cdots$

SOLUTION.

: Let$I = \int \left(\frac{x^3 + 3x + 4}{\sqrt{x}}\right) dx = \int \left(x^{5/2} + 3x^{1/2} + 4x^{-1/2}\right) dx = \frac{x^{7/2}}{7/2} + \frac{3x^{3/2}}{3/2} + \frac{4x^{1/2}}{1/2} + C = \frac{2}{7}x^{7/2} + 2x^{3/2} + 8\sqrt{x} + C$

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