Solutions of these questions are available on my youtube channel
https://www.youtube.com/channel/UCg4VXE3OqXrfIr4q1MTDBCg
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H.O.T.S (Higher Order Thinking Skill)

1. Prove : $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x,\ \frac{-1}{\sqrt{2}}\le x\le 1.$
2. Prove that, $\tan^{-1}1+\tan^{-1}2+\tan^{-1}3=\pi$
3. . Prove that, $\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)=\frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right).$
4. Prove that, $\tan^{-1}\sqrt{x}=\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right),\ x\in[0,\ 1]$
5. Prove that, $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2},\ x\in\left(0,\ \frac{\pi}{4}\right).$
6. . Solve : $\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\tan^{-1}x,\ x>0.$

   Answer :$x=\frac{1}{\sqrt{3}}$
7. . Solve : $\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}.$

   Answer : 0
8. Simplify $\tan^{-1}\left[\frac{a\cos x-b\sin x}{b\cos x+a\sin x}\right],\ if\ \frac{a}{b}\tan x>-1.$

   Answer : $\sin^{-1}\left(\frac{a}{b}\right)-x$
9. If $\cos^{-1}\frac{x}{2}+\cos^{-1}\frac{y}{3}=\theta$, then prove that $9x^2-12xy\cos\theta+4y^2=36\sin^2\theta.$
10. Find the value of $\tan\frac{1}{2}\left[\sin^{-1}\left(\frac{2x}{1+x^2}\right)+\cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right],\ |X|<1,\ y>0\ and\ xy<1.$

    Answer : $\frac{x+y}{1-xy}$
11. . Prove that, $2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{\theta}{2}\right)=\cos^{-1}\left(\frac{a\cos\theta+b}{a+b\cos\theta}\right).$
12. Solve for x : $\cot^{-1}x-\cot^{-1}(x+2)=\frac{\pi}{12},\ where\ x>0.$

    Answer : $x=\sqrt{3}$
13. Solve for x, $\tan^{-1}\left(\frac{2x}{1-x^2}\right)+\cot^{-1}\left(\frac{1-x^2}{2x}\right)=\frac{\pi}{3},\ x>0.$

    Answer :$x=2-\sqrt{3}$

14. Solve for x, $\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\frac{8}{31}$.

Answer : $x=-8$ *or* $\frac{1}{4}$

15. If $\tan^{-1}x+\tan^{-1}y=\frac{\pi}{4}$, then prove that, $x+y+xy=1$.

16. . Evaluate : $\tan\left[\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{3}\right]$.

Answer : $\frac{3-\sqrt{5}}{2}$

17. Prove that : $\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right)=\frac{2b}{a}$.

18. . Prove that $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)=\frac{\pi}{4}-\frac{x}{2},\ x\in\left(-\frac{\pi}{2},\ \frac{\pi}{2}\right)$ OR Prove that $\sin^{-1}\left(\frac{8}{17}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\cos^{-1}\left(\frac{36}{85}\right)$

19. Prove that $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2};\ x\in\left(0,\ \frac{\pi}{4}\right)$. OR Prove that $2\tan^{-1}\left(\frac{1}{5}\right)+\sec^{-1}\left(\frac{5\sqrt{2}}{7}\right)+2\tan^{-1}\left(\frac{1}{8}\right)=\frac{\pi}{4}$

20. Solve for x : $\tan^{-1}\left(\frac{1-x}{1+x}\right)-\frac{1}{2}\tan^{-1}x=0,\ x>0.$

Answer :$x=\frac{1}{\sqrt{3}}$

21. Prove that : $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2},\ x\in\left(0,\ \frac{\pi}{4}\right)$

22. . Solve for x : $\tan^{-1}x+2\cot^{-1}x=\frac{2\pi}{3}$.

Answer :$x=\sqrt{3}$

23. Prove that : $\tan^{-1}\sqrt{x}=\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right),\ x\in[0,\ 1]$

24. Prove that : $\cos^{-1}\left(\frac{12}{13}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\sin^{-1}\left(\frac{56}{65}\right)$.

25. . Prove that : $\tan^{-1}x+\tan^{-1}\left(\frac{2x}{1-x^2}\right)=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$

26. Prove that : $\cos\left[\tan^{-1}\left\{\sin(\cot^{-1}x\right\}\right]=\sqrt{\frac{1+x^2}{2+x^2}}$

27. Prove that : $\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x,\ \ \frac{-1}{\sqrt{2}}\le x\le 1$

28. Find the value of $\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\left(\frac{x-y}{x+y}\right)$

Answer : $\frac{\pi}{4}$.

29. . Solve for x : $\tan^{-1}(x+2)+\tan^{-1}(x-2)=\tan^{-1}\left(\frac{8}{79}\right)$

Answer :$x=\frac{1}{4}$

30. If $sin^{-1x}+sin^{-1}y+sin^{-1}z=\pi$ prove that

$$x\sqrt{1-x^2}+y\sqrt{1-y^2}+z\sqrt{1-z^2}=2xyz$$

31. Show that $2tan^{-1}\{tan\frac{\alpha}{2}tan(\frac{\pi}{4}-\frac{\beta}{2})\}=tan^{-1}\frac{sin\alpha.cos\beta}{cos\alpha+sin\beta}$

32. If $tan^{-1}a+tan^{-1}b+tan^{-1}c=\pi$ prove that $a+b+c=abc$

33. Solve the equation $tan^{-1}\sqrt{x^2+x}+sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{2}$