H.O.T.S (Higher Order Thinking Skill)

1. Prove that $\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3+b^3+c^3-3abc.$

2. Using properties of determinants, prove that $\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix} = 0$

3. Prove that $\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^3$

4. . If p, q, r are not in GP. and $\begin{vmatrix} 1 & q/p & \alpha+q/p \\ 1 & r/p & \alpha+r/p \\ p\alpha+q & q\alpha+r & 0 \end{vmatrix} = 0$, show that $p\alpha^2+2q\alpha+r=0$.

5. Using the properties of determinants, prove that $\begin{vmatrix} {}^mC_1 & {}^mC_2 & {}^mC_3 \\ {}^nC_1 & {}^nC_2 & {}^nC_3 \\ {}^pC_1 & {}^pC_2 & {}^pC_3 \end{vmatrix} = \dfrac{mpn(m-n)(n-p)(p-m)}{12}$

6. Using properties of determinants, prove that $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = (1+abc)(a-b)(b-c)(c-a)$

7. Prove that $\begin{vmatrix} a & a+2b & a+2b+3c \\ 3a & 4a+6b & 5a+7b+9c \\ 6a & 9z+12b & 11a+15b+18c \end{vmatrix} = -a^3$

8. . Using the properties of determinants, prove that $\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2(a+b)(b+c)(c+a)$.

9. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find $A^{-1}$and hence prove that $A^2-4A-5I=0$.

   14. $A^{-1} = \dfrac{-1}{2}\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$

10. Using properties of determinant, prove that $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$

11. Using properties of determinants, show that $\begin{vmatrix} \dfrac{a^2+b^2}{c} & c & c \\ a & \dfrac{b^2+c^2}{a} & a \\ b & b & \dfrac{a^2+c^2}{b} \end{vmatrix} = 4abc.$

12. Using properties of determinants, show that $\begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix} = 9b^2(a+b)$

13. Using properties of determinants, evaluate the following $\begin{vmatrix} 0 & ab^2 & ac^2 \\ a^2b & 0 & bc^2 \\ a^2c & cb^2 & 0 \end{vmatrix}$

$\boxed{\text{Answer: } 2a^3b^3c^3}$

14. Using properties of determinants, prove the following : $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = ab+bc+ca+abc$

15. Using properties of determinants, prove the following: $\begin{vmatrix} x^2+1 & xy & xz \\ xy & y^2+1 & yz \\ xz & yz & z^2+1 \end{vmatrix} = 1+x^2+y^2+z^2$

16. In a triangle ABC, if $\begin{vmatrix} 1 & 1 & 1 \\ 1+sinA & 1+sinB & 1+sinC \\ sinA+sin^2A & sinB+sin^2B & sinC+sin^2C \end{vmatrix} = 0$ then prove that ΔABC is an isosceles triangle.

x = 0 , y = 5 , z = 3

17. If A = $\begin{pmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{pmatrix}$ find A^{-1} and hence solve the following system of linear equations :

$2x+3y+z=11; -3x+2y-4z=4; 5x-4y-2z=-9$

18. Use product $\begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{pmatrix}\begin{pmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{pmatrix}$ to solve the system of equation. $x-y+2z=1; 2y-3z=1; 3x-2y+4z=2$

19. Find the product of matrices $A = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ and use it for solving the equations; $x+y+2z = 1, 3x+2y+z=7, 2x+y+3z=2$.

10. $AB = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ $x=2, y=1, z=-1$

20. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Use it to solve the following system of equations : $2x-3y+5z=16, 3x+2y-4z = -4, x+y-2z=-3$.

Ans. $A^{-1} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}; x=2, y=1, z=3$

21. . Given : $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}, B = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ Find AB and use this result in solving the following system of equations : $x-y+z=4, x-2y-2z=9, 2x+y+3z=1$.

Answer: $AB = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}, x=8, y=-2, z=-1$

22. A school wants to award its students for the value of Honesty, Regularity and Hard work with a total cash award of Rs. 6,000. Three times the award money for Hard work added to that given for honesty amounts to Rs. 11,000. The award money given for Honesty and Hard work together is double the one given for Regularity. Represent the above situation algebraically and find the award money for each value, using matrix method. Apart from these values, namely, Honesty, Regularity and Hard work, suggest one more value which the school must include for awards.

Answer: . $x = Rs.500, y = Rs.2000, z = Rs.3500$

23. Two schools P and Q want to award their selected students on the values of Discipline, Politeness and Punctuality. The school P wants to award Rs. x each, Rs. y each and Rs. z each for the three respective values to its 3, 2 and 1 students with a total award money of Rs. 1,000. School Q wants to spend Rs. 1,500 to award its 4, 1 and 3 students on the respective values (by giving the same award money for the three values as before). If the total amount of awards for one prize on each value isRs. 600, using matrices, find the award money for each value. Apart from the above three values, suggest one more value for awards.

    Answer: $x = 100, y = 200, z = 300$.