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H.O.T.S (Higher Order Thinking Skill)

1. Prove : $\tan^{-1} \left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right) = \frac{\pi}{4} - \frac{1}{2} \cos^{-1} x, \frac{-1}{\sqrt{2}} \leq x \leq 1.$

2. Prove that, $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$

3. . Prove that, $\frac{9\pi}{8} - \frac{9}{4} \sin^{-1} \left(\frac{1}{3} \right) = \frac{9}{4} \sin^{-1} \left(\frac{2\sqrt{2}}{3} \right).$

4. Prove that, $\tan^{-1} \sqrt{x} = \frac{1}{2} \cos^{-1} \left(\frac{1-x}{1+x} \right), x \in [0, 1]$

5. Prove that, $\cot^{-1} \left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}} \right) = \frac{x}{2}, x \in \left(0, \frac{\pi}{4} \right).$

6. . Solve : $\tan^{-1} \left(\frac{1-x}{1+x} \right) = \frac{1}{2} \tan^{-1} x, x > 0.$

Answer : $x = \frac{1}{\sqrt{3}}$

7. . Solve : $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}.$

Answer : 0

8. Simplify $\tan^{-1} \left[\frac{a \cos x - b \sin x}{b \cos x + a \sin x} \right],$ if $\frac{a}{b} \tan x > -1.$

Answer : $\sin^{-1} \left(\frac{a}{b} \right) - x$

9. If $\cos^{-1} \frac{x}{2} + \cos^{-1} \frac{y}{3} = \theta,$ then prove that $9x^2 - 12xy \cos \theta + 4y^2 = 36 \sin^2 \theta.$

10. Find the value of $\tan \frac{1}{2} \left[\sin^{-1} \left(\frac{2x}{1+x^2} \right) + \cos^{-1} \left(\frac{1-y^2}{1+y^2} \right) \right], |X| < 1, y > 0$ and $xy < 1.$

Answer : $\frac{x+y}{1-xy}$

11. . Prove that, $2 \tan^{-1} \left(\sqrt{\frac{a-b}{a+b}} \tan \frac{\theta}{2} \right) = \cos^{-1} \left(\frac{a \cos \theta + b}{a + b \cos \theta} \right).$

12. Solve for $x : \cot^{-1} x - \cot^{-1}(x+2) = \frac{\pi}{12},$ where $x > 0.$

Answer : $x = \sqrt{3}$

13. Solve for $x, \tan^{-1} \left(\frac{2x}{1-x^2} \right) + \cot^{-1} \left(\frac{1-x^2}{2x} \right) = \frac{\pi}{3}, x > 0.$

Answer : $x = 2 - \sqrt{3}$

Inverse Trigonometric Functions

14. Solve for x , $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1} \frac{8}{31}$.

Answer : $x = -8$ or $\frac{1}{4}$

15. If $\tan^{-1}x + \tan^{-1}y = \frac{\pi}{4}$, then prove that, $x + y + xy = 1$.

16. . Evaluate : $\tan \left[\frac{1}{2} \cos^{-1} \frac{\sqrt{5}}{3} \right]$.

Answer : $\frac{3 - \sqrt{5}}{2}$

17. Prove that : $\tan \left(\frac{\pi}{4} + \frac{1}{2} \cos^{-1} \frac{a}{b} \right) + \tan \left(\frac{\pi}{4} - \frac{1}{2} \cos^{-1} \frac{a}{b} \right) = \frac{2b}{a}$.

18. . Prove that $\tan^{-1} \left(\frac{\cos x}{1 + \sin x} \right) = \frac{\pi}{4} - \frac{x}{2}$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2} \right)$ OR Prove that $\sin^{-1} \left(\frac{8}{17} \right) + \sin^{-1} \left(\frac{3}{5} \right) = \cos^{-1} \left(\frac{36}{85} \right)$

19. Prove that $\cot^{-1} \left(\frac{\sqrt{1 + \sin x} + \sqrt{1 - \sin x}}{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}} \right) = \frac{x}{2}$; $x \in \left(0, \frac{\pi}{4} \right)$. OR Prove that $2 \tan^{-1} \left(\frac{1}{5} \right) + \sec^{-1} \left(\frac{5\sqrt{2}}{7} \right) + 2 \tan^{-1} \left(\frac{1}{8} \right) = \frac{\pi}{4}$



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20. Solve for x : $\tan^{-1} \left(\frac{1-x}{1+x} \right) - \frac{1}{2} \tan^{-1} x = 0, x > 0.$

Answer : $x = \frac{1}{\sqrt{3}}$

Inverse Trigonometric Functions

21. Prove that : $\cot^{-1} \left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}} \right) = \frac{x}{2}, x \in \left(0, \frac{\pi}{4}\right)$

22. . Solve for x : $\tan^{-1}x + 2\cot^{-1}x = \frac{2\pi}{3}$.

Answer : $x = \sqrt{3}$

23. Prove that : $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x \in [0, 1]$

24. Prove that : $\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{56}{65}\right)$.

25. . Prove that : $\tan^{-1}x + \tan^{-1}\left(\frac{2x}{1-x^2}\right) = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$

26. Prove that : $\cos[\tan^{-1}\{\sin(\cot^{-1}x)\}] = \sqrt{\frac{1+x^2}{2+x^2}}$

27. Prove that : $\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right] = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x, \frac{-1}{\sqrt{2}} \leq x \leq 1$

28. Find the value of $\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right)$

Answer : $\frac{\pi}{4}$.

29. . Solve for x : $\tan^{-1}(x+2) + \tan^{-1}(x-2) = \tan^{-1}\left(\frac{8}{79}\right)$

Answer : $x = \frac{1}{4}$

30. If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$ prove that

$$x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = 2xyz$$

31. Show that $2\tan^{-1}\left\{\tan\frac{\alpha}{2}\tan\left(\frac{\pi}{4} - \frac{\beta}{2}\right)\right\} = \tan^{-1}\frac{\sin\alpha\cos\beta}{\cos\alpha + \sin\beta}$

32. If $\tan^{-1}a + \tan^{-1}b + \tan^{-1}c = \pi$ prove that $a + b + c = abc$

33. Solve the equation $\tan^{-1}\sqrt{x^2+x} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$