## MULTIPLE CHOICE TYPE QUESTIONS

1. If A and B are symmetric matrices, prove that AB$-$BA is a skew symmetric matrix.

   (a) skew symmetric
   (b) symmetric
   (c) null matrix
   (d) none of these

   SOLUTION

   Given. : A and B are symmetric matrices, therefore $A' = A, B' = B$.
   : $(AB - BA)' = (AB)' - (BA)$
   $= B'A'$ - $A'B' = BA - AB$
   = - $(AB$ - $BA) So, AB - BA$ is a skew-symmetric matrix.

2. The points $(a,b),(a',b'),(a-a',b-b')$ are collinear iff

   (a) $a'b = 2ab'$.
   (b) $2a'b = ab'$.
   (c) $a'b = ab'$.
   (d) $ab = a'b'$.

   SOLUTION:

   The given points are collinear iff $\begin{vmatrix} a & b & 1 \\ a' & b' & 1 \\ a-a' & b-b' & 1 \end{vmatrix} = 0$ Applying $R_3 \to R_3 - R_1 + R_2 \Rightarrow \begin{vmatrix} a & b & 1 \\ a' & b' & 1 \\ 0 & 0 & 1 \end{vmatrix} = 0$, Expanding along $R_3 \Rightarrow 1(-1)^{3+3}\begin{vmatrix} a & b \\ a' & b' \end{vmatrix} = 0 \Rightarrow ab' - a'b = 0 \Rightarrow ab' = a'b$.

3. The vector equation of the line through the point $(5,2-4)$ and which is parallel to the vector $3\hat{i}+2\hat{j}-8\hat{k}$.

   (a) $\vec{r} = 5\hat{i} - \hat{j} - 4\hat{k} + \lambda(3\hat{i} + 2\hat{j} - 7\hat{k})$
   (b) $\vec{r} = 5\hat{i} + 8\hat{j} - 4\hat{k} + \lambda(3\hat{i} + 2\hat{j} - 8\hat{k})$
   (c) $\vec{r} = 3\hat{i} - 2\hat{j} - 4\hat{k} + \lambda(3\hat{i} + 2\hat{j} - 8\hat{k})$
   (d) $\vec{r} = 5\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(3\hat{i} + 2\hat{j} - 8\hat{k})$

   SOLUTION

   .: We have, $\vec{a} = 5\hat{i} + 2\hat{j} - 4\hat{k}$ and

   $$\vec{b} = 3\hat{i} + 2\hat{j} - 8\hat{k}$$

   Therefore, the vector equation of the line is $\vec{r} = 5\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(3\hat{i} + 2\hat{j} - 8\hat{k})$

4. If M and N are two events . The probability that exactly one of them occurs, is

   (a) $P(M) + P(N) - P(M \cap N)$
   (b) $P(M) + P(N) + P(M \cap N)$
   (c) $P(M) + P(N))$
   (d) $P(M) + P(N) - 2P(M \cap N)$

   SOLUTION

   Exactly one of them occurs = $P(M \cap N') + P(M' \cap N) = P(M) - P(M \cap N) + P(N) - P(M \cap N)$
   = $P(M) + P(N) - 2P(M \cap N)$

5. $\int \sqrt{x^2-8x+7}dx$ is equal to

(a) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}+9\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

(b) $\frac{1}{2}(x+4)\sqrt{x^2-8x+7}+9\log\left|x+4+\sqrt{x^2-8x+7}\right|+C$

(c) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-3\sqrt{2}\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

(d) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-\frac{9}{4}\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

SOLUTION

(d) Let $I=\int \sqrt{x^2-8x+7}dx=\int \sqrt{(x-4)^2-3^2}dx=\frac{x-4}{2}\sqrt{x^2-8x+7}-\frac{9}{2}\log\left|(x-4)+\sqrt{x^2-8x+7}\right|+C$

6. $\int \frac{dx}{e^x+e^{-x}}$is equal to

(a) $\tan^{-1}(e^x)+C$

(b) $\tan^{-1}(e^{-x})+C$

(c) $\log(e^x-e^{-x})+C$

(d) $\log(e^x+e^{-x})+C$

SOLUTION

(a) : Let $I=\int \frac{dx}{e^x+e^{-x}}=\int \frac{dx}{e^x+\frac{1}{e^x}} \Rightarrow \int \frac{e^x dx}{(e^x)^2+1}$ Put $e^x=t \Rightarrow e^x dx=dt \therefore I=\int \frac{dt}{1+t^2}=\tan^{-1}(t)+C=\tan^{-1}(e^x)+C$

7. $\int_0^1 sin^{-1}(\frac{2x}{1+x^2})dx=$

(a) $\frac{\pi}{2}-2log\sqrt{2}$

(b) $\frac{\pi}{3}+2log\sqrt{2}$

(c) $\frac{\pi}{2}-2log\sqrt{3}$

(d) $\frac{\pi}{6}+2log\sqrt{2}$

SOLUTION

Ans. (a)

Put $x=tan\theta$

$dx=sec^2\theta d\theta$

Also, x = 1 $\Rightarrow \theta=\frac{\pi}{4}$

x =0 , $\theta=0$

$I=\int_0^{\pi/4}\theta sec^2\theta d\theta$

$I=2(\theta tan\theta)_0^{\pi/4}-2\int_0^{\pi/4}tan\theta d\theta$

8. The value of $\int_{-2}^{2}(ax^3+bx+c)$ depends on

(a) The value of a

(b) The value of b

(c) the value of c

(d) the value of a and b

SOLUTION

$[\frac{ax^4}{4}+\frac{bx^2}{2}+cx]_{-2}^{2}=4c$

therefore, the value of given integral depends on c

9. If $\cos(sin^{-1}\frac{2}{5}+cos^{-1}x)=0$ then x equals to

(a) $\frac{1}{5}$

(b) $\frac{1}{8}$

(c) $\frac{2}{15}$

(d) $\frac{2}{5}$

SOLUTION

$\cos(sin^{-1}\frac{2}{5}+cos^{-1}x)=0$

$\Rightarrow sin^{-1}\frac{2}{5}+cos^{-1}x=cos^{-1}(cos\frac{\pi}{2})$

$\Rightarrow sin^{-1}\frac{2}{5}+cos^{-1}x=\frac{\pi}{2}$

$\Rightarrow cos^{-1}x=\frac{\pi}{2}-sin^{-1}\frac{2}{5}$

$\Rightarrow cos^{-1}x=cos^{-1}\frac{2}{5}$

$\Rightarrow x=\frac{2}{5}$

10. The value of $sin[2tan^{-1}(0.75)]$ is

(a) 0.86

(b) 1

(c) 0.96

(d) 0.7

SOLUTION

$sin[2tan^{-1}(0.75)]=sin[2tan^{-1}(\frac{3}{4})]$

$$=sin\left(sin^{-1}\frac{2.\frac{3}{4}}{1+\frac{9}{16}}\right)$$

$$=sin\left[sin^{-1}(\frac{24}{25})\right]=\frac{24}{25}=0.96$$

11. If $x^y=e^{x-y}$ then $\frac{dy}{dx}=$

(a) $\frac{logx}{(1-logx)^2}$

(b) $\frac{3logx}{(3+logx)^2}$

(c) $\frac{logx}{(1+5logx)^2}$

(d) $\frac{logx}{(1+logx)^2}$

SOLUTION

$x^y = e^{x-y}$

Taking log on both sides :

$ylogx = (x-y)log_e e = (x-y)$

$\Rightarrow y = \frac{x}{1+logx}$

On differentiating both sides we get :

$\frac{dy}{dx} = \frac{1+logx - x\times\frac{1}{x}}{(1+logx)^2} = \frac{logx}{(1+logx)^2}$

12. If y $=(1+x^2)tan^{-1}x - x$ then $\frac{dy}{dx}$ is equal to

(a) $2xtan^{-1}x$

(b) $3xcot^{-1}x$

(c) $2xsin^{-1}x$

(d) $xtan^{-1}x$

SOLUTION

y $=(1+x^2)tan^{-1}x - x$

Differentiating both sides w.r.t. x , we get :

$\frac{dy}{dx} = (1+x^2)\frac{1}{1+x^2} + tan^{-1}x(2x) - 1$

$\Rightarrow \frac{dy}{dx} = 2xtan^{-1}x$

13. The Integrating Factor of the differential equation $(1-y^2)\frac{dx}{dy} + yx = ay(-1<y<1)$ is

(a) $\frac{1}{y^2-1}$

(b) $\frac{1}{\sqrt{y^2-1}}$

(c) $\frac{1}{1-y^2}$

(d) $\frac{1}{\sqrt{1-y^2}}$

SOLUTION

(d) The given equation can be written as : $\frac{dx}{dy} + \frac{y}{1-y^2}x = \frac{ay}{1-y^2}$ which is linear equation of type $\frac{dx}{dy} + Px = Q$ Where $P = \frac{y}{1-y^2}, Q = \frac{ay}{1-y^2} \therefore$ I.F. $= e^{\int Pdy} = e^{\int\left(\frac{y}{1-y^2}\right)dy} = e^{\left(-\frac{1}{2}\int\frac{-2y}{1-y^2}dy\right)}$ Let $I = -\frac{1}{2}\int\frac{-2y}{1-y^2}dy \Rightarrow I = -\frac{1}{2}\log\left(1-y^2\right) = \log\left(1-y^2\right)^{-1/2} = \log\left(\frac{1}{\sqrt{1-y^2}}\right) \therefore$ I.F. $= e^{\int Pdx} = e^{\log\left(\frac{1}{\sqrt{1-y^2}}\right)} = \frac{1}{\sqrt{1-y^2}}$

14. Is $g = \{(1,1),(2,3),(3,5),(4,7)\}$ a function ? If g is described by $g(x) = \alpha x + \beta$ then values assigned to $\alpha$ and $\beta$ is :

(a) $\alpha = 2, \beta = 1$
(b) $\alpha = 2, \beta = -1$
(c) $\alpha = 3, \beta = 1$
(d) $\alpha = 4, \beta = 2$

SOLUTION

Since each element of domain has unique image , so g is a function,
now, $g(1) = \alpha + \beta \Rightarrow \alpha + \beta = 1.....(i)$
g(2) = $2\alpha + \beta \Rightarrow 2\alpha + \beta = 3......(ii)$
Solving (i ) and (ii) we get $\alpha = 2, \beta = -1$

## VERY SHORT ANSWER QUESTIONS

15. Differentiate : $sin(x^2+5)$

SOLUTION

.: Let $y = \sin(x^2+5) \Rightarrow \frac{dy}{dx} = \frac{d}{dx}\sin(x^2+5) = \cos(x^2+5)\frac{d}{dx}(x^2+5) = \cos(x^2+5)(2x+0) = 2x\cos(x^2+5)$

16. . Evaluate : $\int_{-1}^{1} 5x^4\sqrt{x^5+1}dx$

SOLUTION

we have $I = \int_{-1}^{1} 5x^4\sqrt{x^5+1}dx$ Put $x^5+1 = t \Rightarrow 5x^4dx = dt$ When $x = -1, t = 0$ and when $x = 1, t = 2 \therefore I = \int_{-1}^{1} 5x^4\sqrt{x^5+1}dx =$
$\int_0^2 (t)^{\frac{1}{2}} dt = \frac{2}{3}\left[(t)^{\frac{3}{2}}\right]_0^2 = \frac{4\sqrt{2}}{3}$

17. Find unit vector in the direction of vector $a = 2\hat{i}+3\hat{j}+4\hat{k}$

SOLUTION

The unit vector in the direction of a vector $\vec{a}$ is given by $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$. Now, $|\vec{a}| = \sqrt{(2)^2+(3)^2+(4)^2} = \sqrt{29}$ Therefore, $\hat{a} =$
$\frac{2\hat{i}+3\hat{j}+4\hat{k}}{\sqrt{29}} = \frac{2}{\sqrt{29}}\hat{i} + \frac{3}{\sqrt{29}}\hat{j} + \frac{4}{\sqrt{29}}\hat{k}$

18. Evaluate : $\int_0^{\frac{\pi}{3}} \frac{\sec x\, tan x}{1+\sec^2 x}dx$

SOLUTION

Let $I = \int_0^{\pi/3} \frac{\sec x \tan x}{1+\sec^2 x}dx$ Put $\sec x = t \Rightarrow \sec x \tan x dx = dt$ When $x = 0, t = 1$ and when $x = \frac{\pi}{3}, t = 2 \therefore I = \int_1^2 \frac{dt}{1+t^2} = \left[\tan^{-1}(t)\right]_1^2$
$= \tan^{-1}2 - \tan^{-1}1 = \tan^{-1}\left(\frac{2-1}{1+2\cdot 1}\right) = \tan^{-1}\left(\frac{1}{3}\right)$

19. $\int \frac{\sin x}{1+\sin x}dx$

SOLUTION

Let $I=\int \frac{\sin x}{1+\sin x}dx \Rightarrow I=\int \frac{\sin x\,(1-\sin x)}{(1+\sin x)\,(1-\sin x)}dx \Rightarrow I=\int \frac{\sin x-\sin^2 x}{1-\sin^2 x}dx=\int \frac{\sin x-\sin^2 x}{\cos^2 x}dx \Rightarrow I=\int \frac{\sin x}{\cos^2 x}dx-\int \frac{\sin^2 x}{\cos^2 x}dx$
$=\int \sec x \tan x dx-\int \tan^2 x dx=\int \sec x \tan x dx-\int \left(\sec^2 x-1\right)dx=\sec x-\tan x+x+C$

20. $(ax+b)^2$ SOLUTION.

: Let $I=\int (ax+b)^2 dx=\frac{(ax+b)^3}{3a}+C$

21. Find the value of , $sin^{-1}(\frac{x}{x+1})+sec^{-1}(\frac{x+1}{x})$

SOLUTION

$sec^{-1}(\frac{x+1}{x})=cos^{1}(\frac{x}{x+1})$

therefore, $sin^{-1}(\frac{x}{x+1})+sec^{-1}(\frac{x+1}{x})=sin^{-1}(\frac{x}{x+1})+cos^{1}(\frac{x}{x+1})=\frac{\pi}{2}$

[ Using $sin^{-1}x+cos^{-1}x=\frac{\pi}{2}$ ]

22. Evaluate $cot^{-1}\{cot^{-1}(-\frac{\pi}{4})\}$

SOLUTION

$cot^{-1}\{cot^{-1}(-\frac{\pi}{4})\}=cot^{-1}[cot\frac{3\pi}{4}]=\frac{3\pi}{4}\in(0,\pi)$

23. Using principal value, evaluate $cos^{-1}(cos\frac{2\pi}{3})+sin^{-1}(sin\frac{2\pi}{3})$

SOLUTION

$cos^{-1}(cos\frac{2\pi}{3})=\frac{2\pi}{3}+sin^{-1}\left(sin(\pi-\frac{\pi}{3})\right)$

$=\frac{2\pi}{3}+sin^{-1}(sin\frac{\pi}{3})$

$=\frac{2\pi}{3}+\frac{\pi}{3}=\pi$

## FILL IN THE BLANKS

24. The value of $cot(tan^{-1}a+cot^{-1}a)$ is $\cdots$

SOLUTION

Since, $cot^{-1}x+tan^{-1}x=\frac{\pi}{2}$

Therefore, $cot(tan^{-1}a+cot^{-1}a)=cot(\frac{\pi}{2})$

= 0

25. The principal value of $cos^{-1}(-\frac{1}{2})$ is $\cdots$

26. The value of $\int\limits_0^\infty \frac{dx}{x^2+1}$ is $\cdots$

SOLUTION

Let $I = \int\limits_0^\infty \frac{dx}{1+x^2} = \left[\tan^{-1}x\right]_0^\infty = \frac{\pi}{2} - 0 = \frac{\pi}{2}$

27. . The value of $\int\limits_0^1 \frac{2x}{1+x^2}dx$ is $\cdots$

SOLUTION

. Let $I = \int\limits_0^1 \frac{2x}{1+x^2}dx = \left[\log\left|1+x^2\right|\right]_0^1 = \log 2 - \log 1 = \log 2$

28. Integration of $\int x^2\left(1-\frac{1}{x^2}\right)dx$ is $\cdots$

SOLUTION.

: Let$I = \int x^2\left(1-\frac{1}{x^2}\right)dx = \int \left(x^2-1\right)dx = \frac{x^3}{3} - x + C$

29. . Integration of $\int \left(ax^2+bx+c\right)dx$ is $\cdots$

SOLUTION.

: Let$I = \int \left(ax^2+bx+c\right)dx = \frac{ax^3}{3} + \frac{bx^2}{2} + cx + C$

30. The slope of the tangent to the curve $y = x^3 - 3x + 2$ at the point whose $x-$coordinate is 3. $\cdots$

SOLUTION

.: We have, $y = x^2 - 3x + 2 \ldots$(i) Differentiating (i) w.r.t $x$, we get $\frac{dy}{dx} = 3x^2 - 3 \therefore$ Slope of tangent at $x = 3$ is $\left(\frac{dy}{dx}\right)_{x=3} = 3\times 3^2 - 3 = 24$.

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