MULTIPLE CHOICE TYPE QUESTIONS

1. The rate of change of the area of a circle with respect to its radius r when $r = 3$ cm is

   (a) $3\pi \text{cm}^2/\text{cm}$
   (b) $2\pi \text{cm}^2/\text{cm}$
   (c) $6\pi \text{cm}^2/\text{cm}$
   (d) $4\pi \text{cm}^2/\text{cm}$

   SOLUTION

   (c) Let $A = \pi r^2 \ldots (1)$ (where A denotes the area of the circle when its radius is r) Differentiating (1), w.r.t. r, we get $= \frac{dA}{dr} = \pi(2r) = 2\pi r \left(\frac{dA}{dr}\right)_{r=3\text{cm}} = 2\pi(3)\text{cm} = 6\pi \text{cm}^2/\text{cm}$

2. The restriction on n, k and p if X, Y,Z,W,P are matrices having order, $X_{2\times n}, Y_{3\times k}, Z_{2\times p}, W_{n\times 3}, P_{p\times k}$, so that PY + WY will be defined are $\cdots$

   (a) k = 3, p= n
   (b) k is arbitrary, p = 2
   (c) p is arbitrary, k = 3
   (d) k = 2, p= 3

   SOLUTION

   (a) Given : $X_{2\times n}, Y_{3\times k}, Z_{2\times p}, W_{n\times 3}, P_{p\times k}$ Now, $PY + WY = P_{p\times k} \times Y_{3\times k} + Q_{n\times 3} \times Y_{3\times k}$ Clearly, k = 3 and p = n

3. The planes: $2x - y + 4z = 5$ and $5x - 2.5y + 10z = 6$ are

   (a) Perpendicular
   (b) Parallel
   (c) Intersect y-axis
   (d) Pass through $\left(0, 0, \frac{5}{4}\right)$

   SOLUTION

   (b) : The direction ratios of planes are $< 2, -1, 4 >$ and $< 5, -2.5, 10 >$ Here, $\frac{2}{5} = \frac{-1}{-2.5} = \frac{4}{10}$ which is true. $\therefore$ The given planes are parallel.

4. If $\frac{d}{dx} f(x) = 4x^2 - \frac{3}{x^4}$,such that $f(2) = 0$ then, $f(x)$ is

   (a) $x^4 + \frac{1}{x^3} - \frac{129}{8}$
   (b) $x^3 + \frac{1}{x^4} + \frac{129}{8}$
   (c) $x^4 + \frac{1}{x^3} + \frac{129}{8}$
   (d) $x^3 + \frac{1}{x^4} - \frac{129}{8}$

SOLUTION

(a) $f(x)=\int\left(4x^3-\frac{3}{x^4}\right)dx=4\int x^3dx-3\int x^{-4}dx=\frac{4x^4}{4}-\frac{3x^{-3}}{-3}+C=x^4+\frac{1}{x^3}+C\therefore f(2)=(2)^4+\frac{1}{(2)^3}+C=0\Rightarrow C=-16-\frac{1}{8}=-\frac{129}{8}\therefore f(x)=x^4+\frac{1}{x^3}-\frac{129}{8}$

5. On which of the following intervals is the function f given by $f(x)=x^{100}+\sin x-1$ strictly decreasing?
   (a) $(0,1)$
   (b) $\left(\frac{\pi}{2},\pi\right)$
   (c) $\left(0,\frac{\pi}{2}\right)$
   (d) None of these

SOLUTION

(d) We have, $f(x)=x^{100}+\sin x-1\ldots$(i) Differentiating (i) w.r.t. x, we get $f'(x)=100x^{99}+\cos x$ (A $f'(x)$ assumes only $+ve$ values in $(0,1)\therefore f$ is strictly increasing in $(0,1)$ (B) $f'(x)>0$ for all $x\in\left(\frac{\pi}{2},\pi\right)$, therefore f is strictly increasing in $x\in\left(\frac{\pi}{2},\pi\right)$ (C) $f'(x)>0$ for all $x\in\left(0,\frac{\pi}{2}\right)$, therefore f is strictly increasing in $\left(0,\frac{\pi}{2}\right)$.

6. $\int_1^2 e^x(\frac{1}{x}-\frac{1}{x^2})dx=$
   (a) $\frac{e^2}{2}-e$
   (b) $\frac{e^2}{2}+e$
   (c) $3\frac{e^2}{2}-e$
   (d) $\frac{e^2}{2}-2e$

SOLUTION

Let $\frac{1}{x}=t\Rightarrow-\frac{1}{x^2}dx=dt$

$\int_1^2 e^x(\frac{1}{x}-\frac{1}{x^2})dx$

$=e^x\frac{1}{x}+c$ [Using : $\int e^x(f(x)+f'(x))dx=e^xf(x)+c$]

7. If n= p, X and Z are matrices having order , $X_{2\times n},Z_{2\times p}$then the order of the matrix 7X−5Z is
   (a) p × 2
   (b) 2 × n
   (c) n × 3
   (d) p × n

SOLUTION

(B) $7X-5Z=7X_{2\times n}-5Z_{2\times p}$ We can add two matrices if their order is same. $n=p\therefore$ Order of 7X−5Z is 2 × n.

8. $\int_{\pi/3}^{\pi/2} \frac{\sqrt{1+cosx}}{(1-cosx)^{5/2}} dx =$

   (a) $\frac{3}{2}$

   (b) $\frac{3}{5}$

   (c) $\frac{1}{2}$

   (d) $\frac{3}{7}$

   SOLUTION

   $\int_{\pi/3}^{\pi/2} \frac{\sqrt{1+cosx}}{(1-cosx)^{5/2}} \times \frac{\sqrt{1-cosx}}{\sqrt{1-cosx}} dx$

   $= \int_{\pi/3}^{\pi/2} \frac{sinx}{(1-cosx)^3} dx$

   Put $1-cosx = t \Rightarrow sinxdx = dt$

   $\Rightarrow = \int_{\pi/3}^{\pi/2} \frac{sinx}{(1-cosx)^3} dx = \int_{\pi/3}^{\pi/2} \frac{dt}{t^3}$

   $= [\frac{t^{-2}}{(-2)}]_{1/2}^{1}$

   $= \frac{3}{2}$

9. $\int_1^2 \frac{1}{x^2} e^{-1/x} dx =$

   (a) $\frac{\sqrt{e}+1}{2e}$

   (b) $\frac{\sqrt{e}-1}{e}$

   (c) $\frac{\sqrt{2e}-5}{e}$

   (d) $\frac{\sqrt{e}+1}{3e}$

   SOLUTION

   Let $-\frac{1}{x} = t \Rightarrow \frac{1}{x^2} dx = dt$

   $\int_1^2 \frac{1}{x^2} e^{-1/x} dx = \int_{-1}^{-1/2} e^t dt$

   $= e^{-1/2} - e^{-1}$

   $= \frac{\sqrt{e}-1}{e}$

10. If $sin^{-1}x + sin^{-1}y = \frac{\pi}{2}$ then $\frac{dy}{dx}$ is equal to

   (a) $\frac{-x}{y}$

   (b) $\frac{x}{y}$

   (c) $\frac{2x}{y}$

   (d) $\frac{-x}{2y}$

SOLUTION

$sin^{-1}x + sin^{-1}y = \frac{\pi}{2}$

$\Rightarrow sin^{-1}x = \frac{\pi}{2} - sin^{-1}y$

$\Rightarrow sin^{-1}x = cos^{-1}y$

$\Rightarrow y = \sqrt{1-x^2}$

On differentiating both sides :

$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{1-x^2}}(-2x) = -\frac{x}{y}$

11. If $P(A) = \frac{4}{5}$ and $P(A \cap B) = \frac{7}{10}$ then $P(B/A)$ is equal to

 (a) $\frac{1}{8}$
 (b) $\frac{7}{8}$
 (c) $\frac{7}{9}$
 (d) $\frac{1}{7}$

SOLUTION

P$(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{7/10}{4/5} = \frac{7}{8}$

12. If $y = \sqrt{sinx + \sqrt{sinx + \sqrt{sinx \cdots \infty}}}$ then $\frac{dy}{dx}$ is equal to

 (a) $\frac{cosx}{2y-1}$
 (b) $\frac{2cosx}{2y+1}$
 (c) $3\frac{2cosx}{2y+1}$
 (d) $\frac{3cosx}{2y+1}$

SOLUTION

$y = \sqrt{sinx + \sqrt{sinx + \sqrt{sinx \cdots \infty}}}$

$y = \sqrt{sinx + y}$

Squaring both sides we get :

$y^2 = sinx + y$

Differentiating both sides we get :

$2y\frac{dy}{dx} = cosx + \frac{dy}{Dx}$

$\Rightarrow \frac{dy}{dx} = \frac{cosx}{2y-1}$

VERY SHORT ANSWER QUESTIONS

13. Find the equation of the plane that pass through three points $(1,1,0),(1,2,1),(-2,2,-1)$

SOLUTION

Any plane through $(1,1,0)$ is $a(x-1)+b(y-1)+cz=0$ ...(1) Since the plane passes through the points $(1,2,1)$ and $(-2,2,-1)$ $\therefore a(1-1)+b(2-1)+c(1)=0$ and $a(-2-1)+b(2-1)+c(-1)=0 \Rightarrow b+c=0 \ldots$.(2) and $-3a+b-c=0 \ldots$.(3) Solving (2) and (3), $\frac{a}{-1-1}=\frac{b}{-3+0}=\frac{c}{0+3} \Rightarrow \frac{a}{-2}=\frac{b}{-3}=\frac{c}{3}=k$ (say), where $k \neq 0 \therefore a=-2k, b=-3k, c=3k$ Putting these values of a,b,c in (1), we get $-2k(x-1)-3k(y-1)+3kz=0 \Rightarrow -2(x-1)-3(y-1)+3z=0 \Rightarrow -2x+2-3y+3+3z=0 \Rightarrow$ $2x+3y-3z=5$,which is the required equation.

14. . Find the value of x,y and z so that the vectors $a=3\hat{i}+4\hat{j}-z\hat{k}$ and $b=x\hat{i}+y\hat{j}+2\hat{k}$ are equal.

SOLUTION

.. Note that two vectors are equal if and only if their corresponding components are equal. Thus, the given vectors $\vec{a}$ and $\vec{b}$ will be equal if and only if $x=3, y=4, z=-2$.

15. Prove that the function $f(x)=x^n$ is continuous at x = n, where n is a positive integer.

SOLUTION

Given, $f(x)=x^n, n \in N$ So, f (x) is a polynomial function and domain of f is R. $\lim_{x \to n} f(x)=\lim_{x \to n} x^n = x^n = f(n) \Rightarrow$ f is continuous at n∈N.

16. Evaluate : $tan^{-1}\sqrt{3}-sec^{-1}(-2)$

SOLUTION

$tan^{-1}\sqrt{3}-sec^{-1}(-2)$

$=tan^{-1}(tan\frac{\pi}{3})-sec^{-1}(-sec\frac{\pi}{3})$

$=\frac{\pi}{3}-sec^{-1}[sec(\pi-\frac{\pi}{3})]$

$=\frac{\pi}{3}-sec6-1(sec\frac{2\pi}{3})$

$=\frac{\pi}{3}-\frac{2\pi}{3}=-\frac{\pi}{3}$

17. Evaluate $tan(tan^{-1}(-4))$

SOLUTION

$tan(tan^{-1}(-4))=-4$ $[\because tan(tan^{-1}x)=x; x \in R)]$

18. Find the value of $sin^{-1}(cos(\frac{43\pi}{5}))$

SOLUTION

$sin^{-1}(cos(\frac{43\pi}{5}))$

$=sin^{-1}(cos(8\pi+\frac{3\pi}{5}))$

$=sin^{-1}(cos\frac{3\pi}{5})$

$=sin^{-1}(sin(\frac{\pi}{2}-\frac{3\pi}{5}))$

$=sin^{-1}(sin(\frac{-\pi}{10}))$

$=-\frac{\pi}{10}$

19. Evaluate $\int\left(e^{x\log a}+e^{a\log x}+e^{a\log a}\right)dx$

SOLUTION

$I=\int\left(e^{x\log a}+e^{a\log x}+e^{a\log a}\right)dx=\int\left(e^{\log a^x}+e^{\log x^a}+e^{\log a^a}\right)dx=\int(a^x+x^a+a^a)dx=\dfrac{a^x}{\log a}+\dfrac{x^{a+1}}{a+1}+a^ax+C$

20. Find the intercepts cut off by the plane$2x+y-z=5$.

SOLUTION

The given equation of the plane is$2x+y-z=5$. $\Rightarrow \dfrac{x}{5/2}+\dfrac{y}{5}+\dfrac{z}{-5}=1$ Comparing it with$\dfrac{x}{a}+\dfrac{y}{b}+\dfrac{z}{c}=1$, we get the intercepts on the axes as $\dfrac{5}{2},5$ and -5.

21. $\int\dfrac{2^x+3^x}{5^x}dx$

SOLUTION

Let $I=\int\dfrac{2^x+3^x}{5^x}dx\Rightarrow I=\int\dfrac{2^x}{5^x}dx+\int\dfrac{3^x}{5^x}dx=\int\left(\dfrac{2}{5}\right)^x dx+\int\left(\dfrac{3}{5}\right)^x dx\Rightarrow I=\dfrac{\left(\frac{2}{5}\right)^x}{\log_e\left(\frac{2}{5}\right)}+\dfrac{\left(\frac{3}{5}\right)^x}{\log_e\left(\frac{3}{5}\right)}+C$

22. . Show that the given differential equation is homogeneous $y'=\dfrac{x+y}{x}$

SOLUTION

$y'=\dfrac{x+y}{x}$ or $\dfrac{dy}{dx}=1+\dfrac{y}{x}\ldots(1)$ Since R.H.S. is of the form $g(y/x)$, so it is a homogeneous function of degree zero. Therefore, equation (1) is a homogeneous differential equation.

23. Prove that the relation R in the set A = $\{1,2,3,4,5\}$ given by R =$\{(a,b):|a-b|$ is even $\}$ is an equivalence relation

SOLUTION

Reflexive : $|a-a|=0$ which is even, therefore the relation is reflexive.

Symmetric : $|a-b|$ is even , therefore $|b-a|$ is also even, hence relation is symmetric.

Transitive : $|a-b|$ is even, $|b-c|$ is even $\Rightarrow a-b+b-c=$ is also even $\Rightarrow a-c$ is also even. Hence transitive.

FILL IN THE BLANKS

24. The slope of the tangent to the curve $y=\dfrac{x-1}{x-2},x\neq 2$ at $x=10$. is $\cdots$

SOLUTION

We have, $y=\dfrac{x-1}{x-2},x\neq 2\ldots$(i) Differentiating (i) w.r.t. x, we get $\dfrac{dy}{dx}=\dfrac{(x-2)\cdot 1-(x-1)\cdot 1}{(x-2)^2}=\dfrac{-1}{(x-2)^2}$

$\therefore$ Slope of tangent at $x=10$ is $\left(\dfrac{dy}{dx}\right)_{x=10}=\dfrac{-1}{(10-2)^2}=-\dfrac{1}{64}$

25. The angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 2 respectively, having $\vec{a}\cdot\vec{b}=\sqrt{6}$. is $\cdots$

SOLUTION

If θ be the angle between $\vec{a}$ and $\vec{b}$, then $\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|} = \frac{\sqrt{6}}{\sqrt{3}(2)} = \frac{(\sqrt{3}).(\sqrt{2})}{(\sqrt{3})(2)} = \frac{1}{\sqrt{2}} = \cos\frac{\pi}{4}$. Hence, $\theta = \frac{\pi}{4}$.

26. If $sin(sin^{-1}\frac{1}{2}+cos^{-1}x) = 1$ then the value of x is $\cdots$

SOLUTION

$sin(sin^{-1}\frac{1}{2}+cos^{-1}x) = 1$

$\Rightarrow sin^{-1}\frac{1}{2}+cos^{-1}x = sin\frac{\pi}{2}$

$\Rightarrow sin^{-1}\frac{1}{2}+cos^{-1}x = \frac{\pi}{2}$

$\frac{\pi}{6}+cos^{-1}(x) = \frac{\pi}{2}$

$\Rightarrow cos^{-1}(x) = \frac{\pi}{3}$

$\therefore x = \frac{1}{2}$

27. If $tan^{-1}x+tan^{-1}y = \frac{\pi}{4}$ then the value of x +y +xy is $\cdots$

28. . Integration of $\int(2x^2+e^x)\,dx$ is $\cdots$

SOLUTION.

: Let $I = \int(2x^2+e^x)\,dx = \frac{2x^3}{3}+e^x+C$

29. Integration of $\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 dx$ is $\cdots$

SOLUTION.

: Let$I = \int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 dx = \int\left(x+\frac{1}{x}-2\right)dx = \frac{x^2}{2}+\log x-2x+C$

30. Integration of $\int\frac{x^3+5x^2-4}{x^2}dx$ is $\cdots$

SOLUTION.

: Let$I = \int\left(\frac{x^3+5x^2-4}{x^2}\right)dx = \int(x+5-4x^{-2})\,dx = \frac{x^2}{2}+5x-\frac{4x^{-1}}{-1}+C = \frac{x^2}{2}+5x+\frac{4}{x}+C$